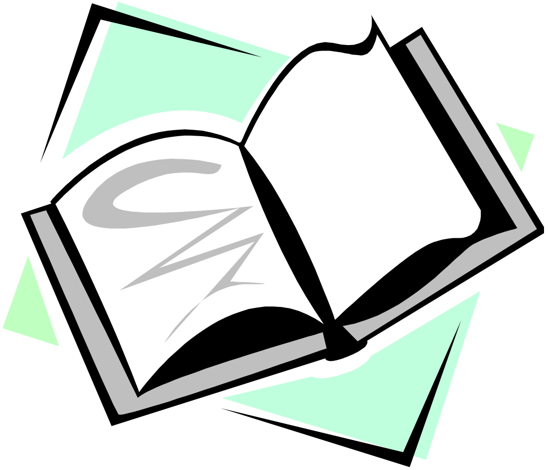
Information Security CS 526

Topic 6: Public Key Encryption and Digital Signatures

Readings for This Lecture

- Required: On Wikipedia
  - Public key cryptography
  - RSA
  - Diffie–Hellman key exchange
  - ElGamal encryption



- Required:
  - Differ & Hellman: “New Directions in Cryptography” IEEE Transactions on Information Theory, Nov 1976.

Review of Secret Key (Symmetric) Cryptography

- Confidentiality
  - stream ciphers (uses PRNG)
  - block ciphers with encryption modes
- Integrity
  - Cryptographic hash functions
  - Message authentication code (keyed hash functions)
- Limitation: sender and receiver must share the same key
  - Needs secure channel for key distribution
  - Impossible for two parties having no prior relationship
  - Needs many keys for n parties to communicate

Concept of Public Key Encryption

- Each party has a pair (K, K^{-1}) of keys:
  - K is the **public** key, and used for encryption
  - K^{-1} is the **private** key, and used for decryption
  - Satisfies $\mathbf{D}_{K^{-1}}[\mathbf{E}_K[M]] = M$
- Knowing the public-key K, it is computationally infeasible to compute the private key K^{-1}
  - How to check (K,K^{-1}) is a pair?
  - Offers only computational security. Secure PK Encryption impossible when P=NP, as deriving K^{-1} from K is in NP.
- The public-key K may be made publicly available, e.g., in a publicly available directory
  - Many can encrypt, only one can decrypt
- Public-key systems aka *asymmetric* crypto systems

Public Key Cryptography Early History

- Proposed by Diffie and Hellman, documented in “New Directions in Cryptography” (1976)
  1. Public-key encryption schemes
  2. Key distribution systems
     - Diffie-Hellman key agreement protocol
  3. Digital signature

- Public-key encryption was proposed in 1970 in a classified paper by James Ellis
  – paper made public in 1997 by the British Governmental Communications Headquarters

- Concept of digital signature is still originally due to Diffie & Hellman

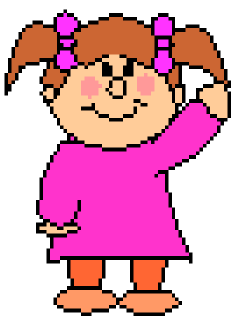
Public Key Encryption Algorithms

- Almost all public-key encryption algorithms use either number theory and modular arithmetic, or elliptic curves
- RSA
  - based on the hardness of factoring large numbers
- El Gamal
  - Based on the hardness of solving discrete logarithm
  - Use the same idea as Diffie-Hellman key agreement

Diffie-Hellman Key Agreement Protocol

Not a Public Key Encryption system, but can allow A and B to agree on a shared secret in a public channel (against passive, i.e., eavesdropping only adversaries)

Setup: p prime and g generator of Z_p^*, p and g public.



$\xrightarrow{g^a \bmod p}$

$\xleftarrow{g^b \bmod p}$

Pick random, secret a

Compute and send g^a mod p

$K = (g^b \bmod p)^a = g^{ab} \bmod p$

Pick random, secret b

Compute and send g^b mod p

$K = (g^a \bmod p)^b = g^{ab} \bmod p$

Diffie-Hellman

- Example: Let p=11, g=2, then

a	1	2	3	4	5	6	7	8	9	10	11
g^a	2	4	8	16	32	64	128	256	512	1024	2048
g^a mod p	2	4	8	5	10	9	7	3	6	1	2

A chooses 4, B chooses 3, then shared secret is

$$(2^3)^4 = (2^4)^3 = 2^{12} = 4 \pmod{11}$$

Adversaries sees $2^3=8$ and $2^4=5$, needs to solve one of $2^x=8$ and $2^y=5$ to figure out the shared secret.

Three Problems Believed to be Hard to Solve

- Discrete Log (DLG) Problem: Given <g, h, p>, computes a such that $g^a = h \bmod p$.
- Computational Diffie Hellman (CDH) Problem: Given <g, $g^a \bmod p$, $g^b \bmod p$> (without a, b) compute $g^{ab} \bmod p$.
- Decision Diffie Hellman (DDH) Problem: distinguish (g^a, g^b, g^{ab}) from (g^a, g^b, g^c), where a,b,c are randomly and independently chosen

- If one can solve the DL problem, one can solve the CDH problem. If one can solve CDH, one can solve DDH.

Assumptions

- DDH Assumption: DDH is hard to solve.
- CDH Assumption: CDH is hard to solve.
- DLG Assumption: DLG is hard to solve

- DDH assumed difficult to solve for large p (e.g., at least 1024 bits).
- Warning:
  - New progress by Joux means solving discrete log for p values with some property can be done quite fast.
  - Look out when you need to use/implement public key crypto
  - May want to consider Elliptic Curve-based algorithms

ElGamal Encryption

- Public key <g, p, h=g^a mod p>
- Private key is a
- To encrypt: chooses random b, computes
  C=[g^b mod p, g^{ab} * M mod p].
  - Idea: for each M, sender and receiver establish a shared secret g^{ab} via the DH protocol. The value g^{ab} hides the message M by multiplying it.
- To decrypt C=[c_1,c_2], computes M where
  - ((c_1^a mod p) * M) mod p = c_2.
    - To find M for x * M mod p = c_2, compute z s.t. x*z mod p =1, and then M = C_2*z mod p
- CDH assumption ensures M cannot be fully recovered.
- IND-CPA security requires DDH.

RSA Algorithm

- Invented in **1978** by Ron **R**ivest, Adi **S**hamir and Leonard **A**dleman
  - Published as R L Rivest, A Shamir, L Adleman, "*On Digital Signatures and Public Key Cryptosystems*", Communications of the ACM, vol 21 no 2, pp120-126, Feb 1978
- Security relies on the difficulty of factoring large composite numbers
- Essentially the same algorithm was discovered in 1973 by Clifford Cocks, who works for the British intelligence

RSA Public Key Crypto System

Key generation:

1. Select 2 large prime numbers of about the same size, p and q

   Typically each p, q has between 512 and 2048 bits

2. Compute $n = pq$, and $\Phi(n) = (q-1)(p-1)$
3. Select e, $1<e<\Phi(n)$, s.t. $\gcd(e, \Phi(n)) = 1$

   Typically e=3 or e=65537

4. Compute d, $1< d< \Phi(n)$ s.t. $ed \equiv 1 \mod \Phi(n)$

   Knowing $\Phi(n)$, d easy to compute.

Public key: (e, n)

Private key: d

RSA Description (cont.)

Encryption

Given a message M, $0 < M < n$ $\quad\quad M \in Z_n - \{0\}$

use public key (e, n)

compute $C = M^e \bmod n$ $\quad\quad C \in Z_n - \{0\}$

Decryption

Given a ciphertext C, use private key (d)

Compute $C^d \bmod n = (M^e \bmod n)^d \bmod n = M^{ed} \bmod n = M$

RSA Example

- $p = 11$, $q = 7$, $n = 77$, $\Phi(n) = 60$
- $d = 13$, $e = 37$ ($ed = 481$; $ed \bmod 60 = 1$)
- Let $M = 15$. Then $C \equiv M^e \bmod n$
  - $C \equiv 15^{37} \pmod{77} = 71$
- $M \equiv C^d \bmod n$
  - $M \equiv 71^{13} \pmod{77} = 15$

RSA Example 2

- Parameters:
  - $p = 3$, $q = 5$, $n = pq = 15$
  - $\Phi(n) = ?$
- Let $e = 3$, what is d?
- Given $M=2$, what is C?
- How to decrypt?

Hard Problems RSA Security Depends on

$C = M^e \bmod (n=pq)$

Plaintext: M $\longrightarrow$ Ciphertext: C

$C^d \bmod n$

1. Factoring Problem: Given n=pq, compute p,q
2. Finding RSA Private Key: Given (n,e), compute d s.t. $ed = 1 \pmod{\Phi(n)}$.
   - Known to be equivalent to Factoring problem.
   - Implication: cannot share n among multiple users
3. RSA Problem: From (n,e) and C, compute M s.t. $C = M^e$
   - Aka computing the e'th root of C.
   - Can be solved if n can be factored

RSA Security and Factoring

- Security depends on the difficulty of factoring n
  - Factor n $\Rightarrow$ compute $\Phi(n)$ $\Rightarrow$ compute d from (e, n)
  - Knowing e, d such that ed = 1 (mod $\Phi(n)$) $\Rightarrow$ factor n
- The length of n=pq reflects the strength
  - 700-bit n factored in 2007
  - 768 bit factored in 2009
- RSA encryption/decryption speed is quadratic in key length
- 1024 bit for minimal level of security today
  - likely to be breakable in near future
- Minimal 2048 bits recommended for current usage
- NIST suggests 15360-bit RSA keys are equivalent in strength to 256-bit
- Factoring is easy to break with quantum computers
- Recent progress on Discrete Logarithm may make factoring much faster

RSA Encryption & IND-CPA Security

- The RSA assumption, which assumes that the RSA problem is hard to solve, ensures that the plaintext cannot be fully recovered.

- Plain RSA does not provide IND-CPA security.
  - For Public Key systems, the adversary has the public key, hence the initial training phase is unnecessary, as the adversary can encrypt any message he wants to.

  - How to break IND-CPA security?

Real World Usage of Public Key Encryption

- Often used to encrypt a symmetric key
  - To encrypt a message M under an RSA public key (n,e), generate a new AES key K, compute
    [K^e mod n, AES-CBC_K(M)]

- One often needs random padding.
  - Given M, chooses random r, and generates F(M,r), and then encrypts as $F(M,r)^e$ mod n
  - From F(M,r), one should be able to recover M
  - This provides randomized encryption

Digital Signatures: The Problem

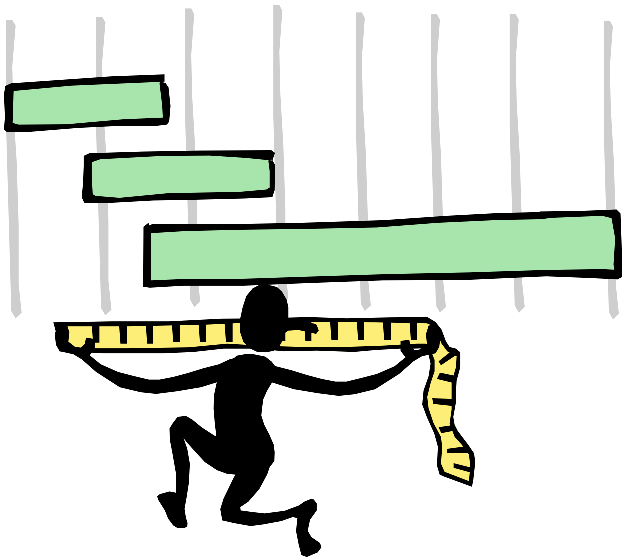
- Consider the real-life example where a person pays by credit card and signs a bill; the seller verifies that the signature on the bill is the same with the signature on the card
- Contracts are valid if they are signed.
- Signatures provide non-repudiation.
  - ensuring that a party in a dispute cannot repudiate, or refute the validity of a statement or contract.
- Can we have a similar service in the electronic world?
  - Does Message Authentication Code provide non-repudiation? Why?

Digital Signatures

- MAC: One party generates MAC, one party verifies integrity.
- Digital signatures: One party generates signature, many parties can verify.
- Digital Signature: a data string which associates a message with some originating entity.
- Digital Signature Scheme:
  - a signing algorithm: takes a message and a (private) signing key, outputs a signature
  - a verification algorithm: takes a (public) verification key, a message, and a signature
- Provides:
  - Authentication, Data integrity, Non-Repudiation

Digital Signatures and Hash

- Very often digital signatures are used with hash functions, hash of a message is signed, instead of the message.
- Hash function must be:
  - Pre-image resistant
  - Weak collision resistant
  - Strong collision resistant



RSA Signatures

Key generation (as in RSA encryption):

- Select 2 large prime numbers of about the same size, p and q
- Compute $n = pq$, and $\Phi = (q - 1)(p - 1)$
- Select a random integer e, $1 < e < \Phi$, s.t. $\gcd(e, \Phi) = 1$
- Compute d, $1 < d < \Phi$ s.t. $ed \equiv 1 \bmod \Phi$

Public key: (e, n)	**used for verification**
Private key: d,	**used for generation**

RSA Signatures with Hash (cont.)

Signing message M

- Verify $0 < M < n$
- Compute $S = h(M)^d \bmod n$

Verifying signature S

- Use public key (e, n)
- Compute $S^e \bmod n = (h(M)^d \bmod n)^e \bmod n = h(M)$

Non-repudiation

- Nonrepudiation is the assurance that someone cannot deny something. Typically, nonrepudiation refers to the ability to ensure that a party to a contract or a communication cannot deny the authenticity of their signature on a document or the sending of a message that they originated.
- Can one deny a signature one has made?
- Does email provide non-repudiation?

The Big Picture

	Secret Key Setting	Public Key Setting
Secrecy / Confidentiality	Stream ciphers Block ciphers + encryption modes	Public key encryption: RSA, El Gamal, etc.
Authenticity / Integrity	Message Authentication Code	Digital Signatures: RSA, DSA, etc.

Coming Attractions …

- User authentication