On the Security of Proofs of Sequential Work in a Post-Quantum World

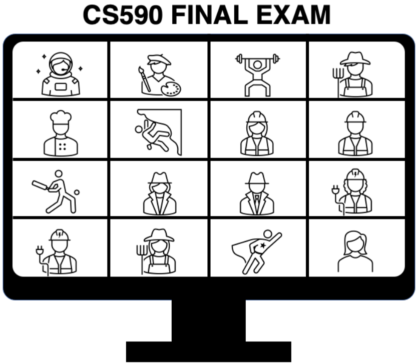
Jeremiah Blocki[1], Seunghoon Lee[1], Samson Zhou[2]

[1] Department of Computer Science, Purdue University
[2] School of Computer Science, Carnegie Mellon University

July 28, 2021

Conference on Information-Theoretic Cryptography (ITC) 2021

Motivation: Online Exams during the Pandemic



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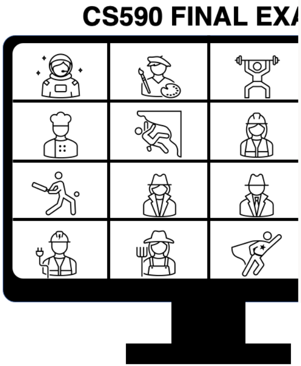

[CS590] 5 mins late - having internet issue

CG Cinseer Goodman
Tue 5/2/2021 9:05 PM
To: Seunghoon Lee

answer-goodman.pdf
157 KB

Dear Professor,

My name is Cinseer Goodman who is taking CS590 this semester.
I hope this email finds you well.
I was not able to submit the final exam to the server on time due to an unexpected internet connectivity loss.
It just went back 5 minutes later so I send you the file via email.
I promise I have not done any extra work after the exam time. I hope it works.
Thank you.

Best,
Cinseer Goodman

Reply | Forward

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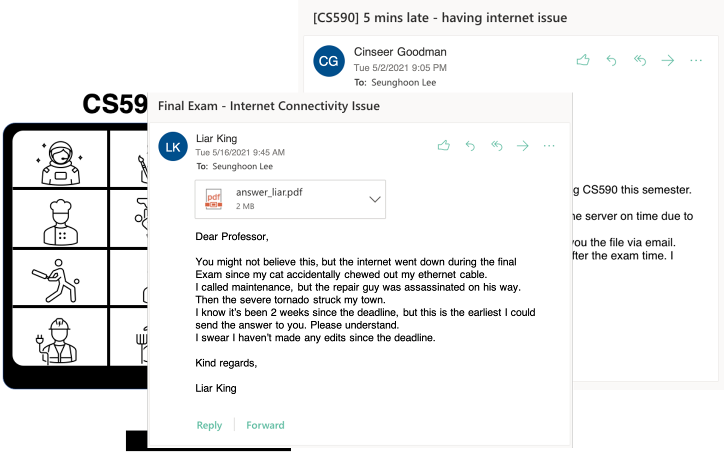

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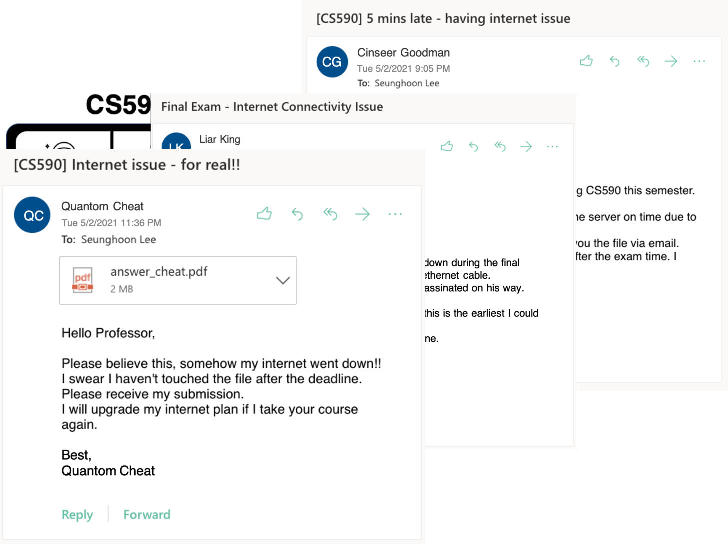

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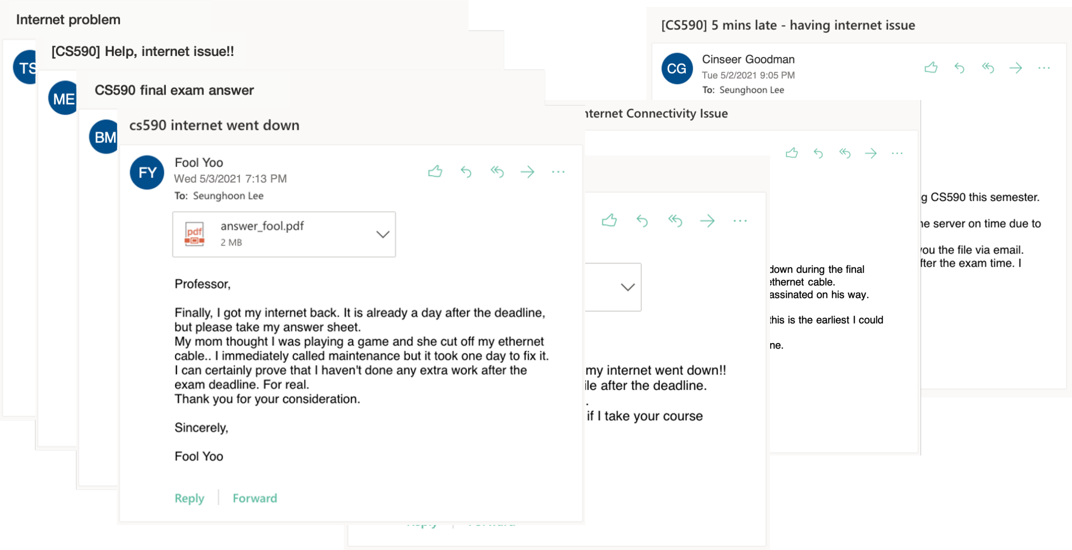

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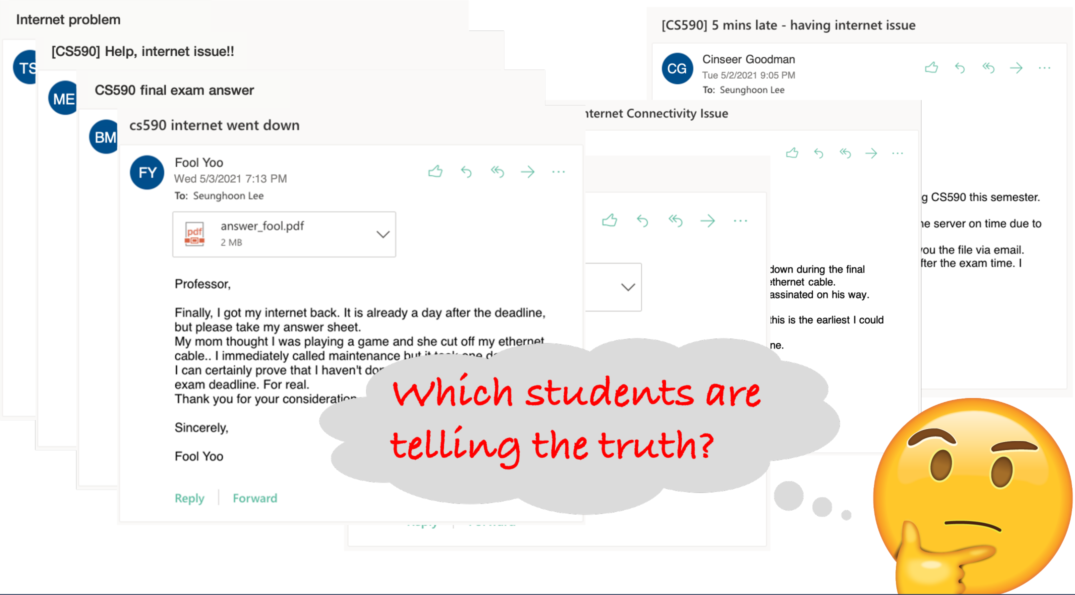

Solution: Proofs of Sequential Work (PoSW)

What is a Proof of Sequential Work? (Informal)

A proof that a large amount (N) of sequential work was performed after a prover committed an initial message, e.g., the solution for the final exam

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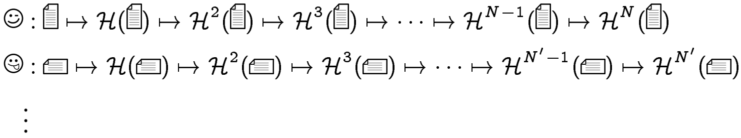

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- Many late students? $\rightarrow$ insufficient computational resources to verify all solutions

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- Soundness: students (prover) should *not* be able to produce a *valid* proof faster (than sequential time $\Omega(N)$, even if running in parallel).

PoSW Constructions

Mahmoody et al. [MMV13]: the first theoretical construction of a PoSW

- Verifier time $\mathtt{polylog}\ N$, and prover time $\Omega(N)$,
- Parallel cheating prover running in sequential time $< N$ cannot fool the verifier, and
- Security proof in the classical ROM.

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Cohen and Pietrzak [CP18]: an improved & practical PoSW construction

- Modular security proof in the classical ROM:
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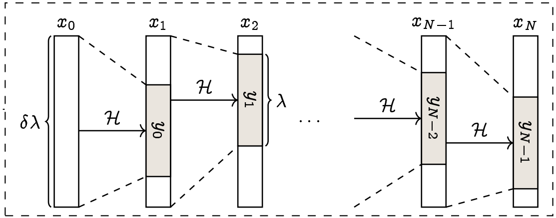

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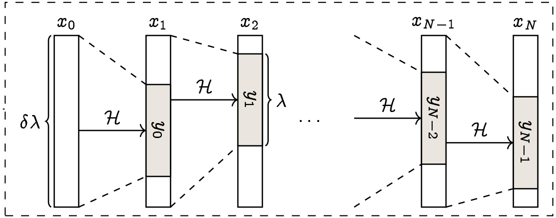

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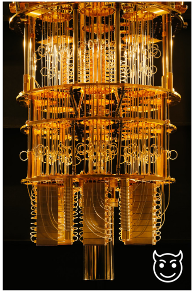
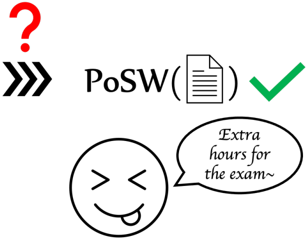
Key Research Questions:

- Can a sequentially time-bounded parallel *quantum* attacker produce a long $\mathcal{H}$-sequence?
- Can a sequentially time-bounded parallel *quantum* attacker produce a valid PoSW?





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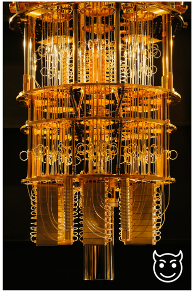

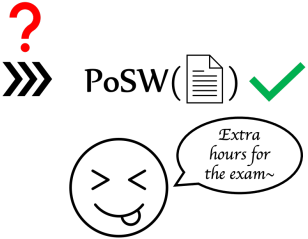

Short answer: NO!

Our Result. Hardness of Producing an $\mathcal{H}$-Sequence/PoSW in a Quantum Setting

Theorem (informal)

A quantum adversary making at most $q \ll 2^{\lambda/3}$ queries over $N-1$ rounds outputs an $\mathcal{H}$-sequence of length N $(x_0, \ldots, x_N$ with $|x_i| \le \delta\lambda$ where $\delta \ge 1)$ with negligible probability $\mathcal{O}\left(\frac{q^3\delta\lambda}{2^\lambda}\right)$.

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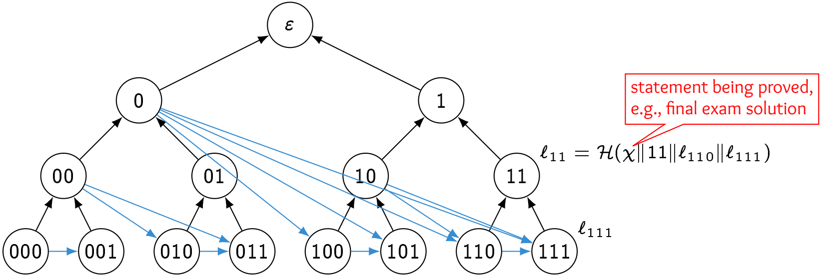
Concurrent/Subsequent Work.

- Chung et al. [CFHL21]: also gave a comparable security bounds for the PoSW in the pqROM

The [CP18] Construction



- For all leaf nodes v, add an edge (u, v) for any u that is a left sibling of a node on the path from v to the root ε

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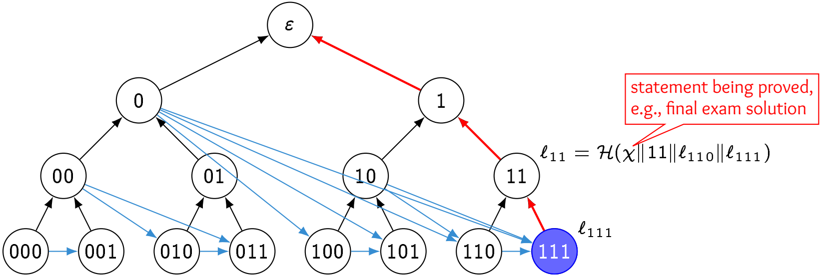

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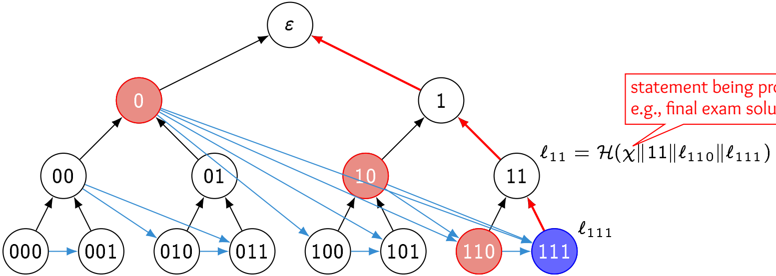

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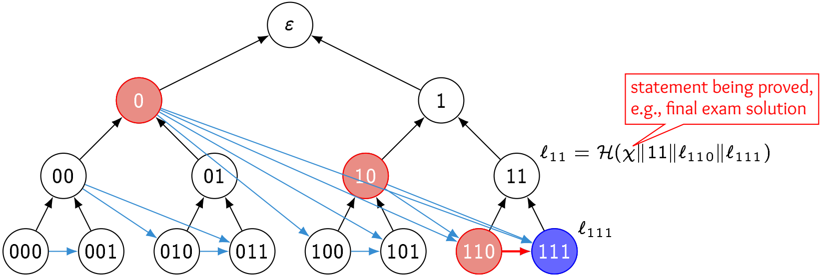

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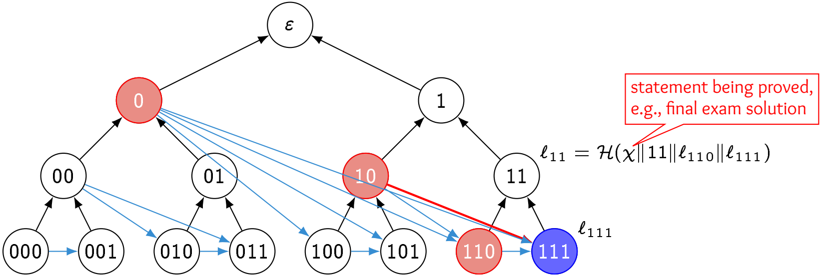

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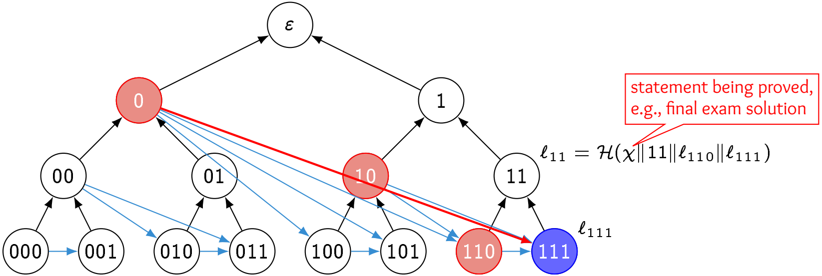

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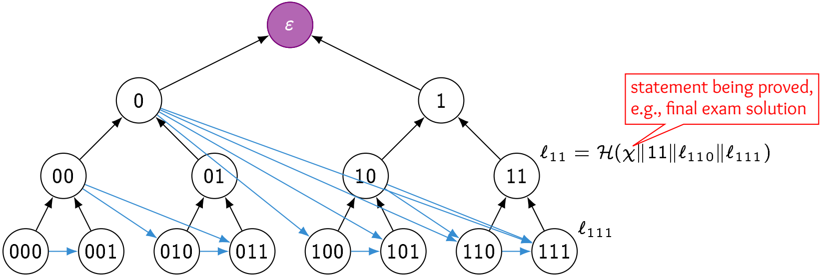

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- The label of root node forms a **Merkle tree commitment** of all the other nodes
  - Verifier can audit the prover by forcing the prover to open certain labels
  - Show that they are locally consistent

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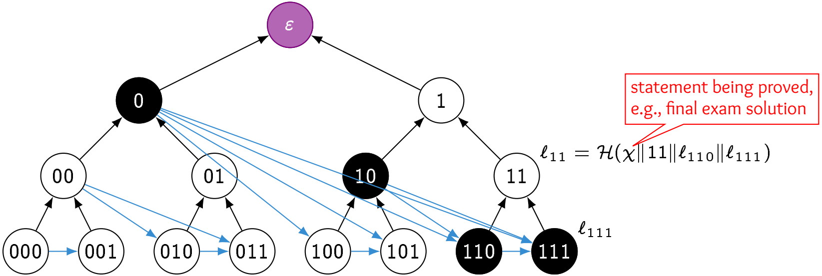

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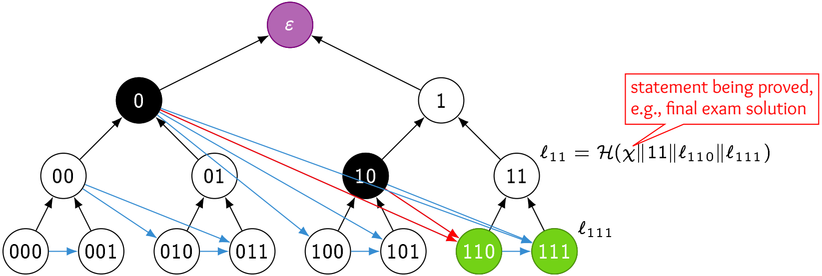

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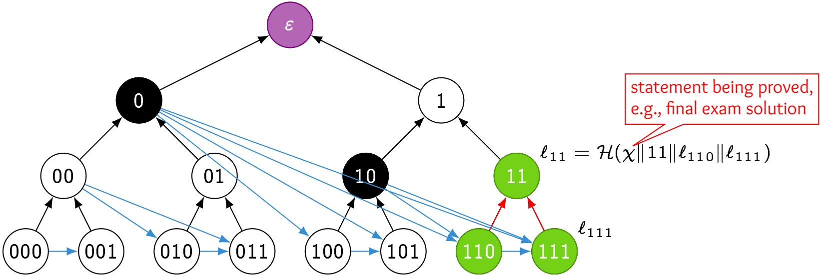

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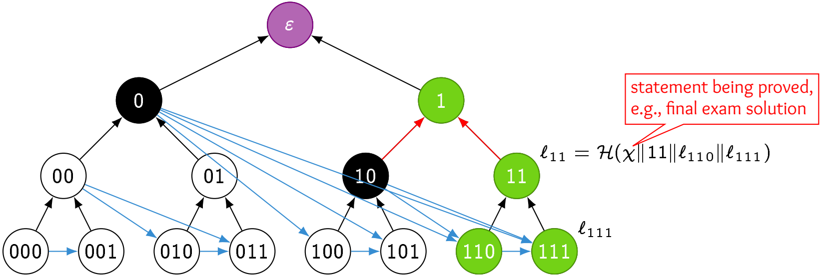

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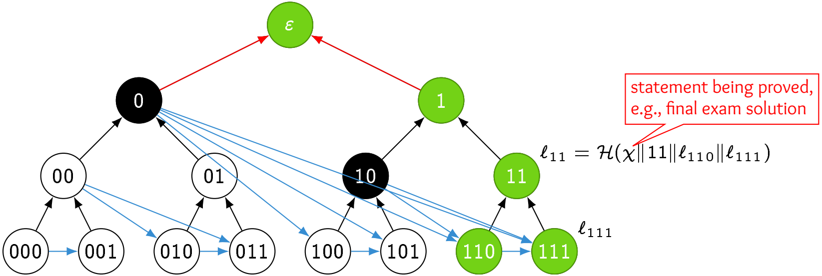

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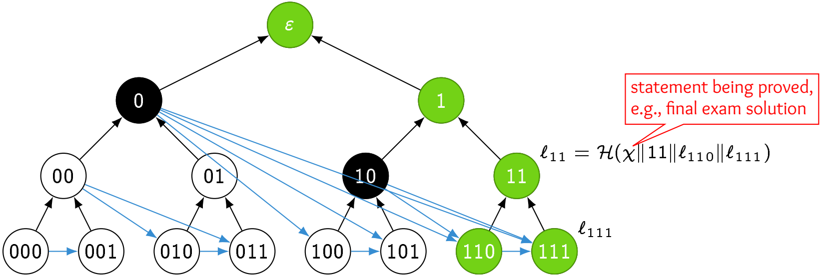

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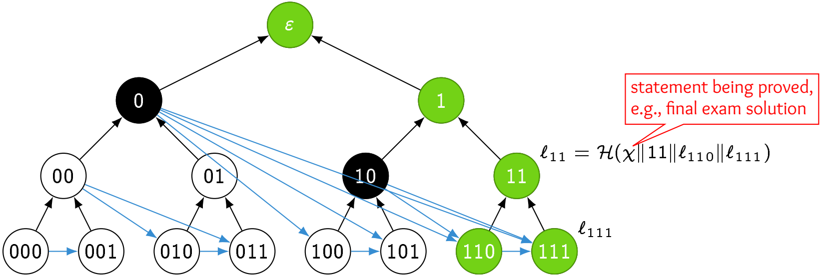

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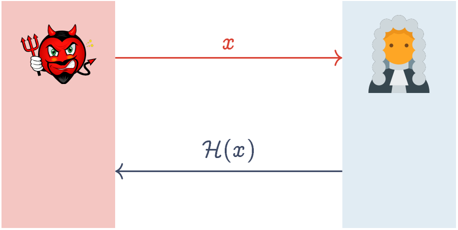
- Any classical ROM attacker that produces a valid PoSW in time $< N$ must produce a long $\mathcal{H}$-sequence

ROM vs qROM [BDF+11]

⟨Classical ROM⟩



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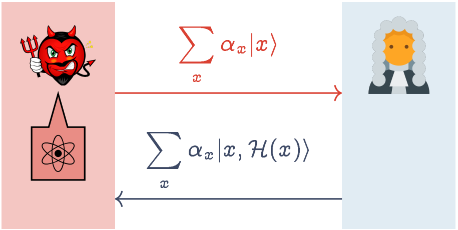
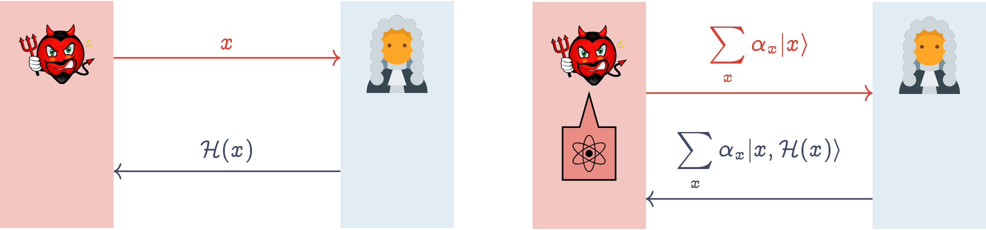


ROM vs qROM [BDF+11]

<Classical ROM> <Quantum ROM>



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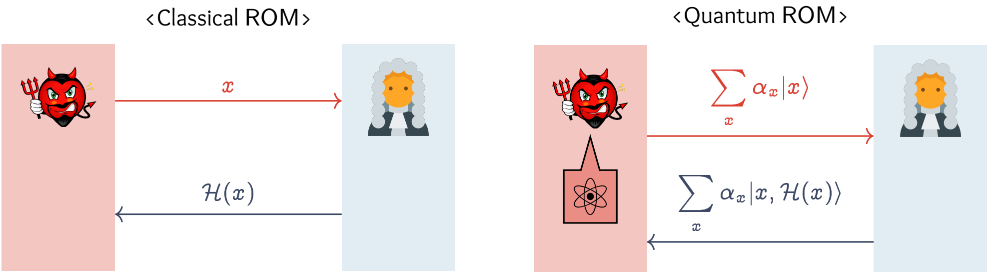

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  - Recording quantum queries?
- Compressed Oracle Technique [Zha19]: change of view (compressed phase oracle (CPhsO))

$$|x, y\rangle \otimes |\mathcal{H}\rangle \mapsto |x, y \oplus \mathcal{H}(x)\rangle \otimes |\mathcal{H}\rangle$$

$$\Updownarrow$$

$$|x, y\rangle \otimes |\mathcal{H}\rangle \mapsto (-1)^{y \cdot \mathcal{H}(x)} |x, y\rangle \otimes |\mathcal{H}\rangle$$

Compressed Phase Oracle (CPhsO)

A database $\mathcal{D} := \{(x_i, y_i), i \geq 1\}$, where $\mathcal{D}(x_i) = y_i$.

How to view a random oracle?

- Classical: databases of known I/O pairs & unknown I/O pairs don't appear
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- a compressed dataset of at most q input/output pairs.

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Example: Single Query (simplest case)

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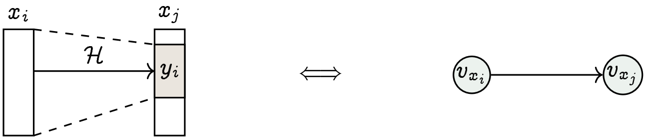
- w_i's range over all possible outputs of $\mathcal{H}(x_i)$'s for each i.

Notations

- Given a database $\mathcal{D} = \{(x_1, y_1), \ldots, (x_q, y_q)\}$, define a directed graph $G_{\mathcal{D}}$ on q nodes $(v_{x_1}, \ldots, v_{x_q})$ such that:



- $\mathsf{PATH}_s := \{\mathcal{D} : G_{\mathcal{D}}$ contains a path of length $s\}$ (set of databases), and
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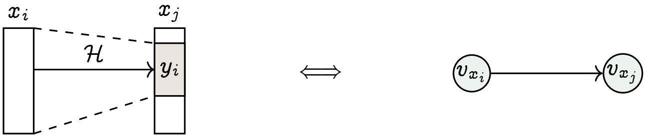

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$\mathcal{D}$ contains an $\mathcal{H}$-sequence of length s $\quad\Longleftrightarrow\quad$ $\mathcal{D} \in \mathsf{PATH}_s$

Proof Ideas: Hardness of Producing an $\mathcal{H}$-sequence in a Quantum Setting

Lemma

$|\varphi\rangle$: *an initial state, and let* $|\varphi'\rangle = \mathsf{CPhsO}^k|\varphi\rangle$. *Then* $L_2(|\varphi'\rangle, \widetilde{\mathsf{PATH}}_{s+1}) - L_2(|\varphi\rangle, \widetilde{\mathsf{PATH}}_s) \leq \frac{4k\sqrt{(q+k)\delta\lambda}}{2^{\lambda/2}}$.

Interpretation/Intuition:

- $L_2(|\varphi\rangle, \widetilde{\mathsf{PATH}}_s)$: 2-norm of the projection of $|\varphi\rangle$ onto $\widetilde{\mathsf{PATH}}_s$, i.e.,

$$|\varphi\rangle = \sum_X \alpha_X |X\rangle \quad \Rightarrow \quad L_2(|\varphi\rangle, \widetilde{\mathsf{PATH}}_s) = \sqrt{\sum_{|X\rangle \in \widetilde{\mathsf{PATH}}_s} |\alpha_X|^2}.$$

- If we start with the state that is nearly orthogonal to $\widetilde{\mathsf{PATH}}_s$, then after applying the oracle CPhsO^k, the resulting state is also nearly orthogonal to $\widetilde{\mathsf{PATH}}_{s+1}$.

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Basic proof idea: split the states into good and bad part. (in this talk, suppose that $x_1, \ldots, x_k \notin \mathcal{D}$ and all distinct for simplicity)

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Proof by example:

$$\mathcal{D} = \{(\overset{x_1}{10101}, \overset{y_1}{0001}), (\overset{x_2}{00011}, \overset{y_2}{0010}), (\overset{x_3}{00010}, \overset{y_3}{0110})$$

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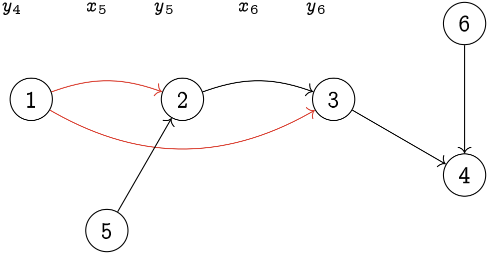

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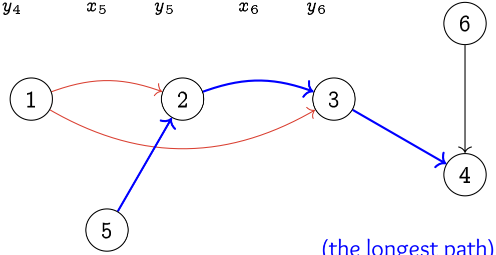

(the longest path)=(5, 2, 3, 4)

$\mathcal{D} \in \mathsf{PATH}_3$ but $\mathcal{D} \notin \mathsf{PATH}_4$

Proof Ideas: Hardness of Producing an $\mathcal{H}$-sequence in a Quantum Setting

Suppose we have one query: $x_7 = 00001$ (where $x_7 \notin \mathcal{D}$). Then the updated database is:

$$\mathcal{D}_1 = \{(10101, 0001), (00011, 0010), (00010, 0110), (01101, 0000), (11110, 0011), (01011, 1101)\} \cup \boxed{\{(00001, w)\}}$$

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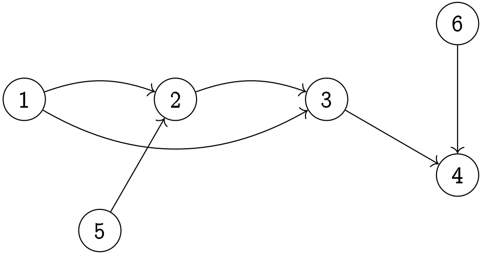

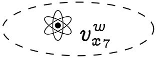

superposition over w's

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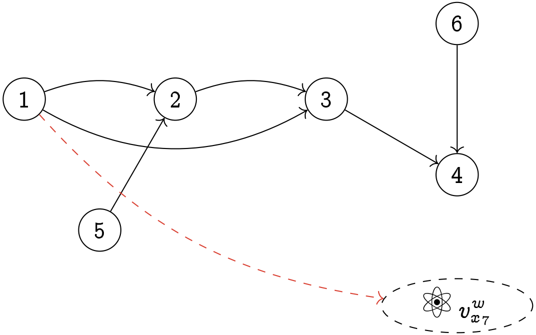

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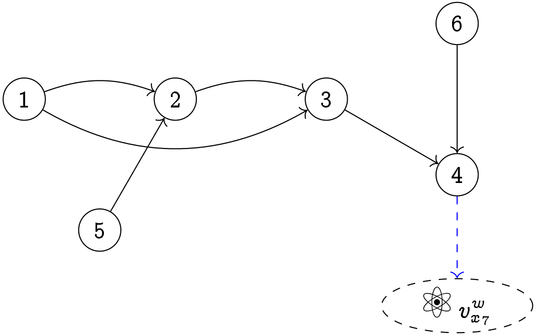

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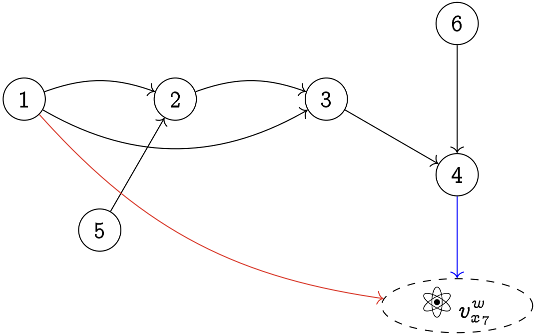

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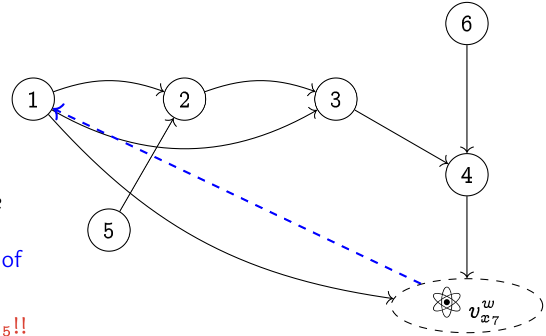

BAD if:

1. back edges from $v^w_{x_7}$ to some $i \in \{1, \dots, 6\}$.
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$\Rightarrow$ $\mathcal{D} \notin \mathrm{PATH}_4$ but $\mathcal{D}_1 \in \mathrm{PATH}_5$!!

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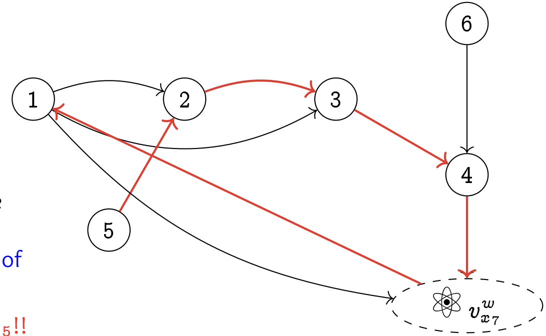

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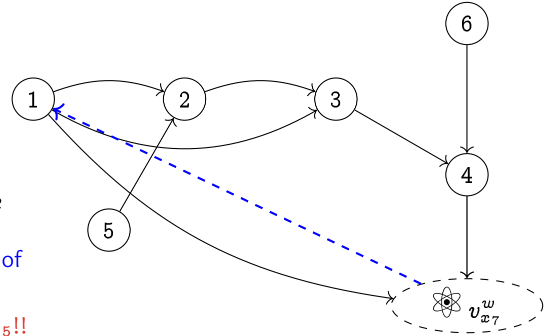

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Key observation:

The fraction of such w's is negligible! ($\mathcal{O}(q\delta\lambda)$ out of 2^λ)

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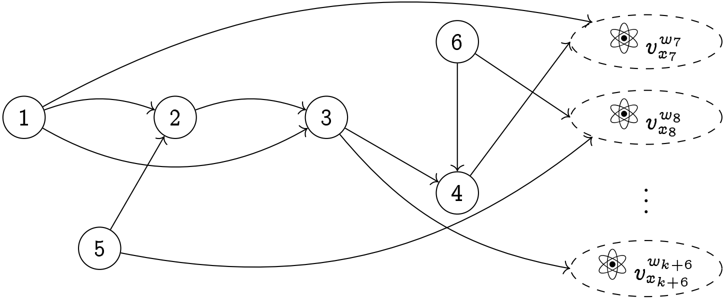

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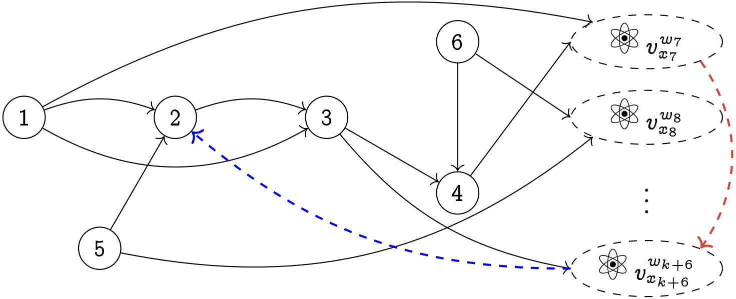

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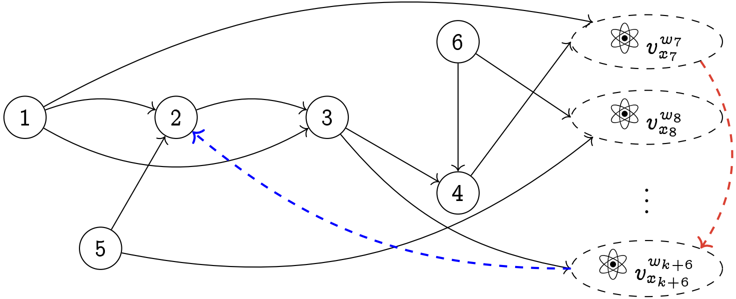
$\Rightarrow$ $\mathcal{D} \notin \text{PATH}_4$ but $\mathcal{D}_k \in \text{PATH}_5$!!

Proof Ideas: Hardness of Producing an $\mathcal{H}$-sequence in a Quantum Setting

For a parallel query: $x_7, \ldots, x_{k+6}$ (where $x_7, \ldots, x_{k+6} \notin \mathcal{D}$ and all x_i's are distinct for simplicity),

$$\mathcal{D}_k = \{(10101, 0001), (00011, 0010), (00010, 0110) \\ , (01101, 0000), (11110, 0011), (01011, 1101)\} \cup \{(x_7, w_7), \ldots, (x_{k+6}, w_{k+6})\}$$

$G_{\mathcal{D}_k}$



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Key observation: The fraction of such $w_7, \ldots, w_{k+6}$'s is negligibly small! ($(q + k)\delta\lambda$ out of 2^λ for each w_i)

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We have shown: k parallel queries in a single round,

$$L_2(|\varphi'\rangle, \widetilde{\mathsf{PATH}}_{s+1}) - L_2(|\varphi\rangle, \widetilde{\mathsf{PATH}}_s) \leq \frac{4k\sqrt{(q+k)\delta\lambda}}{2^{\lambda/2}}.$$

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Security of a non-interactive PoSW: similar argument using the result above - details in the paper (`https://arxiv.org/pdf/2006.10972.pdf`)

Concluding Remarks

Takeaways.

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- Can techniques extend to other primitives, e.g., Proofs of Space, Memory-Hard Functions, etc.?

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