Randomized Exploration for Reinforcement Learning with General Value Function Approximation

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Reinforcement Learning

Learn to interact with an unknown environment through trial and error
Reinforcement Learning

Learn to interact with an unknown environment through trial and error

Action: $a_1, a_2, ..., a_H$

State observations: $s_1, s_2, ..., s_H$
Rewards: $r_1, r_2, ..., r_H$

Goal: maximize cumulative reward for a horizon $H$

Value: $E[r_1 + r_2 + r_3 + \cdots + r_H]$

*Long term effect* needs to be considered.
Reinforcement Learning

$S_0 \xrightarrow{a_0 \xleftarrow{\text{move 1}}} S_1 \xrightarrow{a_1, a_2, \ldots, r_1, r_2 \ldots} S_{89} \xrightarrow{a_{90}, a_{91}, \ldots, r_{90}, r_{91} \ldots} S_{226}$

Wining: $r_{226} \leftarrow 1$
Losing: $r_{226} \leftarrow 0$

Actions? States? Rewards?

OpenAI Arm
Markov Decision Process (MDP)

- Environment is unknown
  - States: \( S \); actions: \( A \)
  - Reward: \( r(s, a) \in [0, 1] \)
  - **Unknown** state transition: \( P_h(\cdot | s, a) \)
  - Horizon: \( H \) (a large number)
  - Goal: optimal policy \( \pi^*: S \rightarrow \Delta_A \)

\[
\max_{\pi} \mathbb{E}[r_1(s_1, \pi(s_1)) + r_2(s_2, \pi(s_2)) + \cdots + r_H(s_H, \pi(s_2))] =: Q^\pi
\]

\[
s_i \sim P(\cdot | s_{i-1}, \pi(s_{i-1}))
\]
Theories of RL on MDP

• Exploration + exploitation [Kearns & Singh 2002, Jaksch et al. 2010]
  • Learn from scratch
  • Exploitation: optimize policy based on existing data
  • Exploration: collect new info about the environment
    • Regret: average error v.s. optimal policy

• Focus has been on Tabular RL
  • Does not scale in practical problem
  • Provides sanity check for exploration algorithm
  • In deep RL, the default is $\epsilon$-greedy exploration

Play current policy $\pi^k$

Data: [history trajectories]

Improve policy to $\pi^{k+1}$
Does tabular algorithm work in practice?

• Number of episodes required to get a good \( \pi \)

\[
\tilde{\Theta}[|S||A|\text{poly}(H)]
\]

[Jin et al’2018] [Azar et al’ 2017][…]

• Curse of Dimensionality

\(|S| = 3^{361}\)

\(|S| \geq 256^{256 \times 240}\)
Function Approximation in Practice

- DQN

Limitations? **Huge** number of training samples. Hard to **understand**. No **theoretical** guarantee.
RL Theory v.s. Practice

• Theory
  • Markov decision process
    • Finite state space $S$
    • Finite action space $A$
    • Finite horizon $H$
  • Many theoretical results
    • Mostly tabular – well understood
    • Not scalable

• Practice
  • Infinite state space
  • Function approximation via Deep Neural Networks
  • Many empirical results
    • Little understanding
    • No guarantee
Function Approximation

- Find a function class to approximate $Q^*(s,a)$ or $\pi^*$

- Generalization ability
  - Infer values/policies for unseen $(s,a)$
Linear Function Approximation

• Need correct features
  • Features are given: $\phi(s, a) \rightarrow R^d$

$$\phi \left( \begin{array}{c}
\text{3 question marks, 1 enemies, 4 bushes, 1 chimney, …}
\end{array} \right)$$

• Bad features requires **exponential** time/sample to learn
  [Du-Kakade-Wang-Yang’ 20] [Van Roy & Dong’ 20] [Lattimore et al’ 20] [Weisz et al’ 20]

• Good features
  • Linear MDP [Yang & Wang’ 19]:
    efficient algorithm: [Jin et al’ 20]
  • Low-bellman error [Zanette et al’ 20]
  • Low-bellman rank [Jiang et al’ 17]
General function approximation

• No features are given

• Function class $\mathcal{F}$
  • Might be parametric
  • $f(s,a)$ may rep. $Q^*(s,a)$

• Used in practice

Goals for RL:
• Efficient algorithms with practical potentials
• Theoretical guarantees for special cases

[Silver et al 2016]
Strategies for Exploration

• Optimism in the face of uncertainty:
  • Upper Confidence Bound (UCB)

• Thompson Sampling
  • One of the oldest heuristics for balancing exploration exploitation trade-off. (Thompson, 1933)
  • Randomly select an action according to the probability of it being the optimal action.
  • PSRL = Thompson Sampling for MDPs. (Strens 2000)
  • Sample MDP from posterior, apply policy for an entire episode.
Randomized value functions

• Key idea: generate approximate posterior samples
  • Use standard value learning algorithms (LSVI, DQN, …)
  • Fit to randomly perturbed data

• Theory for tabular representation + LSVI:
  • Worst-case regret bound for Gaussian noise (Russo 2019)
    \[ \text{Regret}(T) \leq \tilde{O}(H\sqrt{S^3AT}) \]

• Computational results with generalization
  • Parameterized representation for Q(s,a)
  • Scalable unlike UCB based methods or posterior sampling
  • Approximate posterior inference is good enough for efficient exploration.
Current limitations

• No theoretical result for RVF with general function approximation
  • Limited to empirical results only (Bootstrapped DQN, Ensemble sampling)

• Lack of unification between OFU and Thompson Sampling
  • Can we combine both principle for algorithm design?

• Bypassing UCB bonus in applying OFU principle
  • UCB bonus is not scalable
  • For GFA, requires complicated sensitivity sampling scheme [Wang et al, 2020]
LSVI for Online RL with **General VFA**

- Initialize an arbitrary $Q^0 \leftarrow 0$
- For episode $k = 1, 2, \ldots K$:
  - Solve for $Q_h^k$ using LSVI on the history
    \[
    \theta_h^k \leftarrow \arg\min_w \sum_t \left[ f_w(s_t, a_t) - \left( r(s_t, a_t) + \max_a Q^k_{h+1}(s_{t+1}, a) \right) \right]^2
    \]
    \[
    Q^k_h(s, a) = f_{\theta_h^k}(s, a)
    \]
- Collect a trajectory of data
  \[
  \pi^k(s) \leftarrow \arg\max_a Q^k_h(s, a)
  \]
  \[
  (s^k_1, a^k_1, r^k_1) \rightarrow (s^k_2, a^k_2, r^k_2) \rightarrow (s^k_3, a^k_3, r^k_3) \rightarrow \cdots (s^k_H, a^k_H, r^k_H)
  \]
LSVI as Approximate Dynamic Programming (ADP)

• Each iteration solves

\[ Q_h^k = f_{\theta_h^k}(s, a) \]
Optimistic Sampling

Data: \(\{(s_h^\tau, a_h^\tau, r_h^\tau + \xi_{h,k}^{\tau,1})\}\) → Learning Agent → \(\tilde{f}_h^{k,1}\)

\[Q_h^{k,m}(\cdot, \cdot) = \tilde{f}_h^{k,m}(\cdot, \cdot)\]

\[Q_h^k(\cdot, \cdot) = \min_{m \in [M]} \max_{h \in [H]} Q_h^{k,m}(\cdot, \cdot), H - h + 1\]
Theory for General functions

• Assumption: \( r + PV \in \mathcal{F}, \forall V \)
  - Realizability: The function set is the “image” of Bellman projection
  - Corresponding to linear MDP for linear setting

• Eluder dimension [Russo & Van Roy’ 2013]
  - \( d_E \): the longest determination sequence of the function set
  - d-dim linear / generalized linear: \( \approx d \)

Theorem:

LSVI-PHE with optimistic sampling satisfies regret bound of \( O(\text{poly}(d_E H)\sqrt{T}) \) with high probability
Riverswim:

Riverswim12: best run

Episode Return

Training Episodes $\times 10^4$
Deep Sea: M sensitivity
Mountain Car:

Sparse MountainCar: best run

Average Return

Step $\times 10^5$
Summary

• Provably efficient RVF method for RL with general function approximation
  • Sublinear regret
  • Computationally efficient

• Optimistic sampling allows us the unify OFU and Thompson Sampling
Collaborators

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Paper Link:

[Ishfaq, Cui, Nguyen, Ayoub, Yang, Wang, Precup, Yang' ICML 2021] Randomized Exploration for Reinforcement Learning with General Value Function Approximation
https://arxiv.org/abs/2106.07841