

# Abstraction in Reinforcement Learning

David Abel

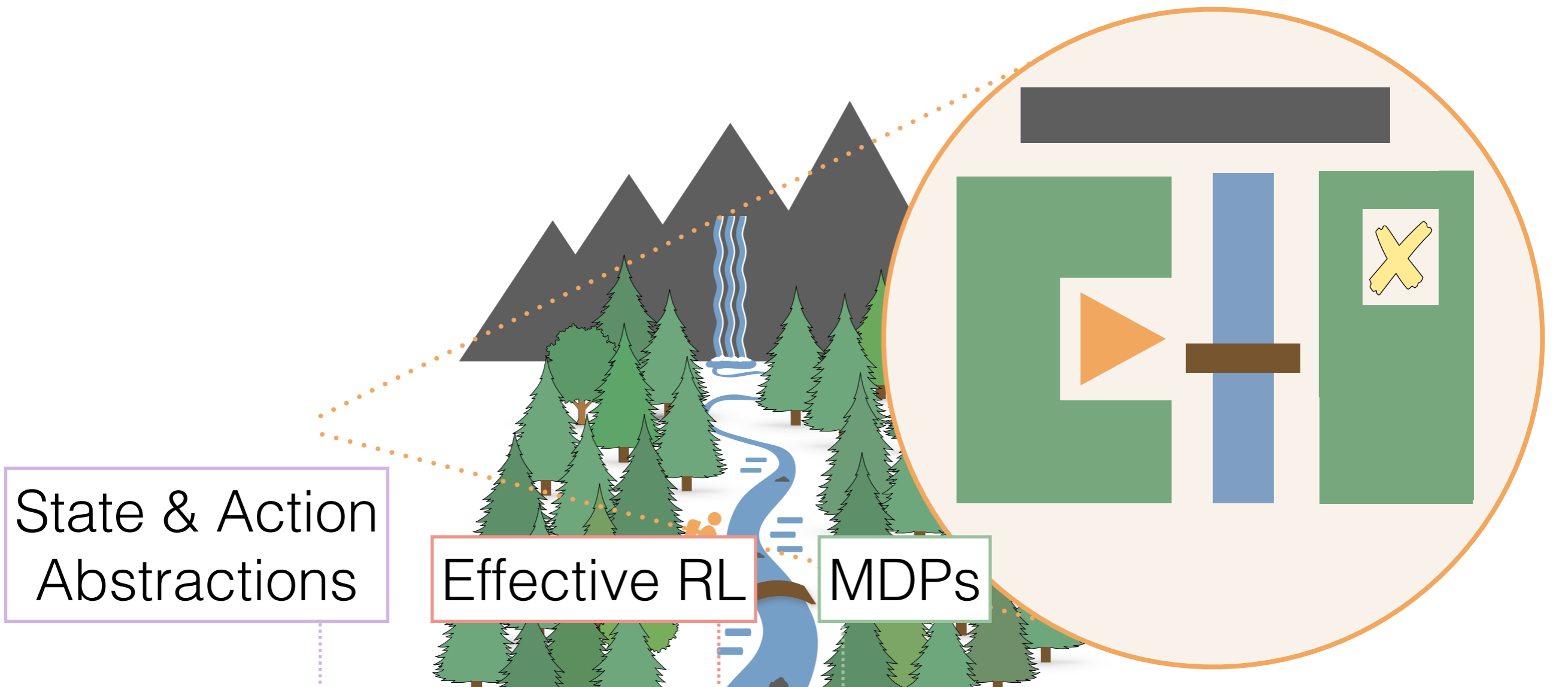
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*Purdue*  
*July 14, 2021*

**Dissertation:** [david-abel.github.io/thesis.pdf](https://david-abel.github.io/thesis.pdf)

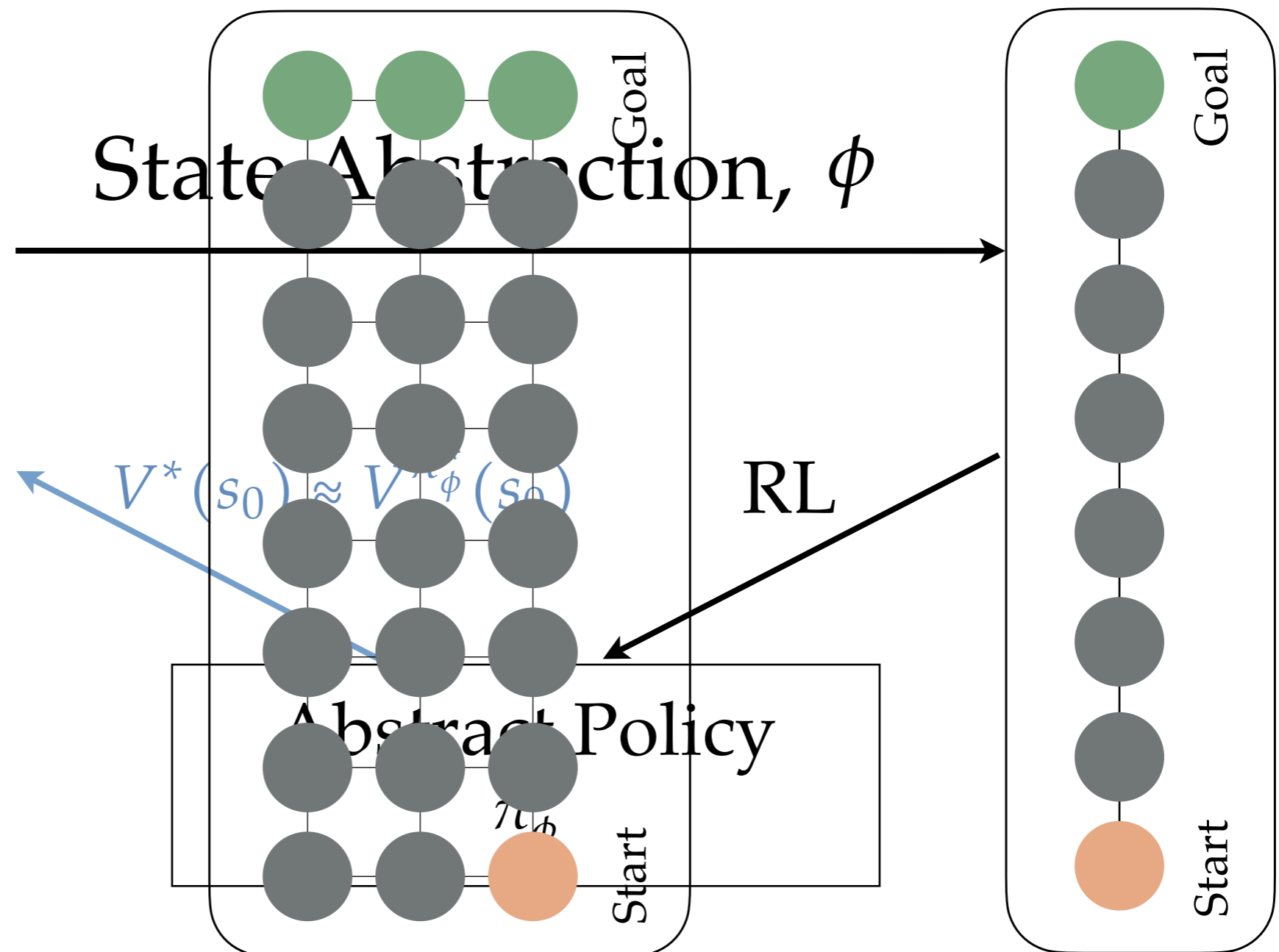
**Contact:** [dmabel@deepmind.com](mailto:dmabel@deepmind.com)

# Abstraction



**Question:** How do *intelligent agents* come up with the right *abstract understanding* of the *worlds* they inhabit?

# State Abstraction



# State Abstraction

**Definition.** A state abstraction is a function  $\phi : \mathcal{S} \rightarrow \mathcal{S}_\phi$  that maps every ground state to an abstract state.

[Fox '73]

[Jong, Stone '05] [Ortner et al. '07, '14, '19]

[Whitt '78]

[Ferns et al., '04, '06] [Hutter '14, '16, '19]

[Singh et al. '95]

[Li et al. '06] [Jiang et al., '14, '15]

[Dean, Givan '97]

[Whiteson et al. '07] [Akrouer et al., '18]

[Dieterich '00]

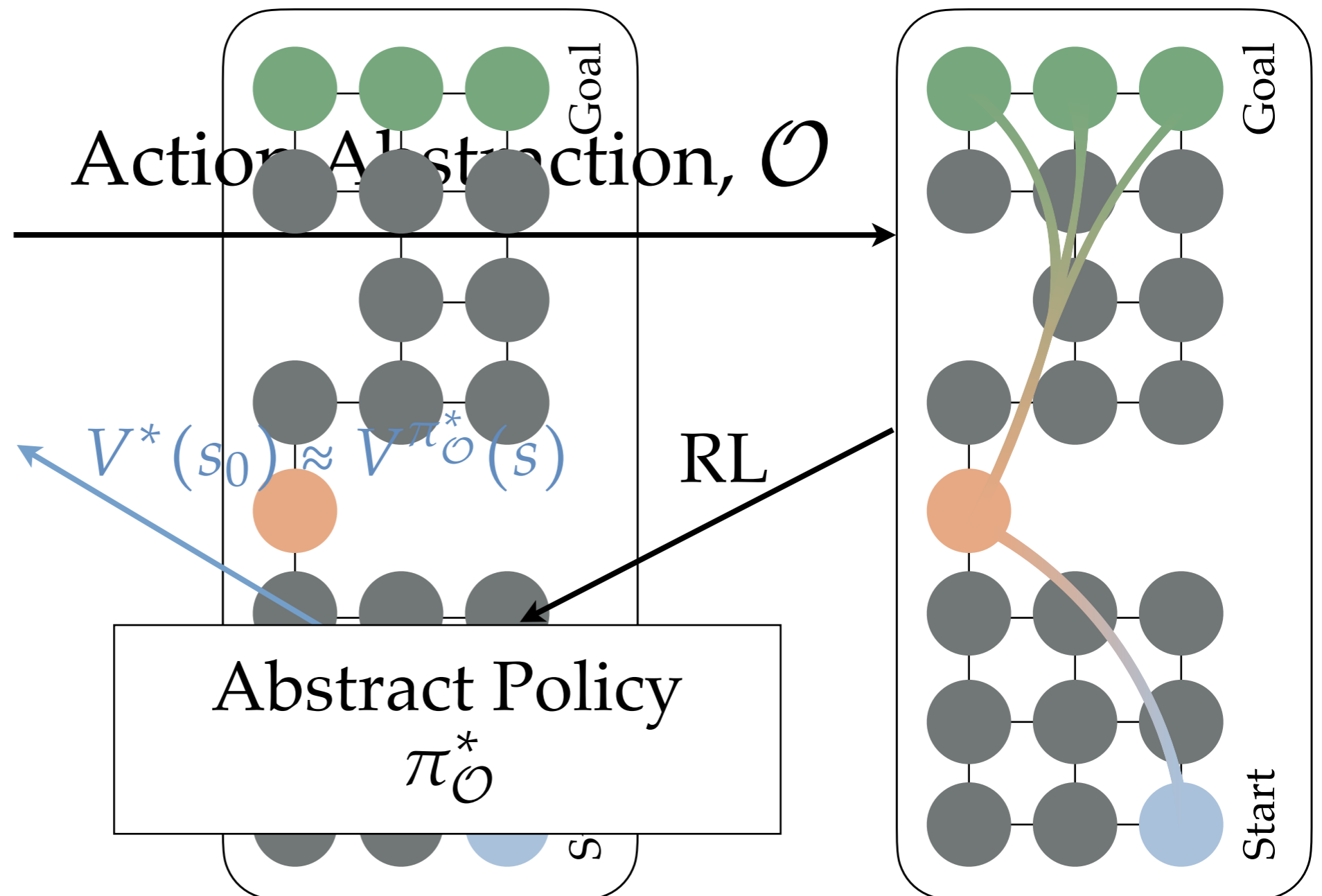
[Castro, Precup '09] [Menashe, Stone '18]

[Andre, Russell '02] [Mugan, Kuipers '12] [Taïga et al. '18]

[Ravindran, Barto '03, '04]

[Hostetler et al. '14, '15, '17]

# Action Abstraction



# Action Abstraction

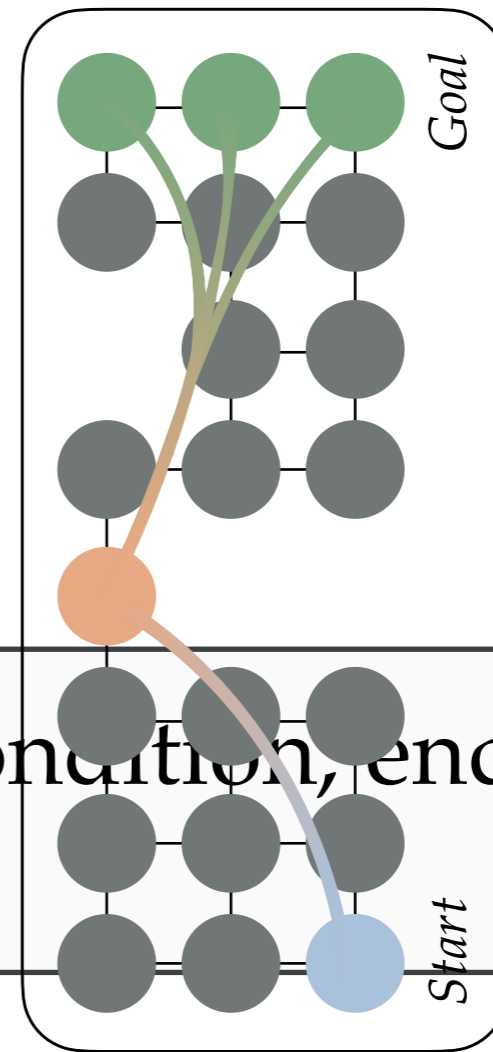
*Example:*

$$o_1 = (\text{blue circle}, \text{orange circle}, \pi_1)$$

*[Sutton, Precup, Singh 1999]*

$$o_2 = (\text{orange circle}, \text{green circle}, \pi_2)$$

**Definition (Option):** A start condition, end condition, and a policy.



# Action Abstraction

**Definition** (Action Abstraction): An action abstraction replaces the primitive actions with the option set  $\mathcal{O}$ .

*[McGovern et al. '97]*

*[Sutton, Precup, Singh '99]*

*[Simsek, Barto, '05, '08]*

*[Jong, Hester, Stone '08]*

*[Brunskill, Li '14]*

*[Ciosek, Silver '15]*

*[Konidaris et al. '06, '07, '09, '10, '18]*

*[Bacon et al. '17, '18]*

*[Fruit et al. '17, '17]*

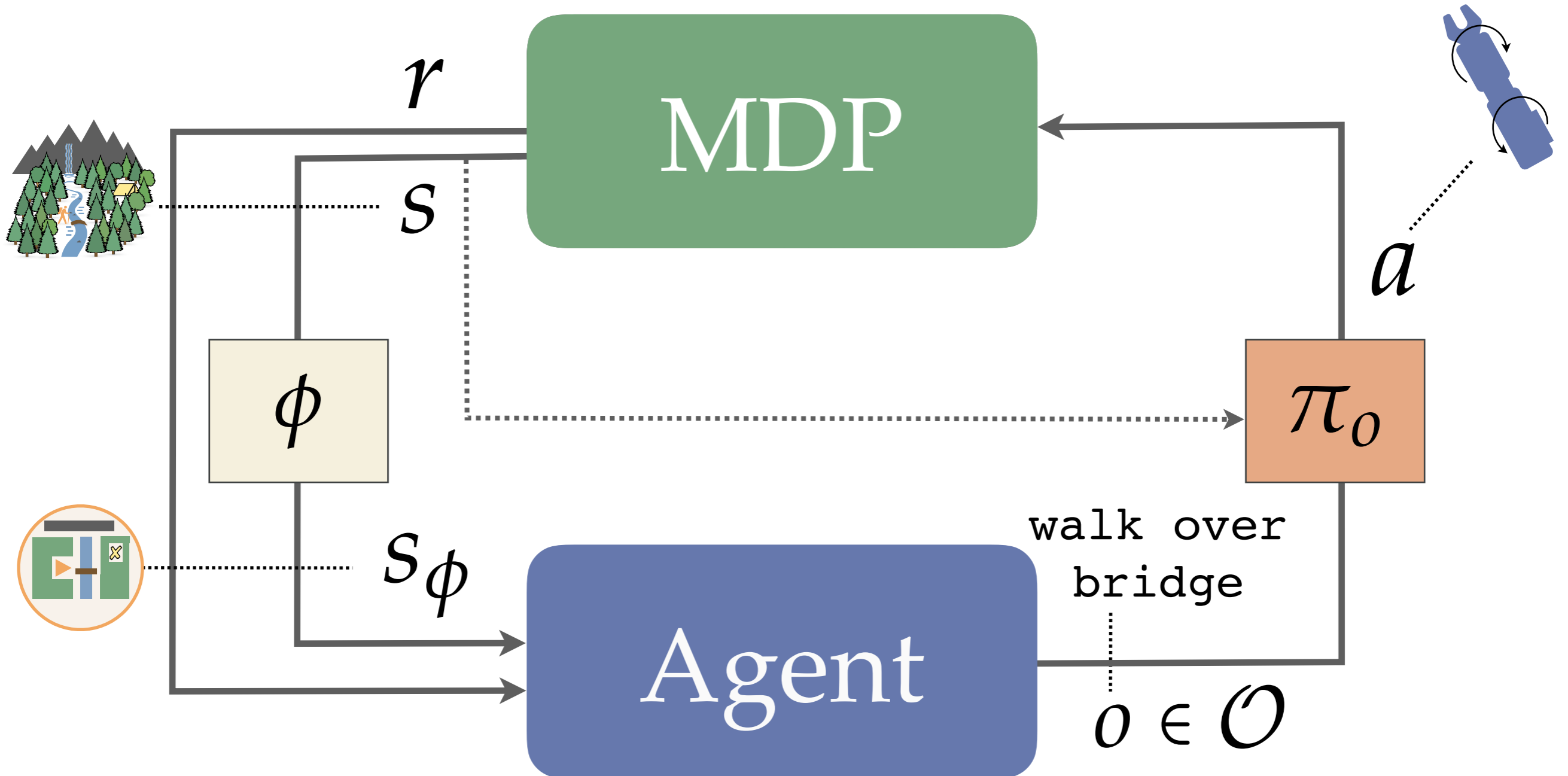
*[Machado et al. '17]*

*[Harutyunyan et al. '18]*

*[Eysenbach et al. '18]*

*[Majeed & Hutter '19]*

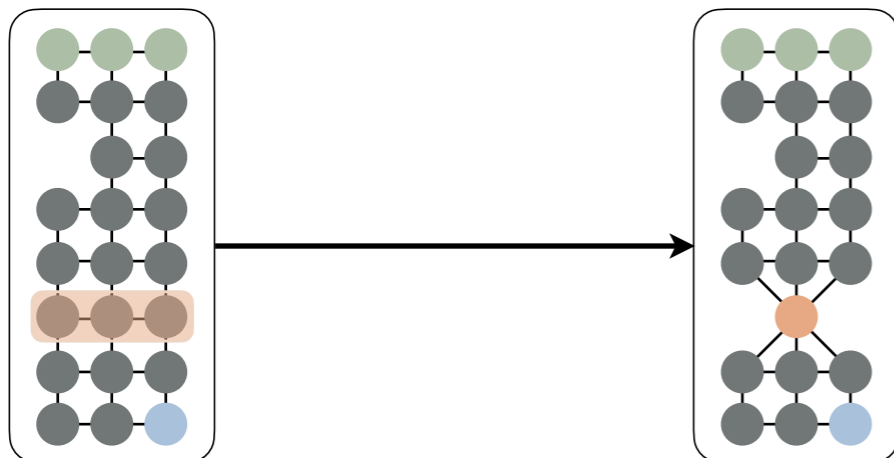
# Abstraction in RL





## Part 1

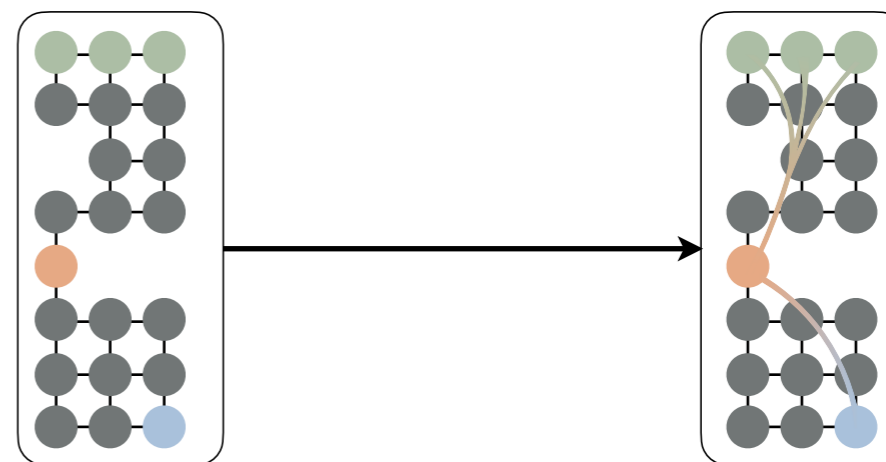
### STATE ABSTRACTION



1. Approximate State Abstraction  
*ICML 2016*
2. State Abstraction In Lifelong RL  
*ICML 2018*
3. State Abstraction As Compression  
*AAAI 2019*

## Part 2

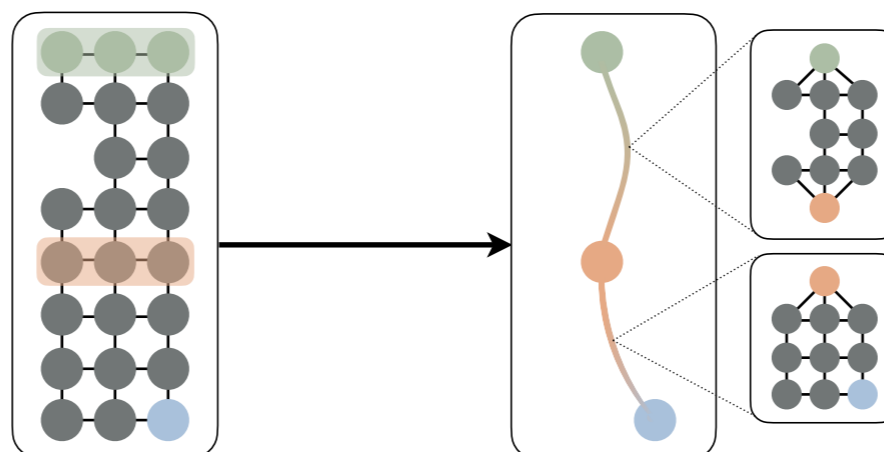
### ACTION ABSTRACTION



4. Options for Planning  
*ICML 2019*
5. Options for Exploration  
*ICML 2019*
6. A New Option Model  
*IJCAI 2019*

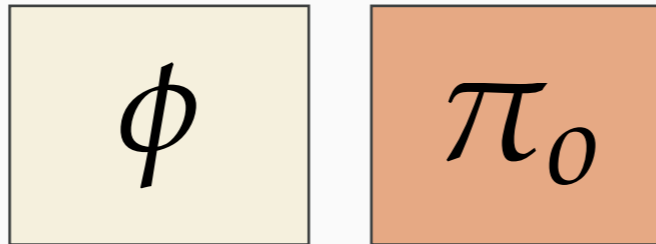
## Part 3

### STATE-ACTION ABSTRACTION



7. Value-Preserving Hierarchies  
*AISTATS 2020*

# Desirable Abstractions



Q: Which kinds of abstractions are desirable?

Easy To Construct

Supports Efficient  
Reinforcement  
Learning

Preserves  
Solution Quality

# Abstraction Desiderata

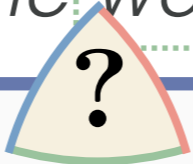
*Easy To Construct*

State & Action  
Abstractions

Effective RL

MDPs

**Question:** How do intelligent agents come up with the right abstract understanding of the worlds they inhabit?

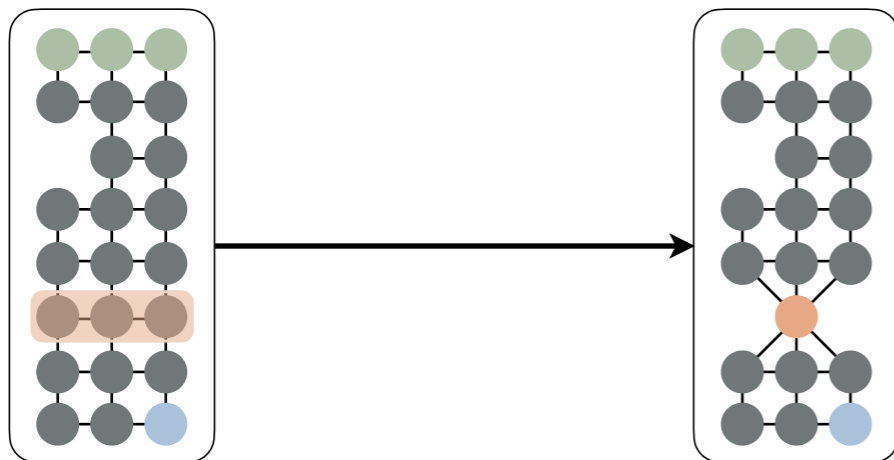


*Supports Efficient  
Reinforcement Learning*

*Preserves  
Solution Quality*

## Part 1

### STATE ABSTRACTION



1. Approximate State Abstraction  
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*ICML 2018*
3. **State Abstraction As Compression**  
*AAAI 2019*

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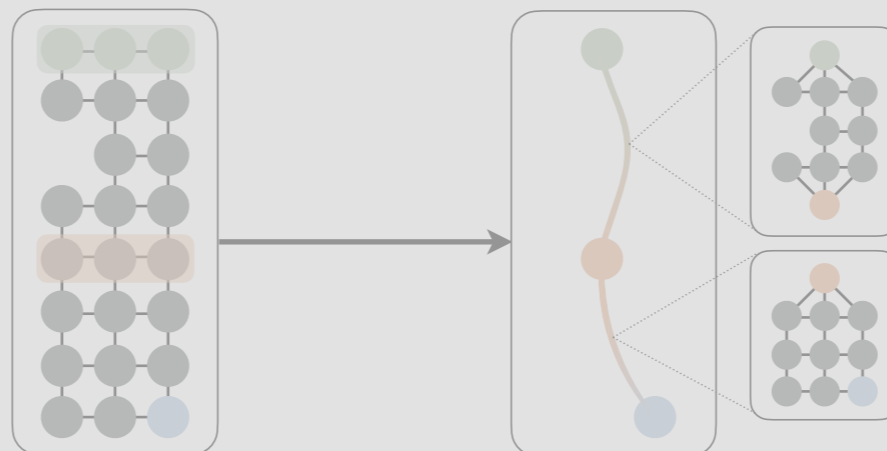
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## Part 3

### STATE-ACTION ABSTRACTION

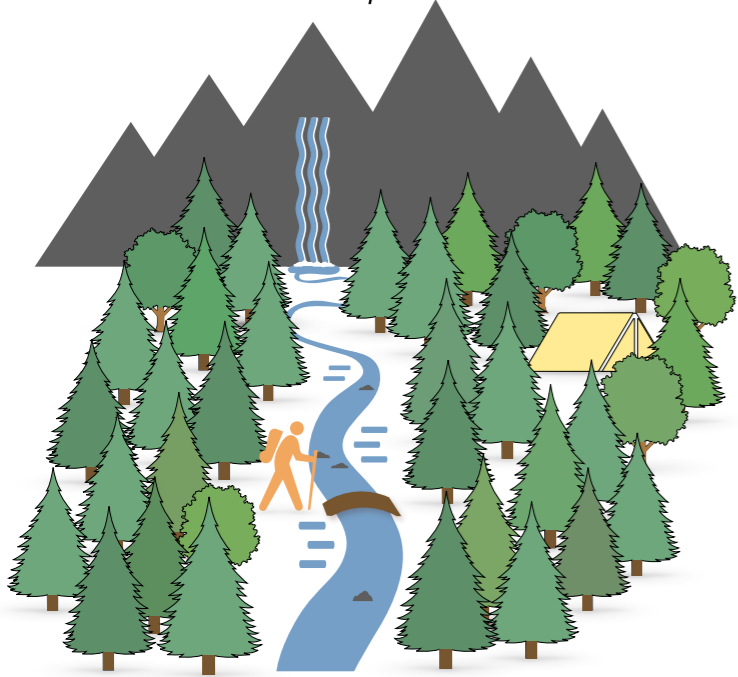


7. Value-Preserving Hierarchies  
*AISTATS 2020*

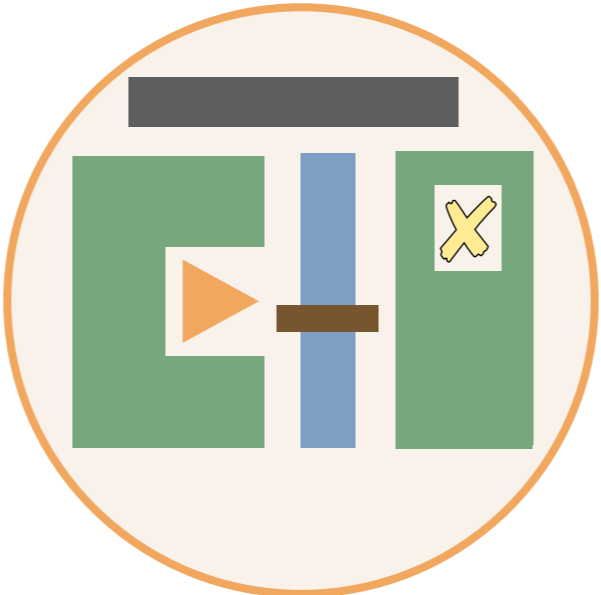
# State Abstraction as Compression

[AAAJWL, AAI 2019]

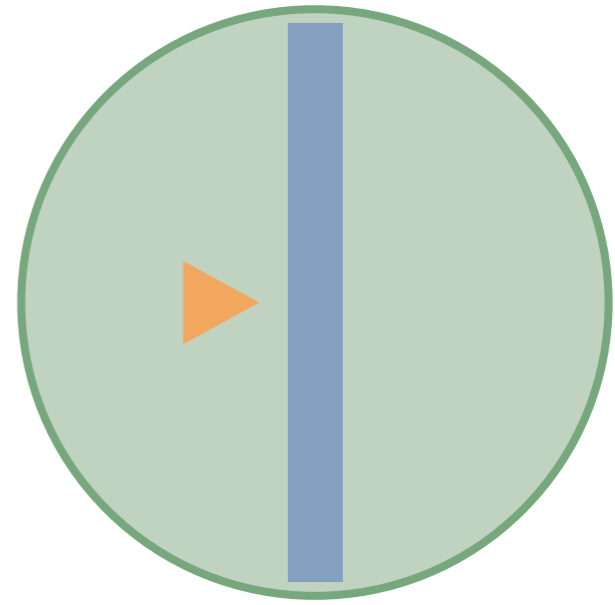
High Value  
No Compression



Some Value  
Some Compression



No Value  
High Compression



**Question:** How can we construct state abstractions that trade-off between compression and representational quality?

Dilip Arumugam

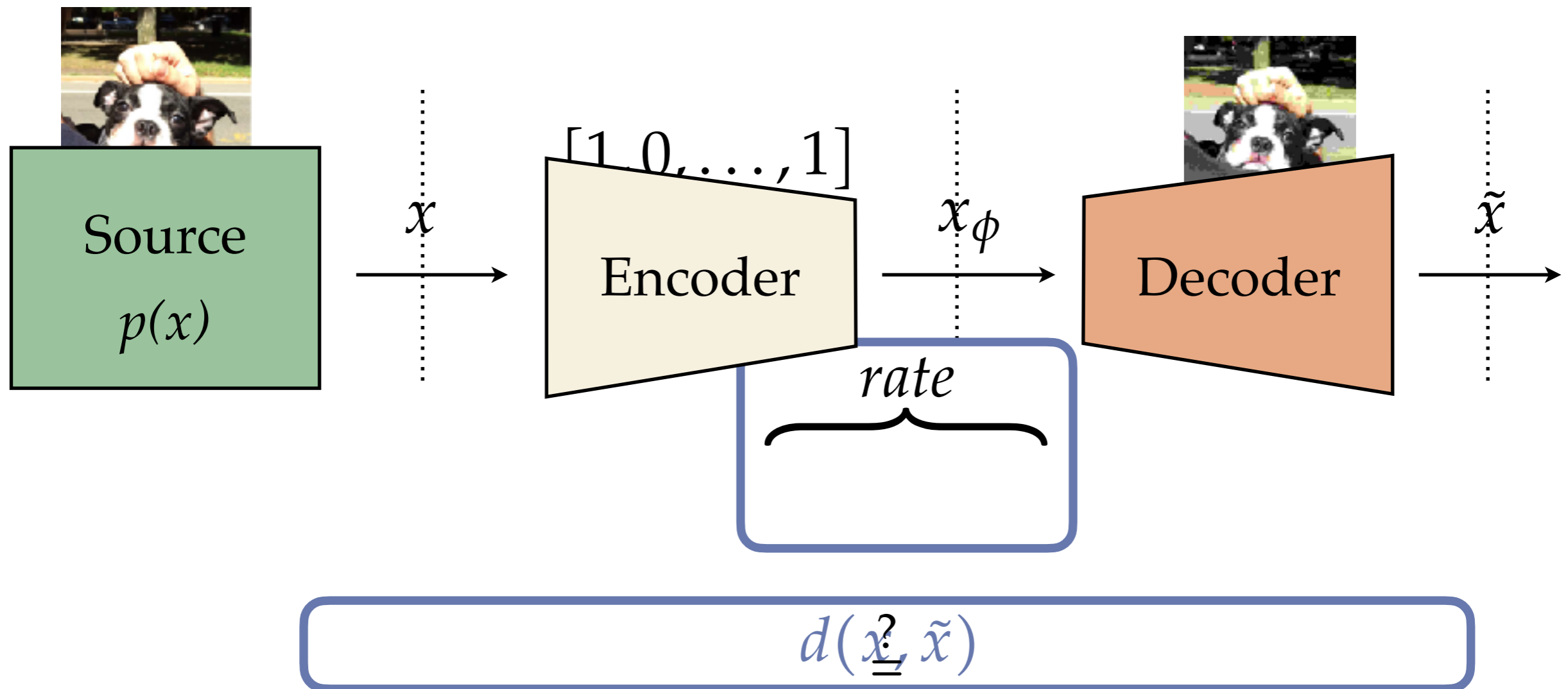
Kavosh Asadi

Yuu Jinnai

Lawson L.S. Wong

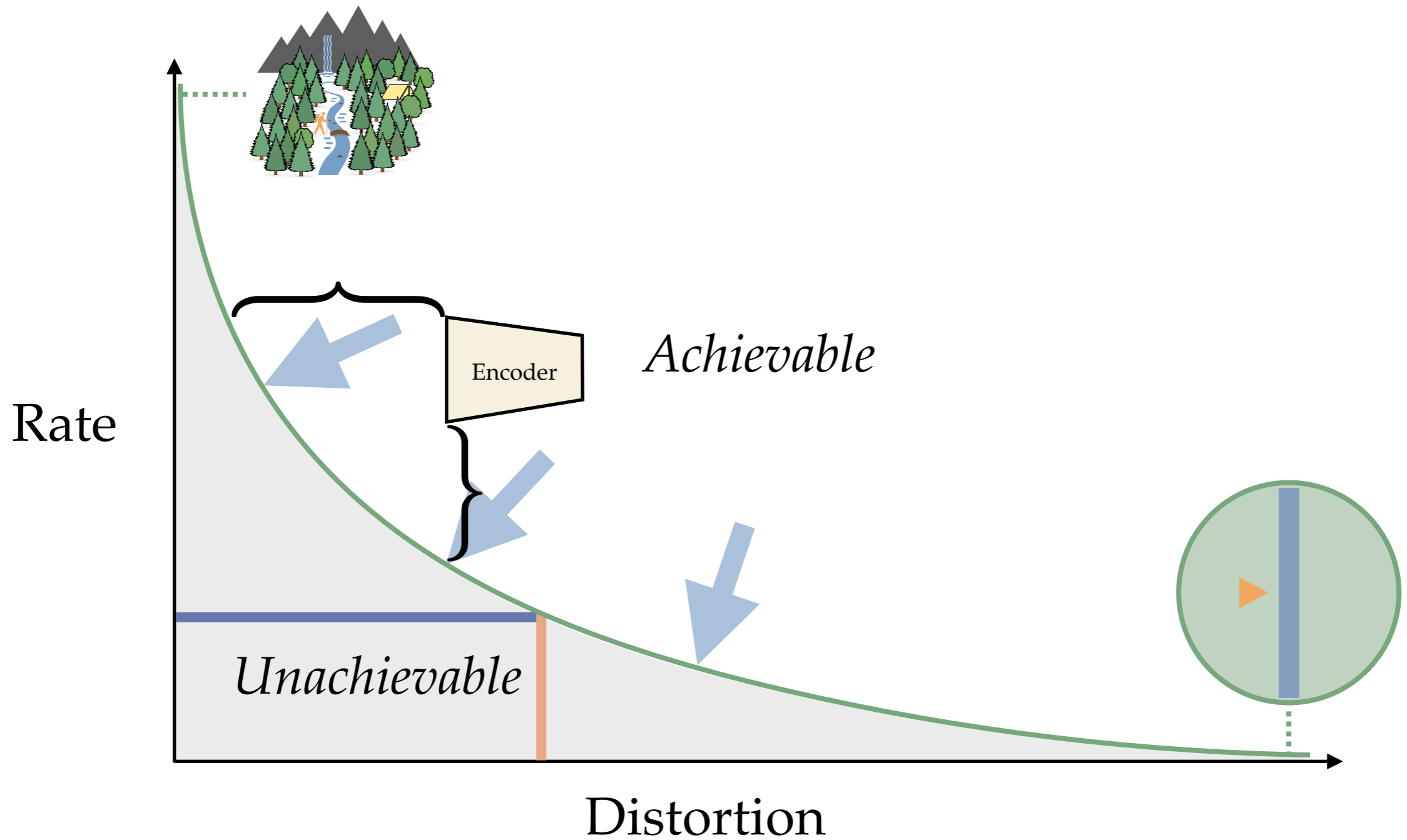
Michael L. Littman

# Rate-Distortion Theory



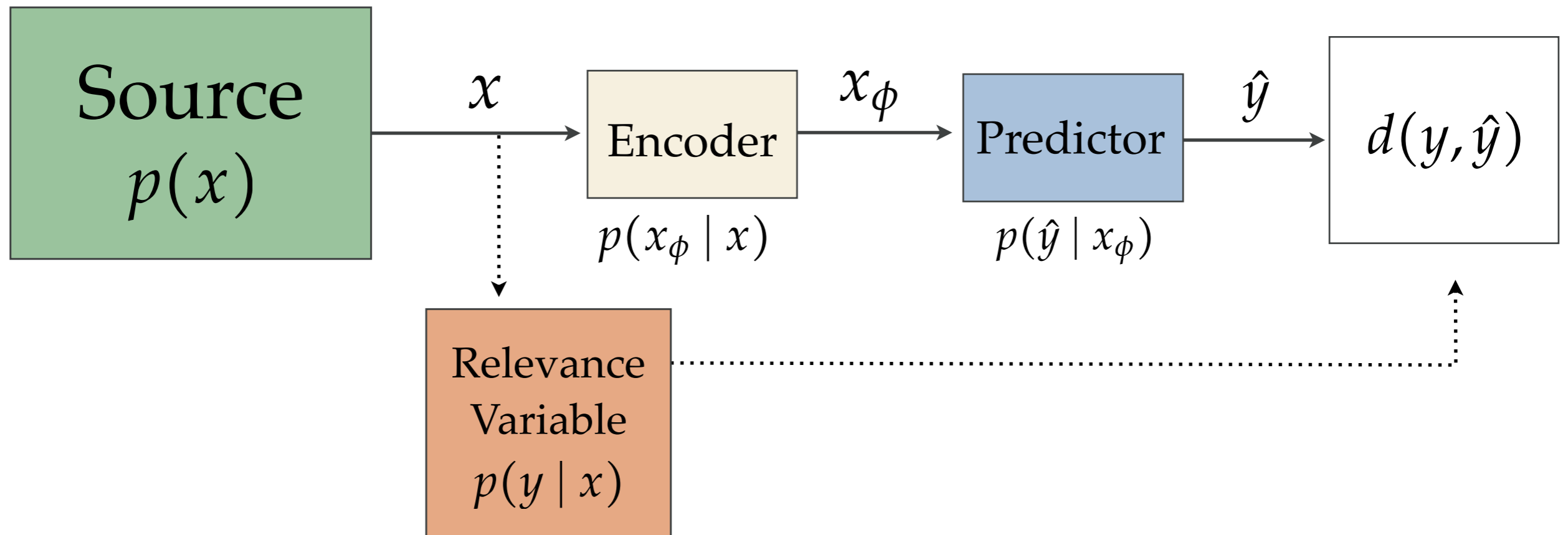
[Shannon '48, Berger '03]

# Rate-Distortion Theory



# Information Bottleneck

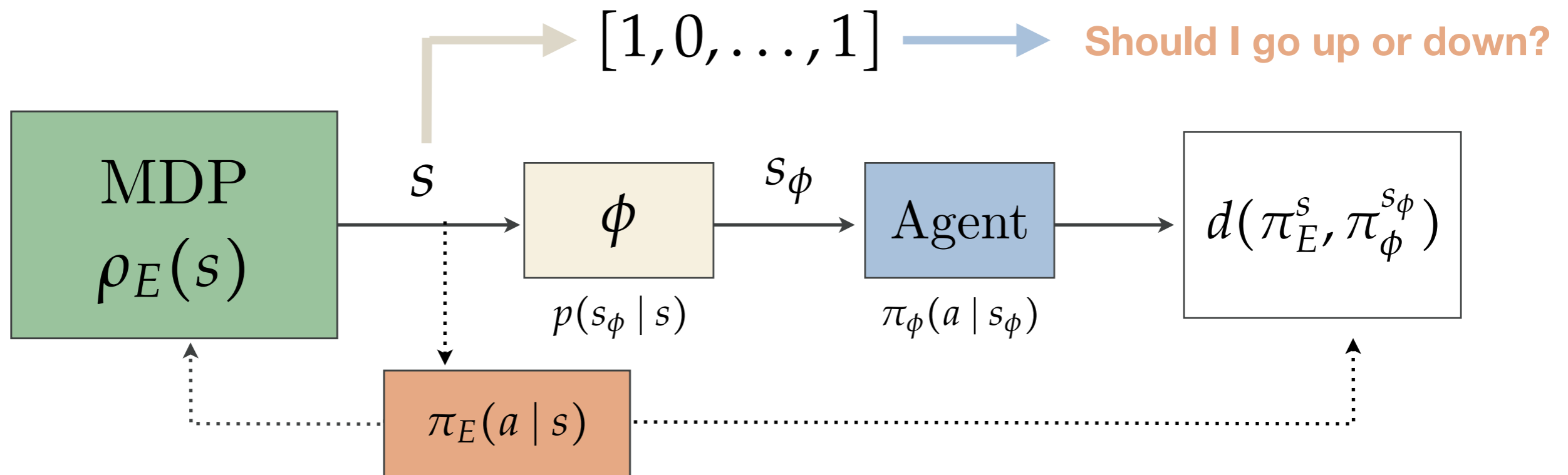
→  $[1, 0, \dots, 1]$  → Dog?  
Boston Terrier?  
Barley?



*[Tishby, Pereira, Bialek '99]*



# State Abstraction as Compression



$$\min_{\phi} \left( |\mathcal{S}_\phi| + \beta \mathbb{E}_{\rho_E(s)} \left[ \underbrace{V^{\pi_E}(s) - V^{\pi_\phi^*}(\phi(s))}_{\text{Value Loss}} \right] \right)$$

*Our Objective*

*Value Loss*

# State Abstraction as Compression

**Theorem.**

$$\min_{\phi} \left( |\mathcal{S}_{\phi}| + \beta \mathbb{E}_{\rho_E(s)} \left[ V^{\pi_E}(s) - V^{\pi_{\phi}^*}(\phi(s)) \right] \right)$$

*Our Objective*

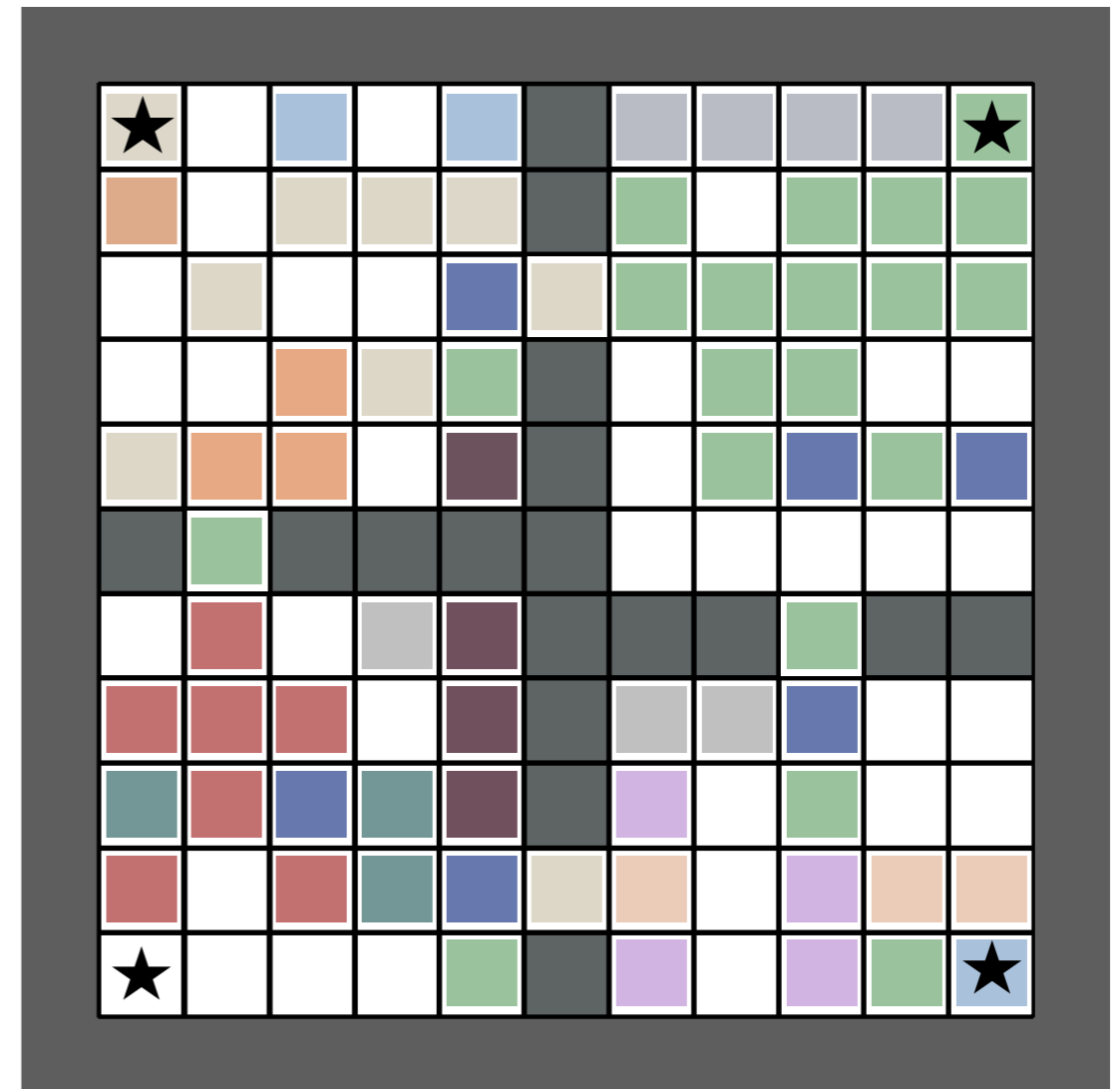
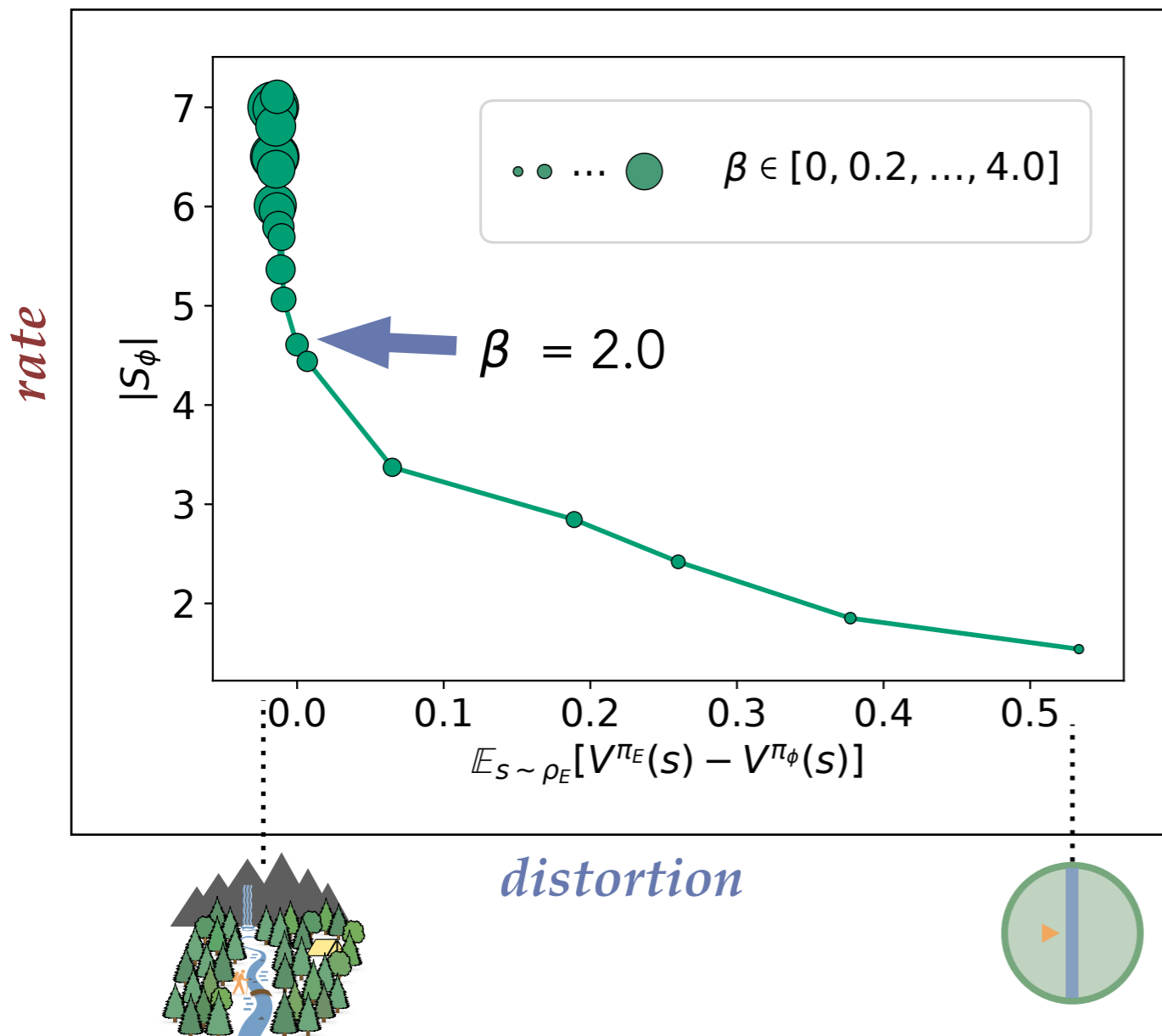
$\leq$

*DIB Objective*

[Strouse & Schwab, '17]

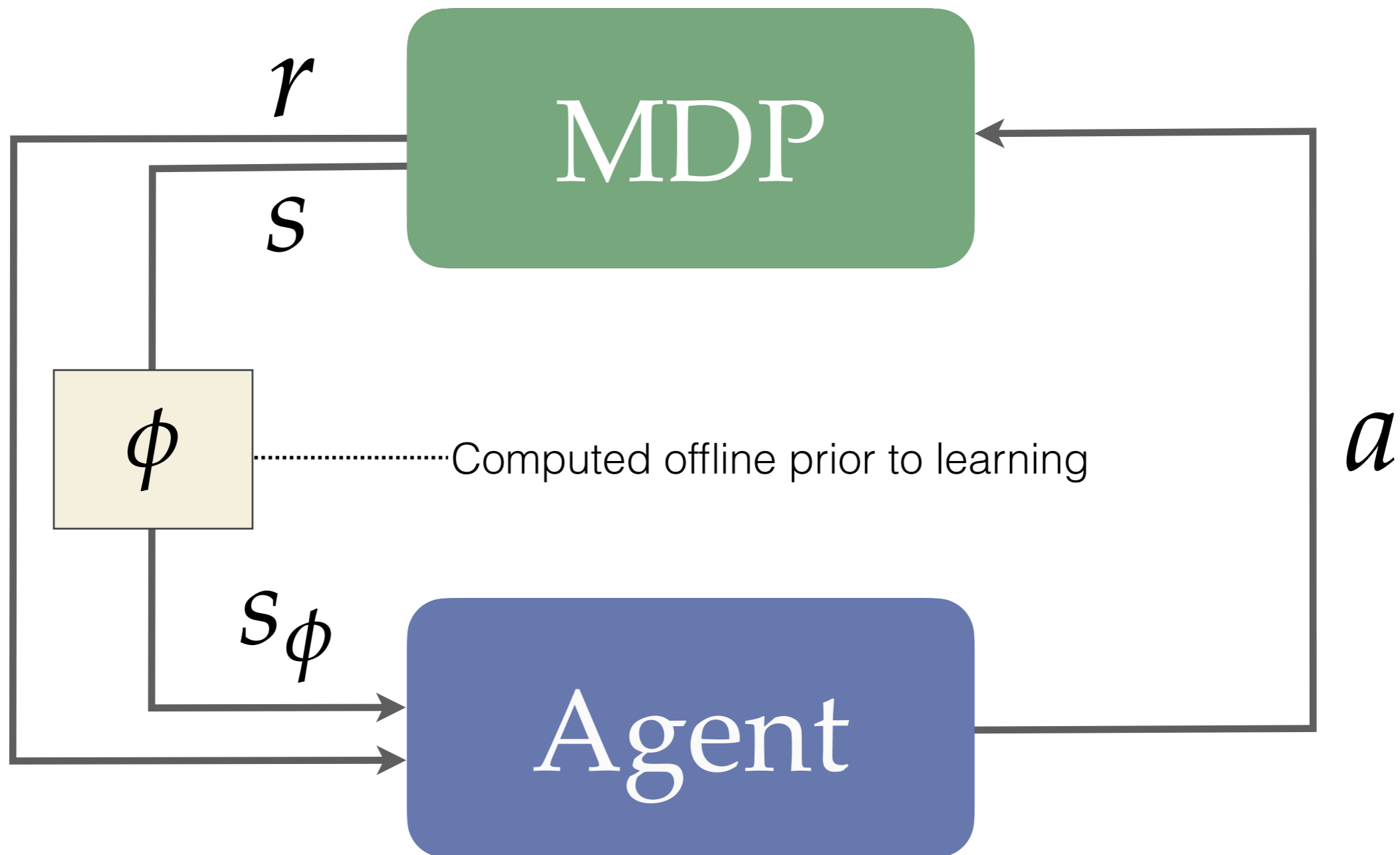
$$\min_{\phi} \left( \frac{H(\rho_{\phi})}{\delta \log \frac{1}{\delta}} + 2VM_{\text{MAX}}\beta \mathbb{E}_{\rho_E(s)} \left[ D_{\text{KL}}(\pi_E(s) \parallel \pi_{\phi}^*(\phi(s))) \right] \right)$$

# State Abstraction as Compression

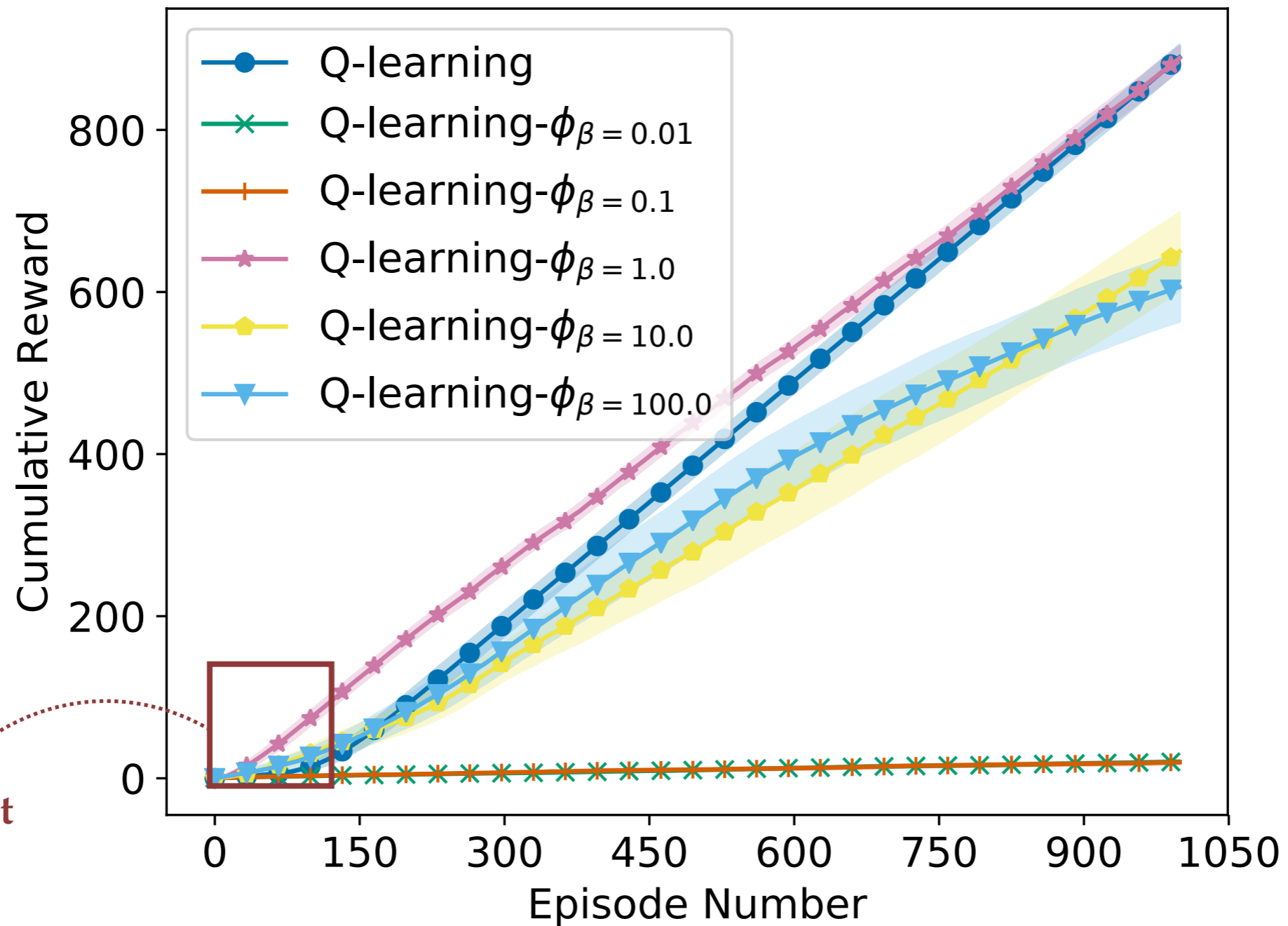


*Multitask Abstraction*

# Learning Experiments

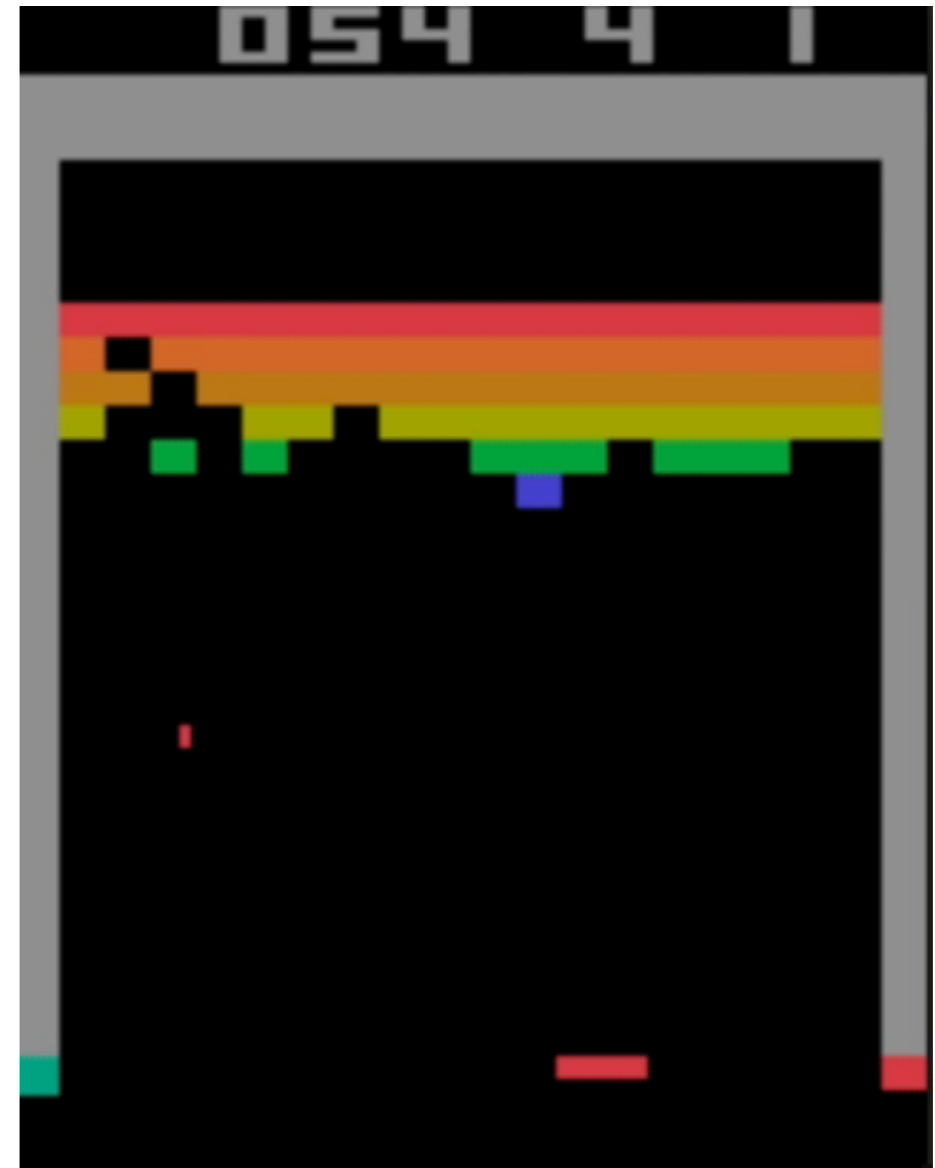
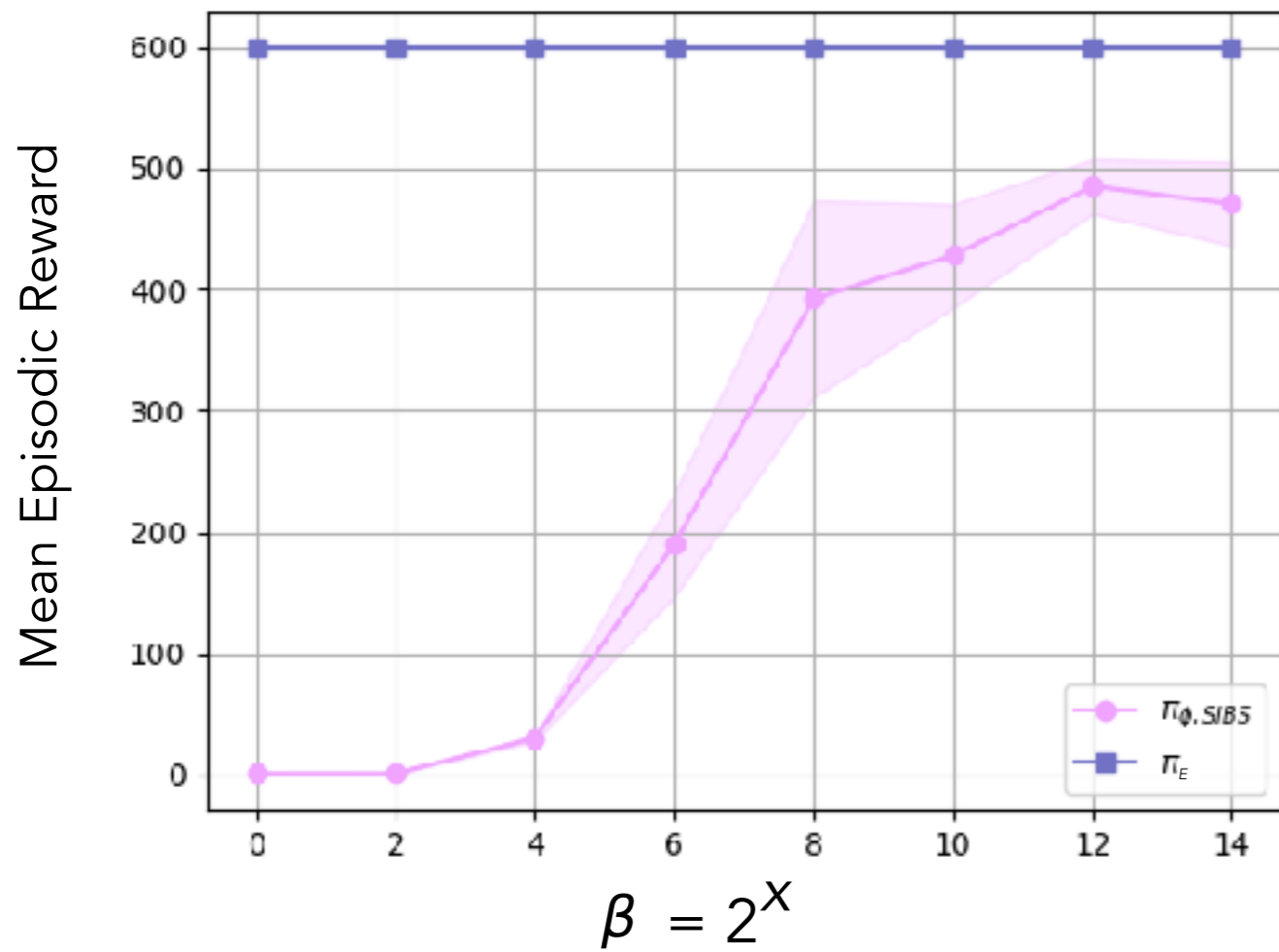


# Experiments: Four Rooms



Previous Plot

# Experiments: Breakout



# Extension: Continuous State

**Theorem.** For any  $\delta \in (0, 1)$ ,  $n$  the size of the training data set,  $\Delta \in \mathbb{R}$  training loss, and  $\rho$  a fixed distribution on states used to train  $\tilde{\phi} \in \Phi$ , with probability at least  $1 - \delta$ :

$$\mathbb{E}_{s \sim \rho} \left[ \left\| \left( \pi^*(\cdot | s) - \pi_{\tilde{\phi}}(\cdot | s) \right) \right\|_1 \right] \leq \frac{\Delta}{2} + 2\sqrt{2} \text{Rad}(\Phi) + \sqrt{\frac{2 \ln \frac{1}{\delta}}{n}}$$

[Barlett, Mendelson '02]

*training error*

*hypothesis  
class richness*

*size of  
training data*

*led  
project* →

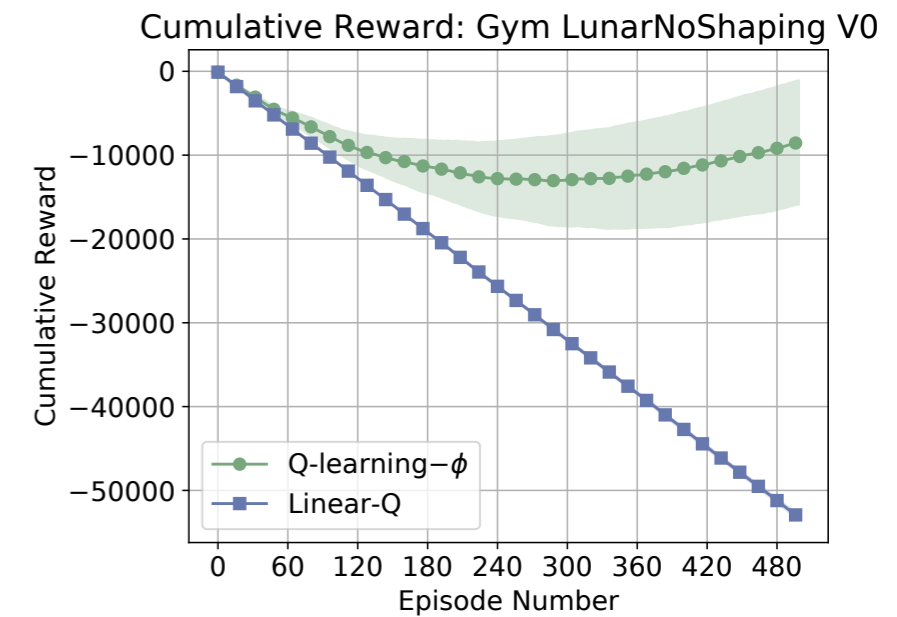
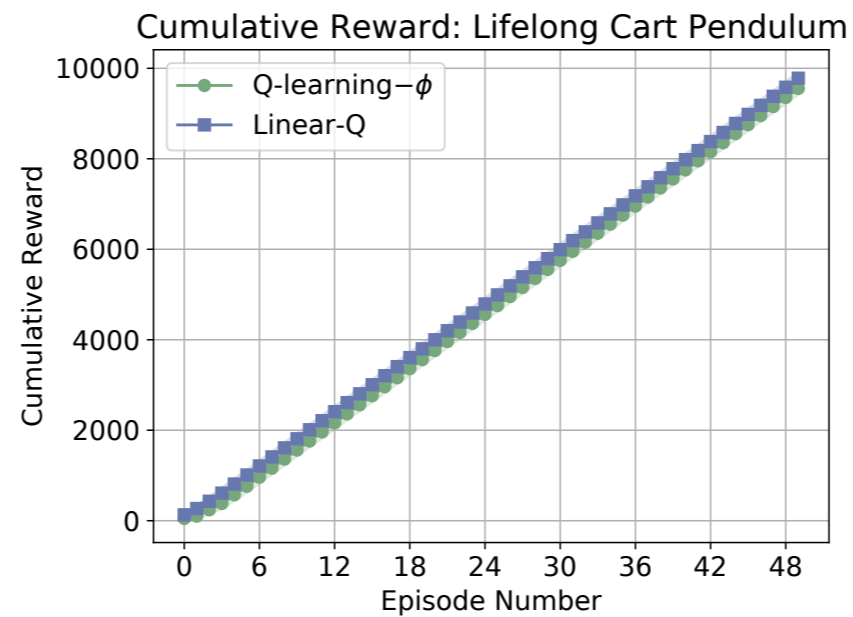
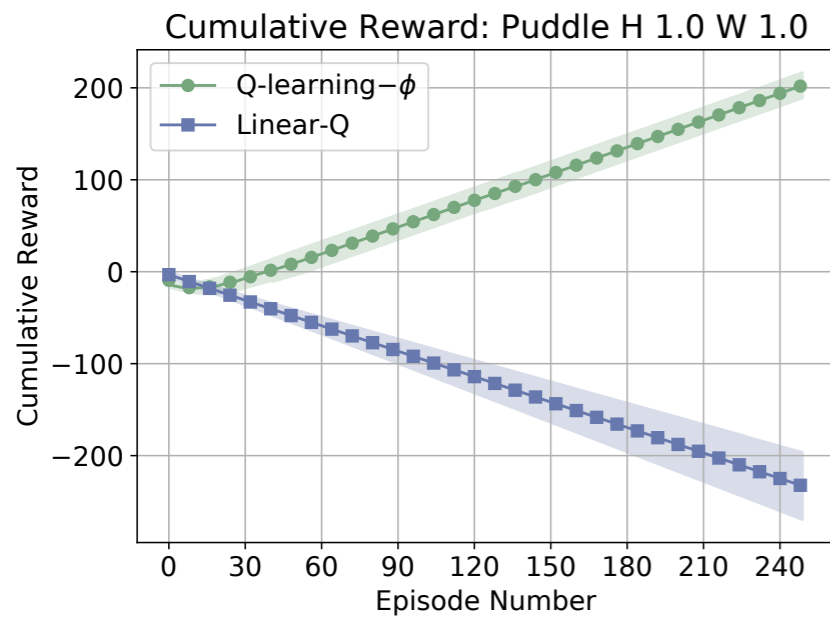
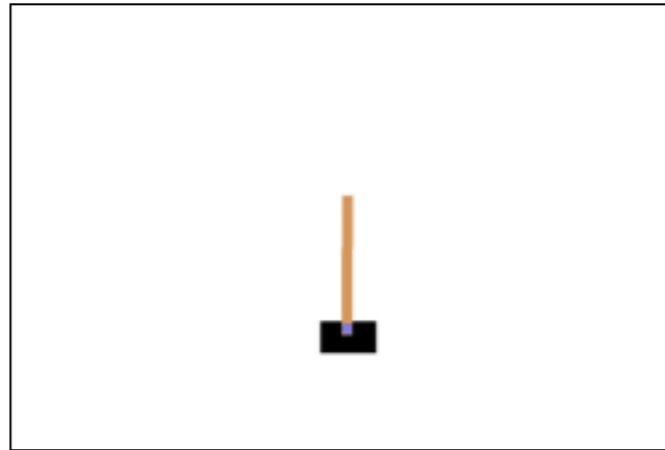
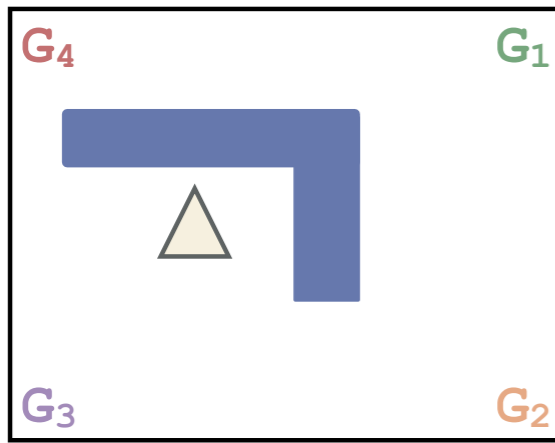


Kavosh  
Asadi



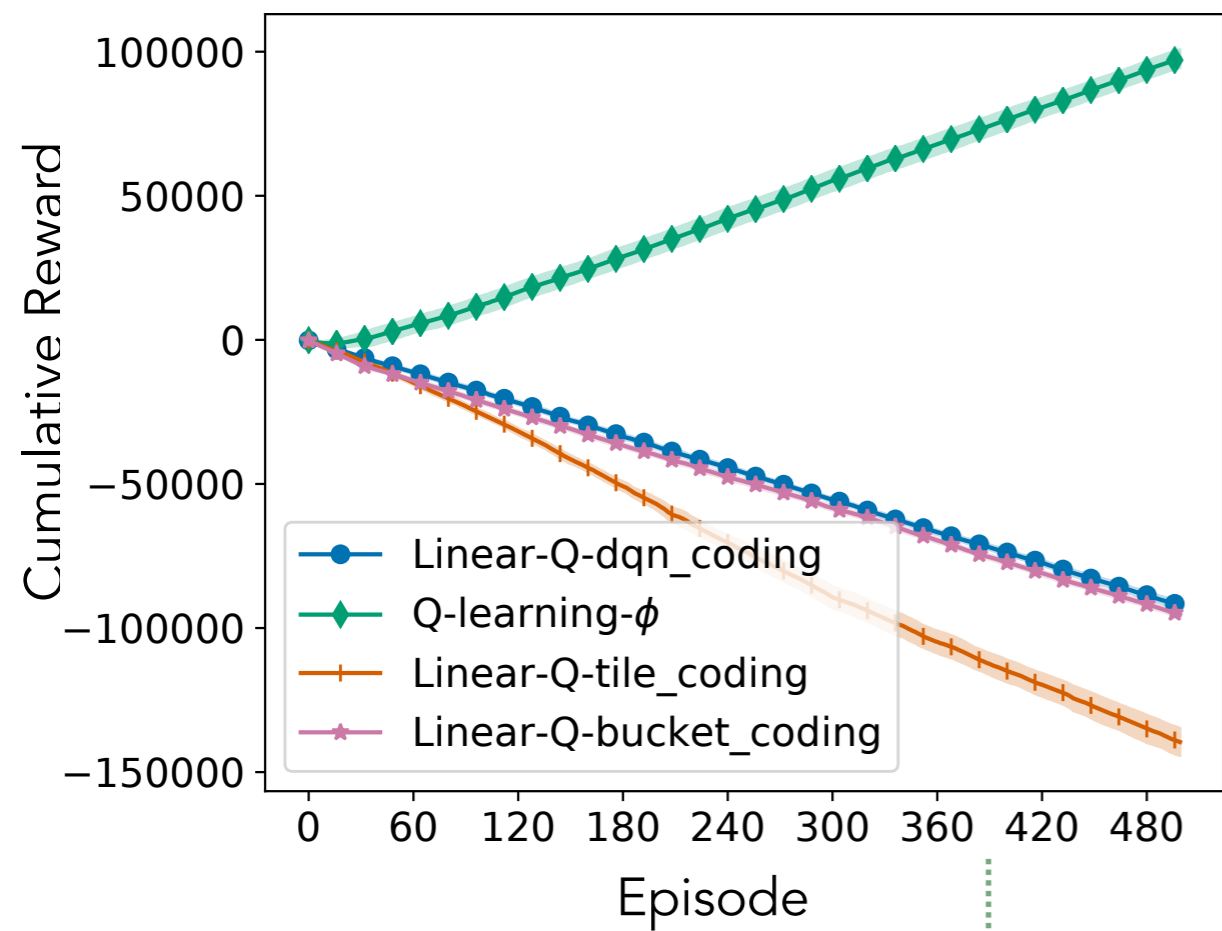
Michael L.  
Littman

# Extension: Continuous State

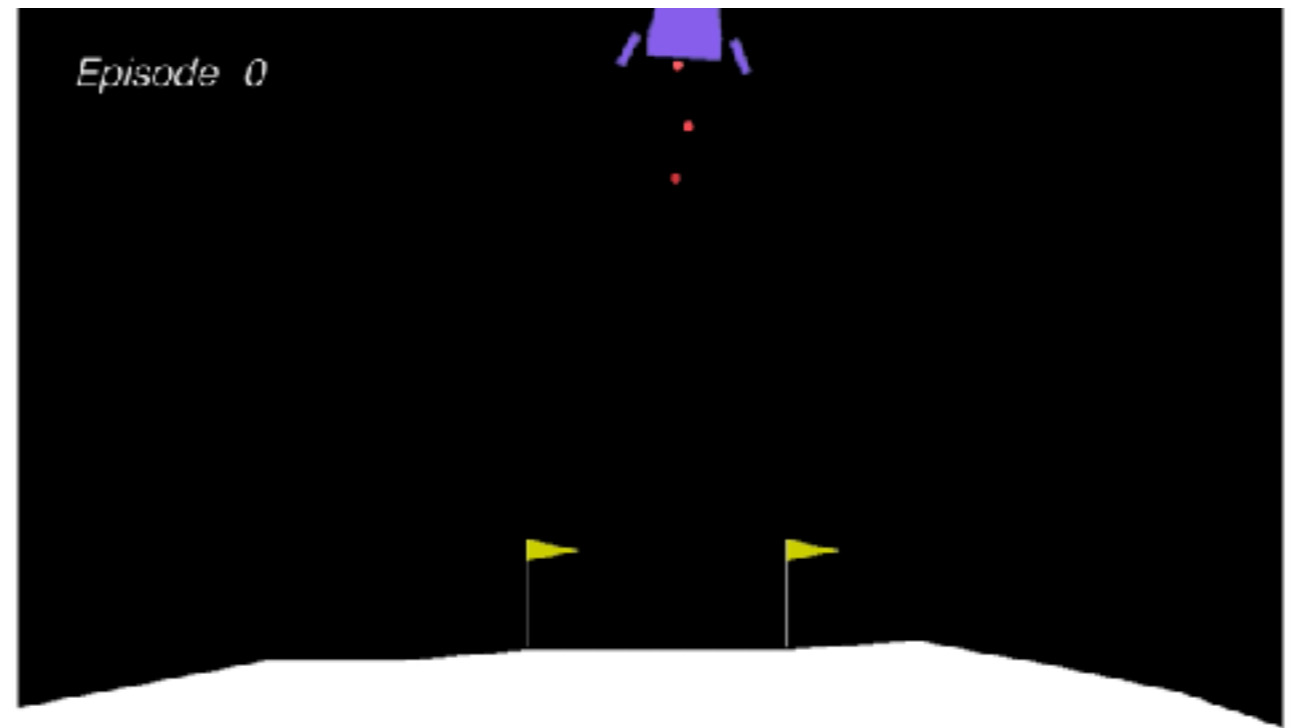




# Experiments: Lunar Lander



[Sutton '96]



Tabular Q-Learning with  $\phi$

## Part 1

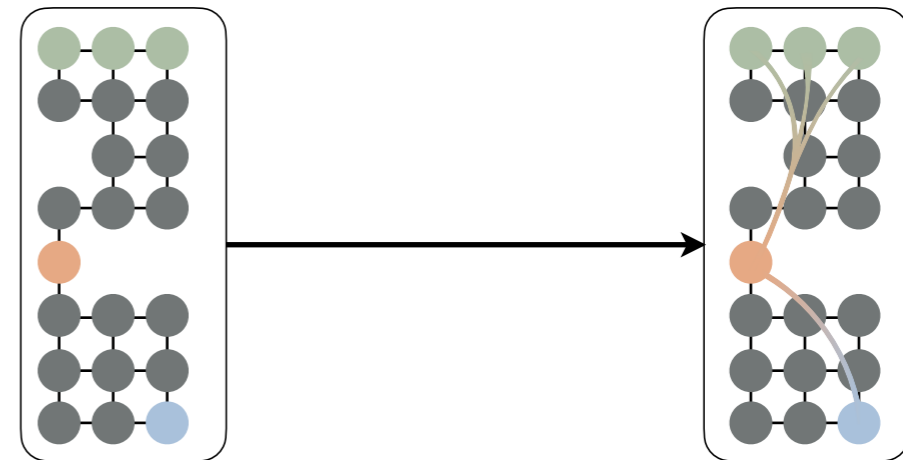
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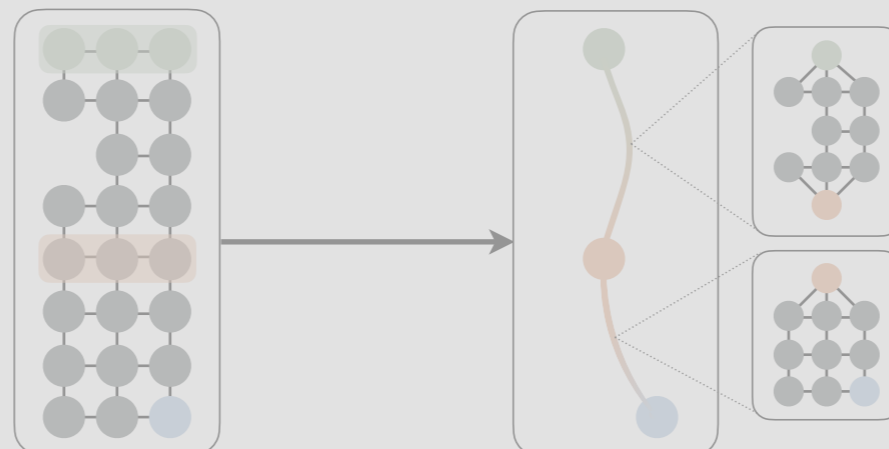
### ACTION ABSTRACTION



4. Options for Planning  
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*IJCAI 2019*

## Part 3

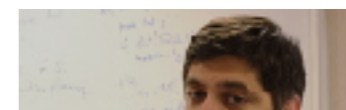
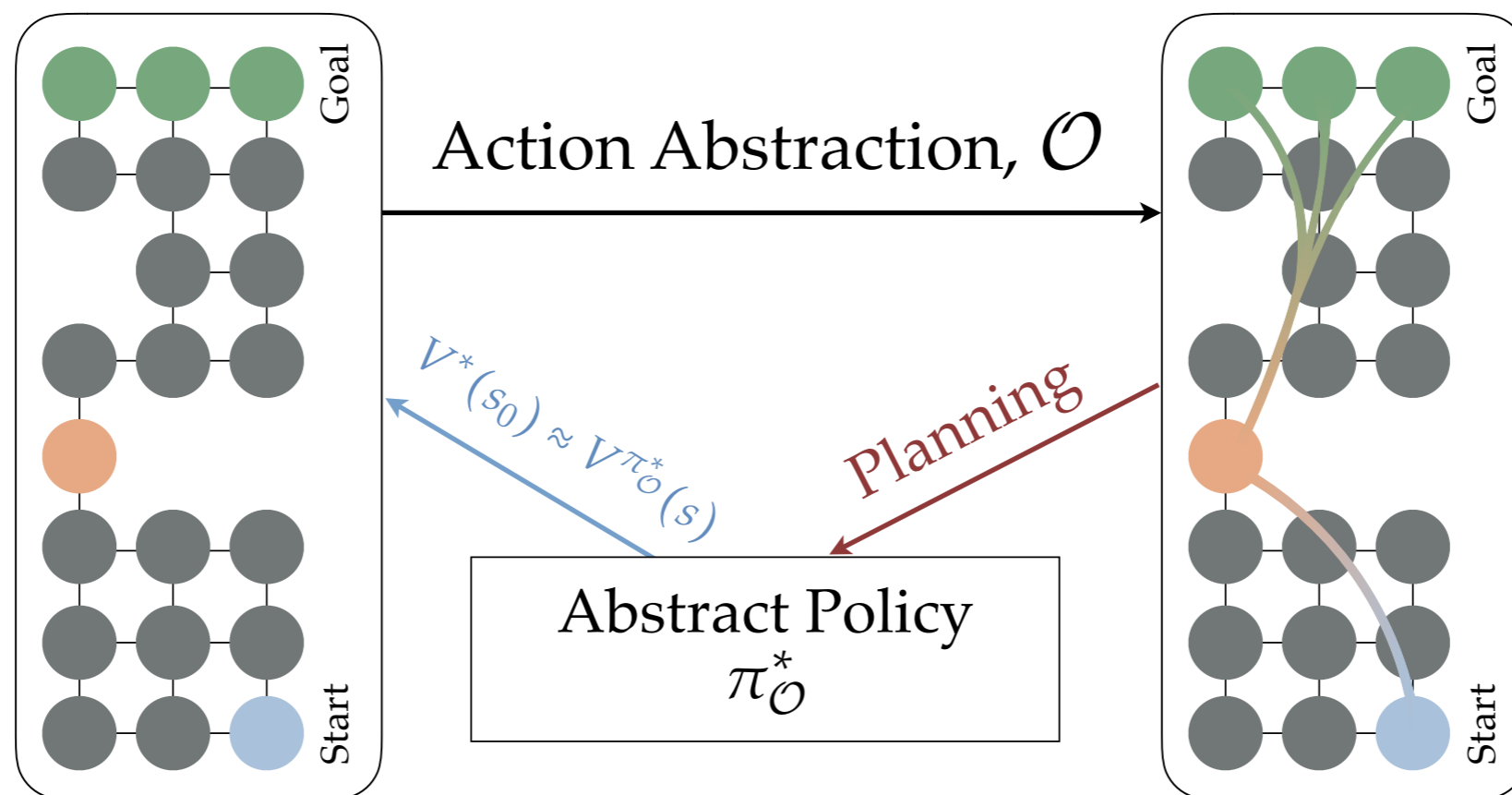
### STATE-ACTION ABSTRACTION



7. Value-Preserving Hierarchies  
*AISTATS 2020*

# Options for Planning

[JAHLK, ICML 2019]



**Question:** How can we find the set of options that make planning as fast as possible?

project



Jinnai

Hershkowitz

Littman

Konidaris

# Options for Planning

[JAHLK, ICML 2019]

**Theorem.** Finding the set of options that minimizes planning time is:

1) NP-hard in general.

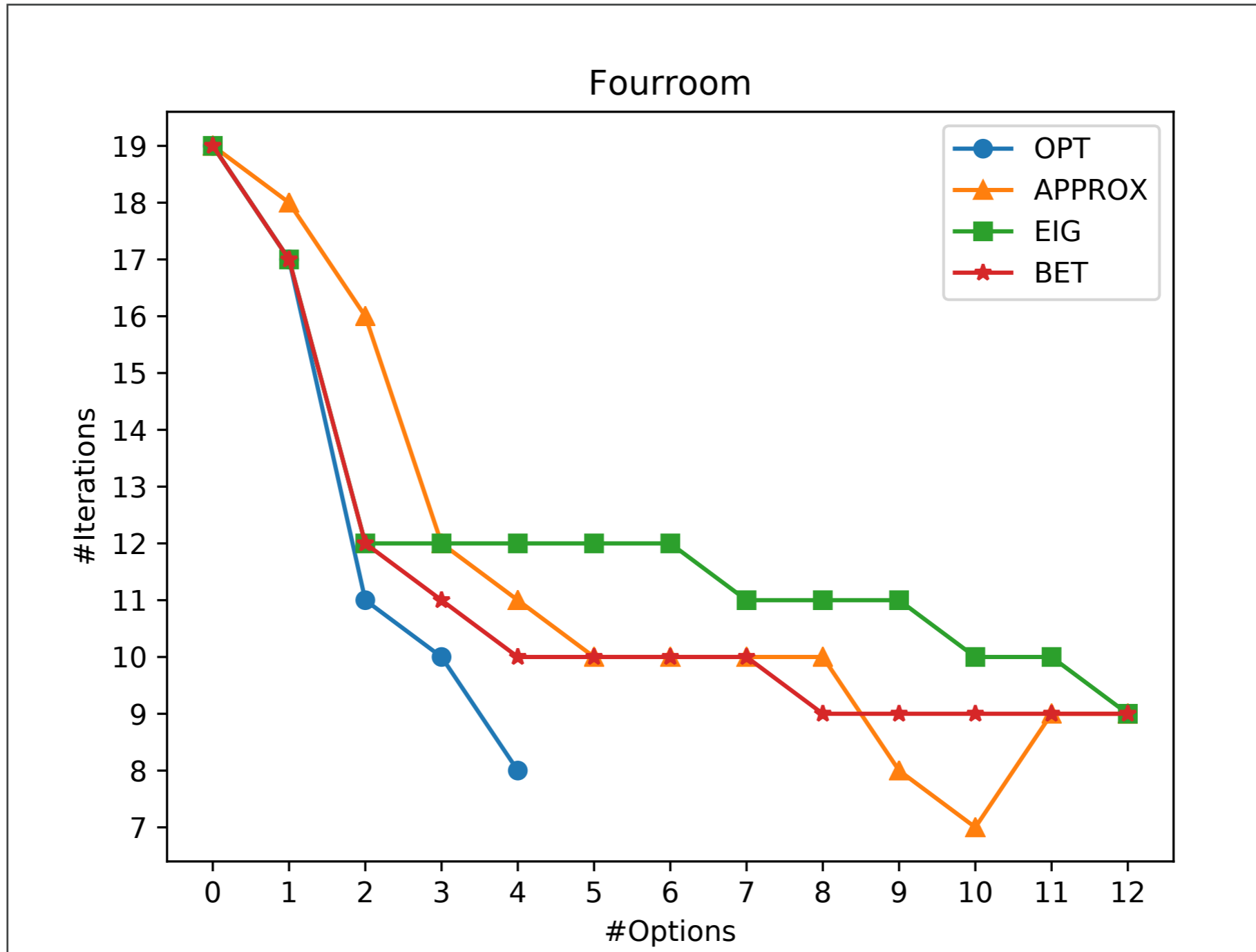
2)  $2^{\log^{1-\varepsilon} n}$ -hard to approximate.<sup>1</sup>

<sup>1</sup>Unless  $\text{NP} \subseteq \text{DTIME}(n^{\text{poly log } n})$  [Dinitz et al. 2012]

**Question:** How can we find the set of options that make **planning** as **fast** as possible?

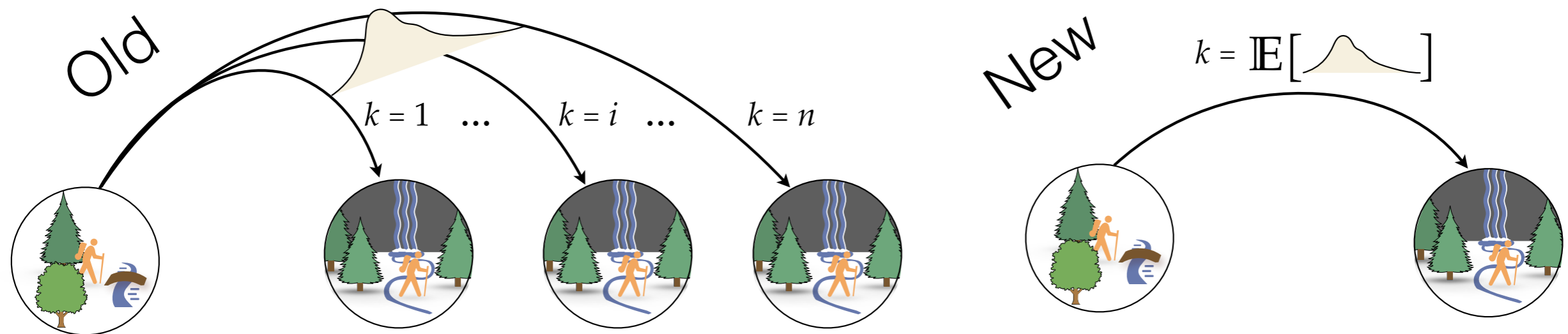
# Empirical Evaluation

[JAHLK, ICML 2019]



# A New Option Model

[AWdL, IJCAI 2019]



*jointly led project* →



John Winder



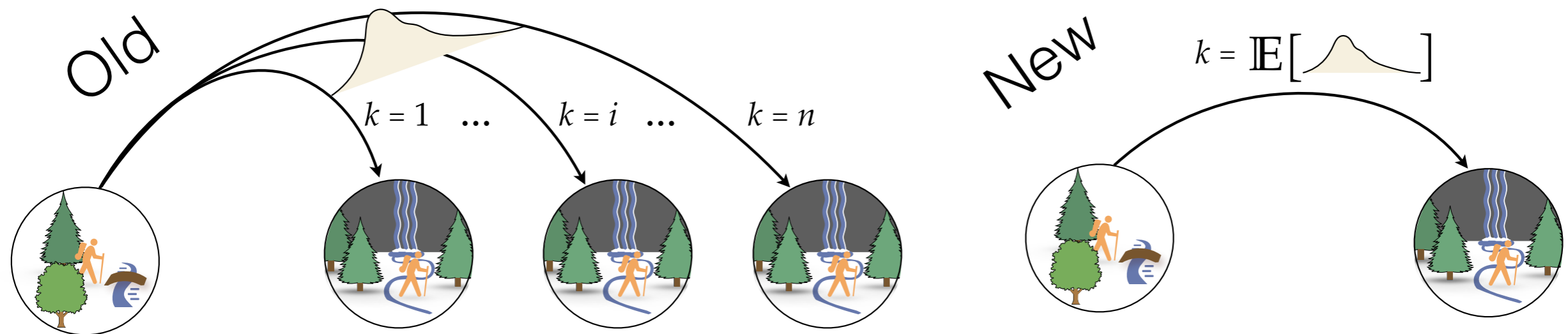
Marie desJardins



Michael L. Littman

# A New Option Model

[AWdL, IJCAI 2019]



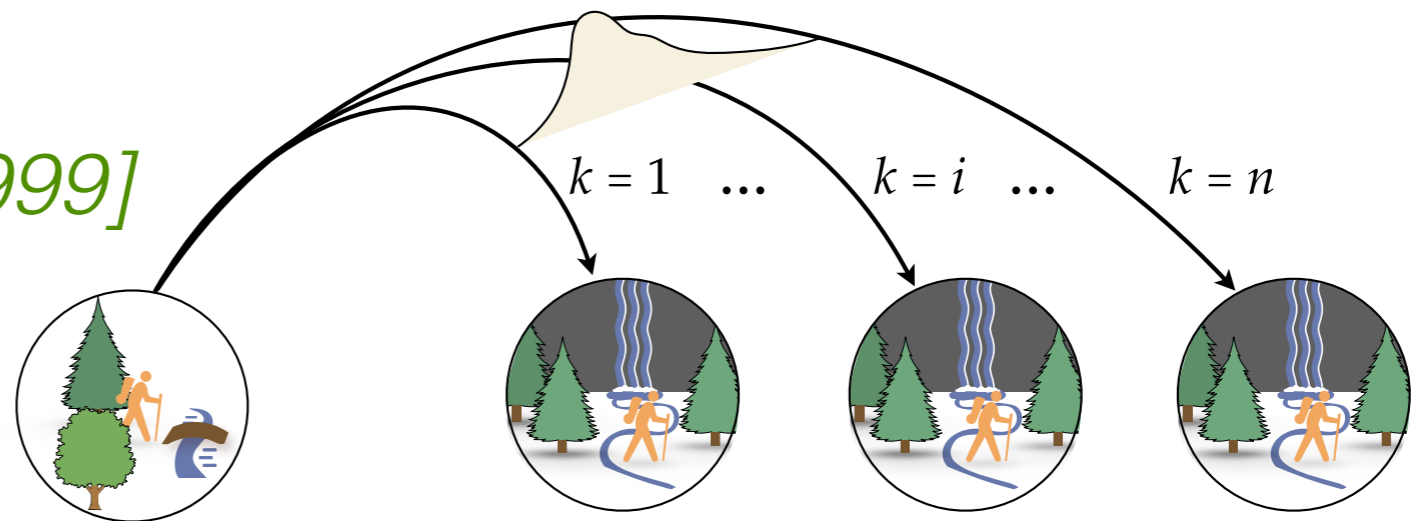
**Question:** How can we efficiently estimate the transition and reward models of options?

# A New Option Model

[AWdL, IJCAI 2019]

## Multi-Time Model

[Sutton, Precup, Singh 1999]



$$T_{\gamma}(s' | s, o) := \sum_{k=0}^{\infty} \gamma^k \beta(s_k) \mathbb{P}(s_k = s' | s, o)$$

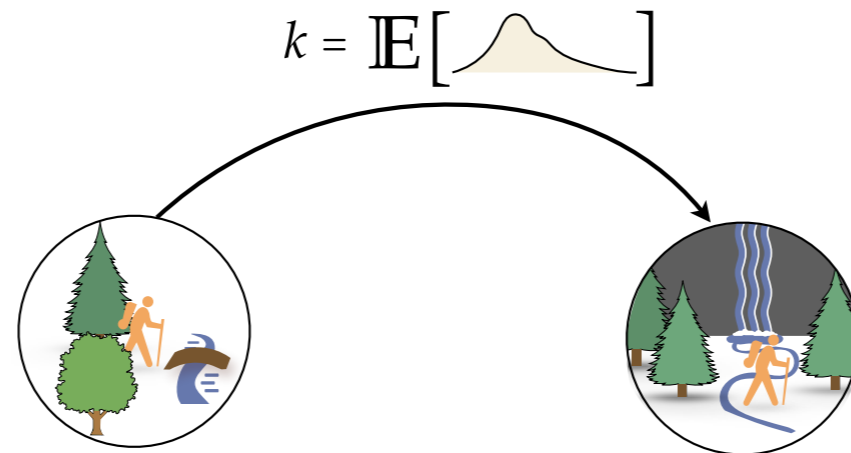
$$R_{\gamma}(s, o) := \mathbb{E}_{k, s_1 \dots k} \left[ r_1 + \gamma r_2 \dots + \gamma^{k-1} r_k \mid s, o \right]$$



# A New Option Model

[AWdL, IJCAI 2019]

## ***Expected Length Model***



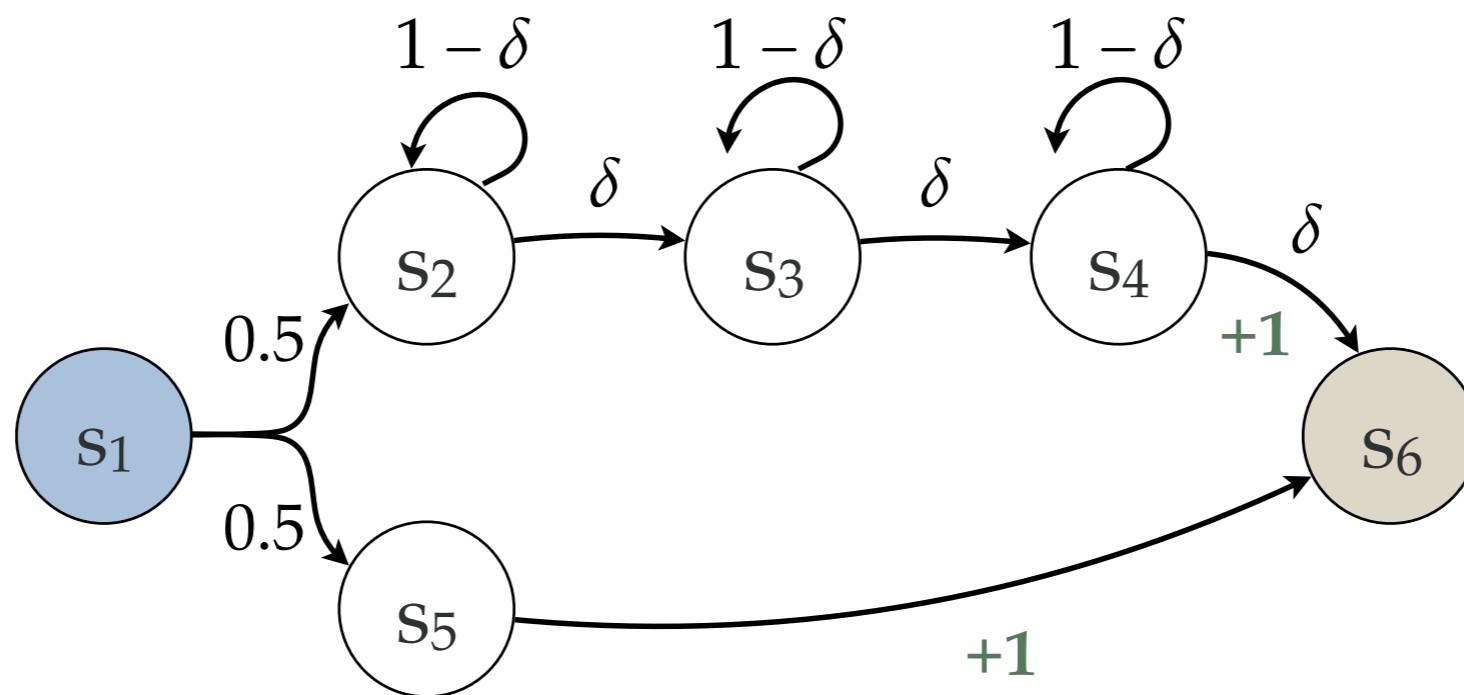
$$T_{\mu_k}(s' | s, o) := \gamma^{\mu_k} p(s' | s, o),$$

$$R_{\mu_k}(s, o, s') := \gamma^{\mu_k} \mathbb{E} [r_1 + r_2 \dots + r_{\mu_k} | s, o],$$

where  $\mu_k = \mathbb{E}[k | s, o]$ .

# A New Option Model

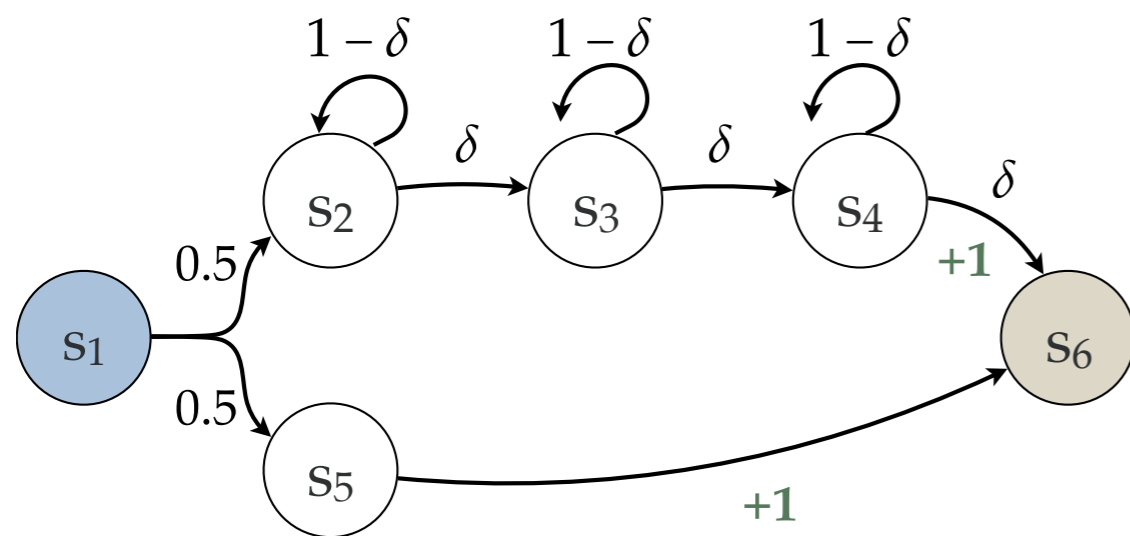
[AWdL, IJCAI 2019]



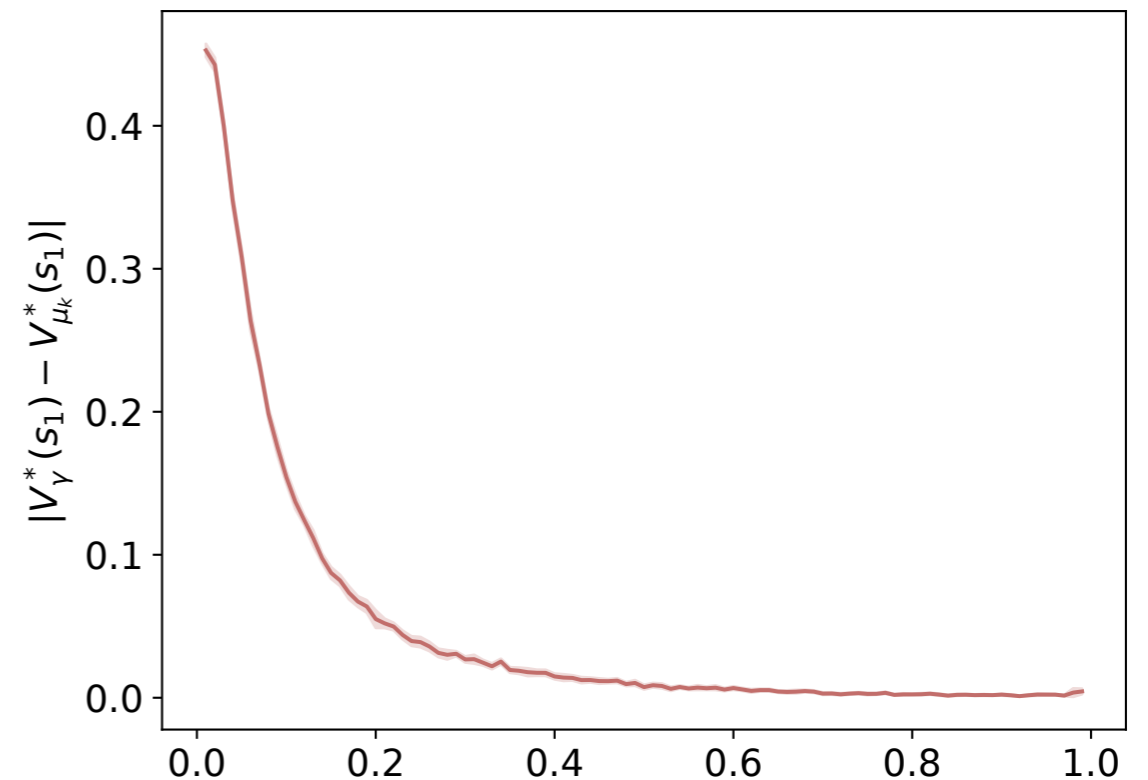
$$\mathbb{P}(s_6, k \mid s, 0)$$

# A New Option Model

[AWdL, IJCAI 2019]



## Value Difference



*more  
stochastic*

$\delta$

*less  
stochastic*

$$\mathbb{P}(s_6, k \mid s, 0)$$

# A New Option Model

[AWdL, IJCAI 2019]

**Lemma.** *There exists a  $\tau \geq 1$  such that*

$$|T_\gamma(s' | s, o) - T_{\mu_k}(s' | s, o)| \leq \gamma^{\mu_{k,o} - \tau} (2\tau + 1) e^{-\beta_{\min}}.$$

**Lemma.** *In stochastic shortest path MDPs,*

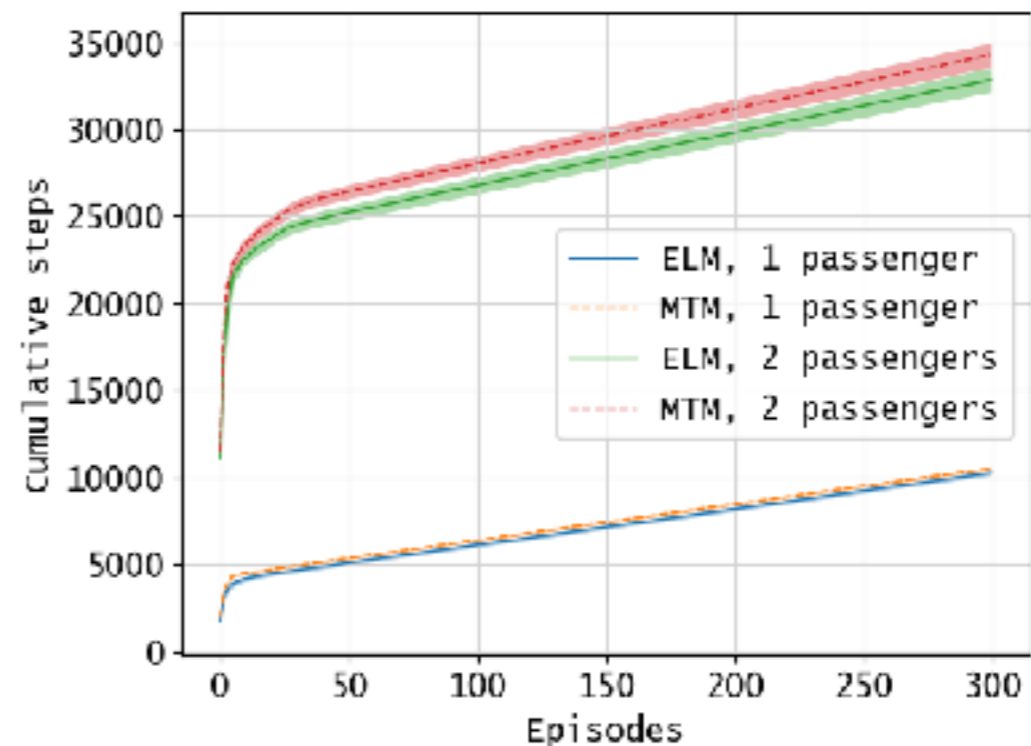
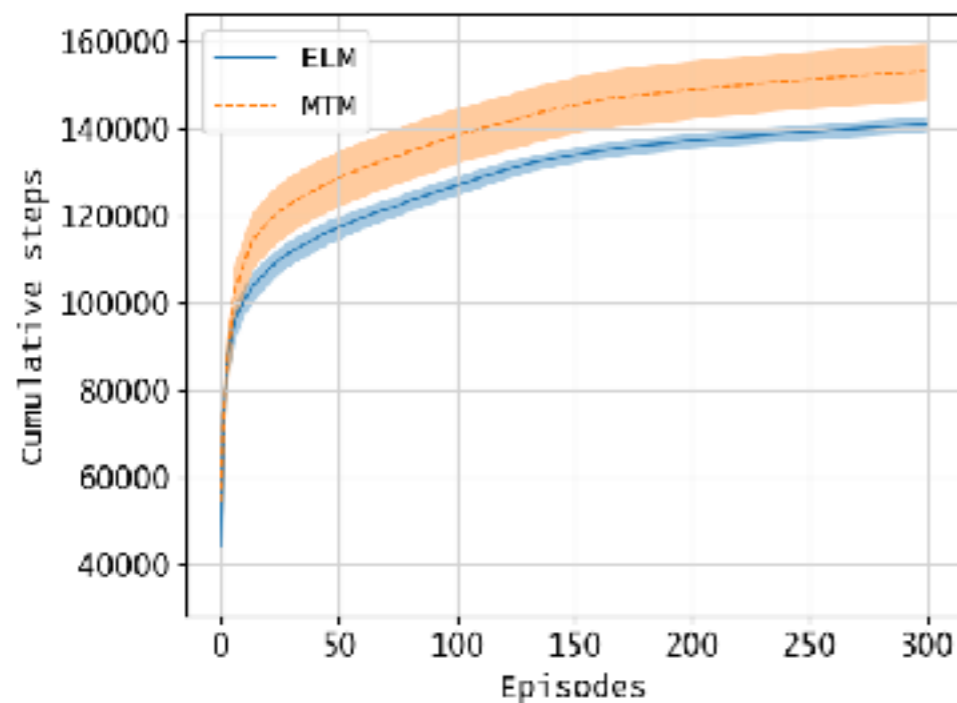
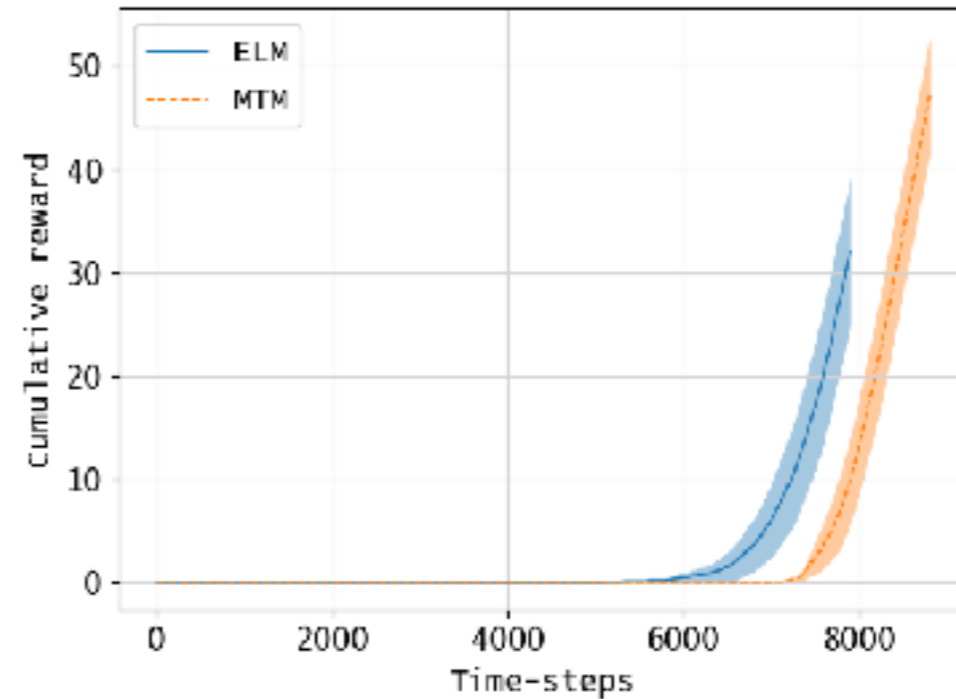
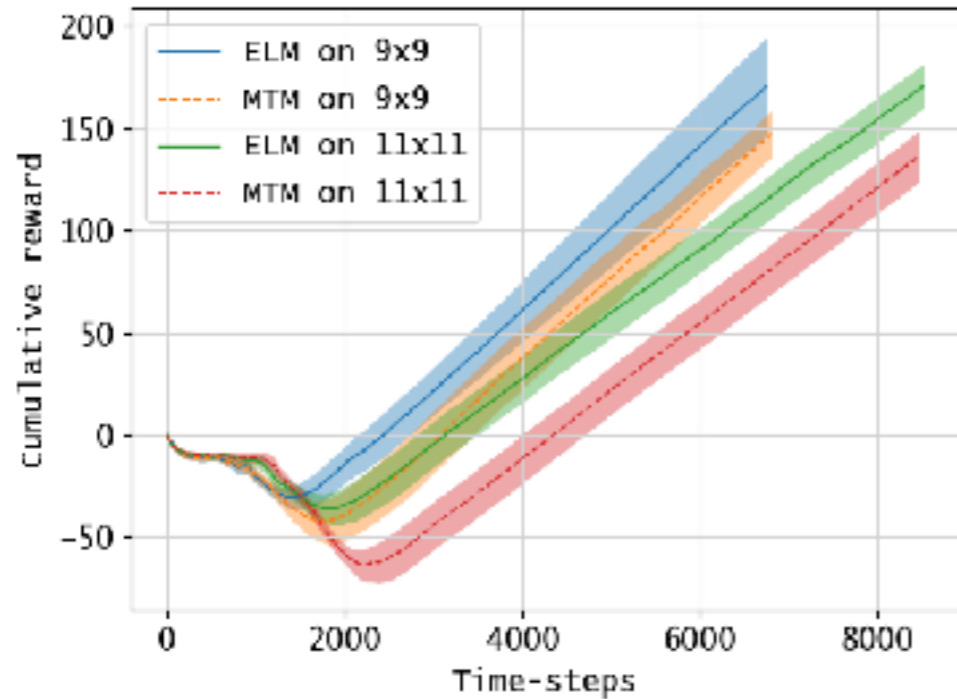
$$|R_\gamma(s, o) - R_{\mu_k}(s, o)| = |T_\gamma(s_g | s, o) - T_{\mu_k}(s_g | s, o)|.$$

**Theorem.** *In stochastic shortest path MDPs,*

$$|V_\gamma^{\pi_o}(s) - V_{\mu_k}^{\pi_o}(s)| \leq \frac{\varepsilon(1 - \gamma^{\mu_k}) + \gamma^{\mu_k} \frac{\varepsilon}{2} \text{RM}_{\text{MAX}}}{(1 - \gamma^{\mu_k})(1 - \gamma^{\mu_k} + \frac{\varepsilon}{2} \gamma^{\mu_k})}.$$

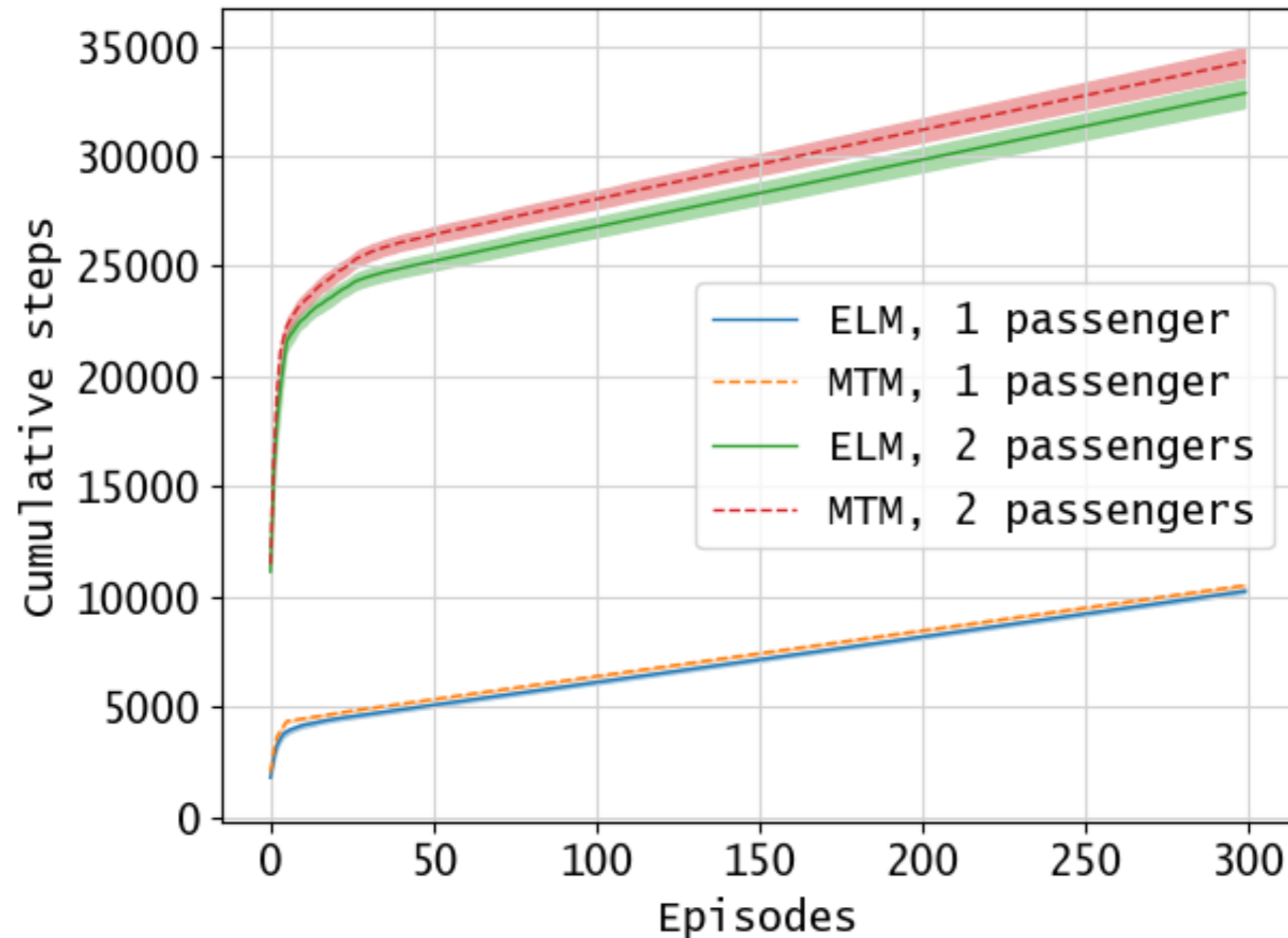
# A New Option Model

[AWdL, IJCAI 2019]



# A New Option Model

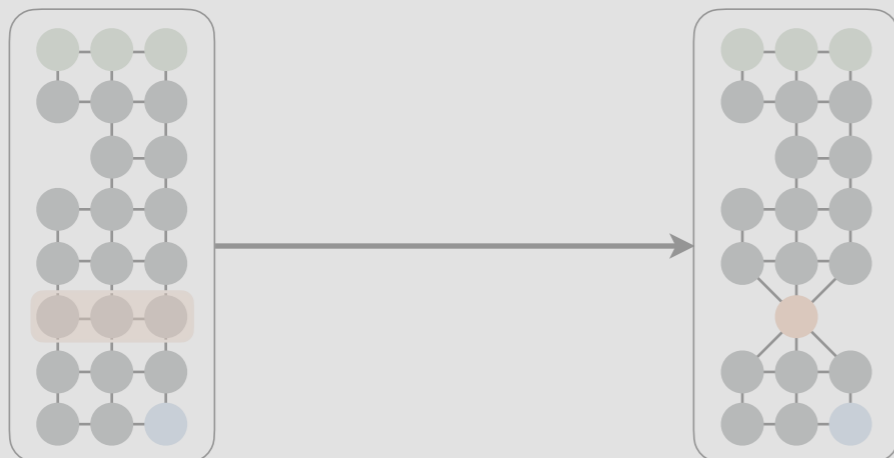
[AWdL, IJCAI 2019]



*Taxi Domain*  
[Dietterich '00]

## Part 1

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*AAAI 2019*

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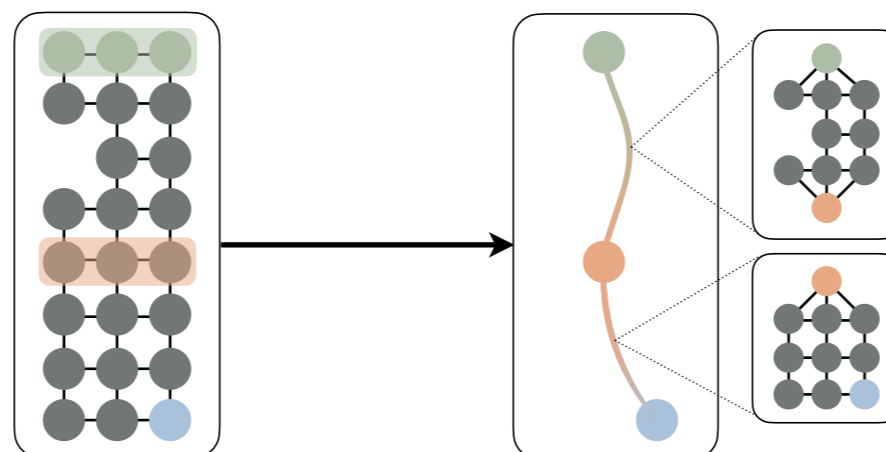
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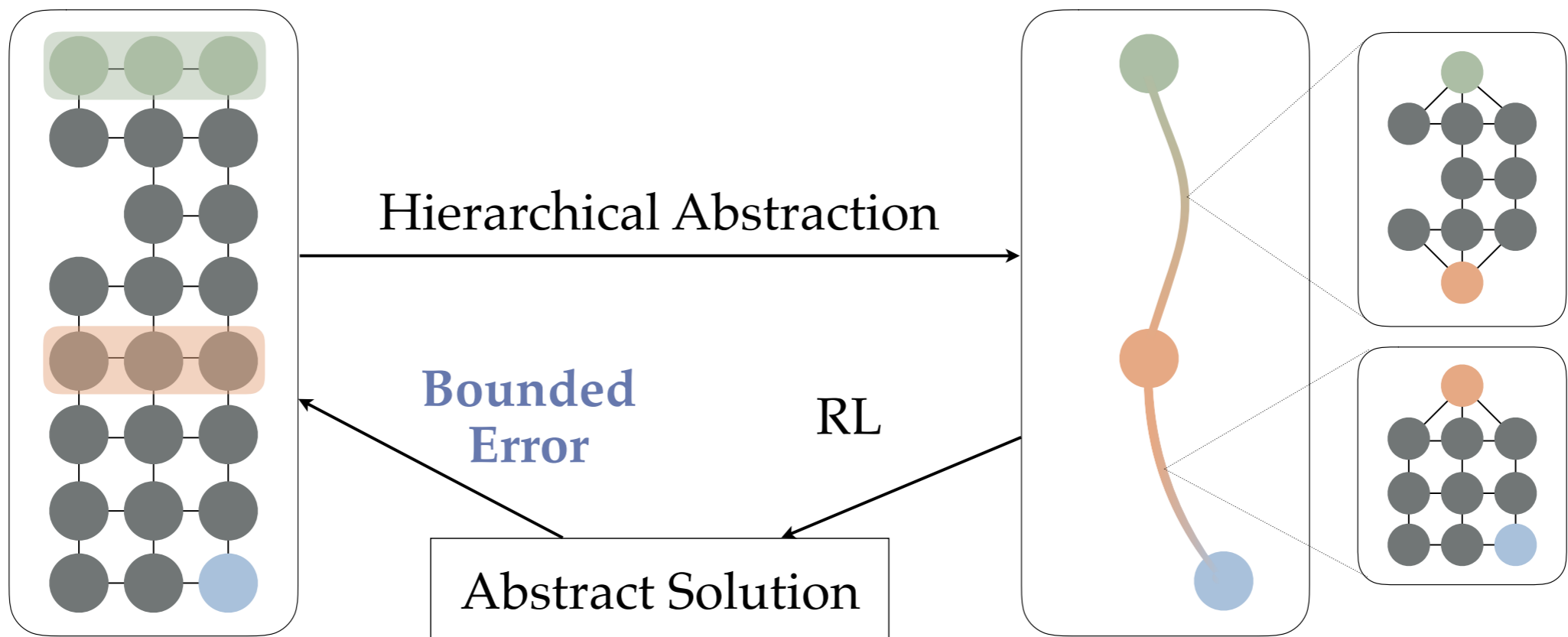
### STATE-ACTION ABSTRACTION



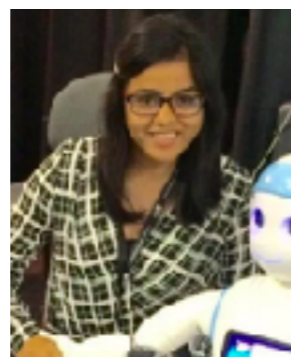
7. Value-Preserving Hierarchies  
*AISTATS 2020*

# Value-Preserving Hierarchies

[**AUKAPL**, AISTATS 2020]



Nathan  
Umbanhowar



Khimya  
Khetarpal



Dilip  
Arumugam



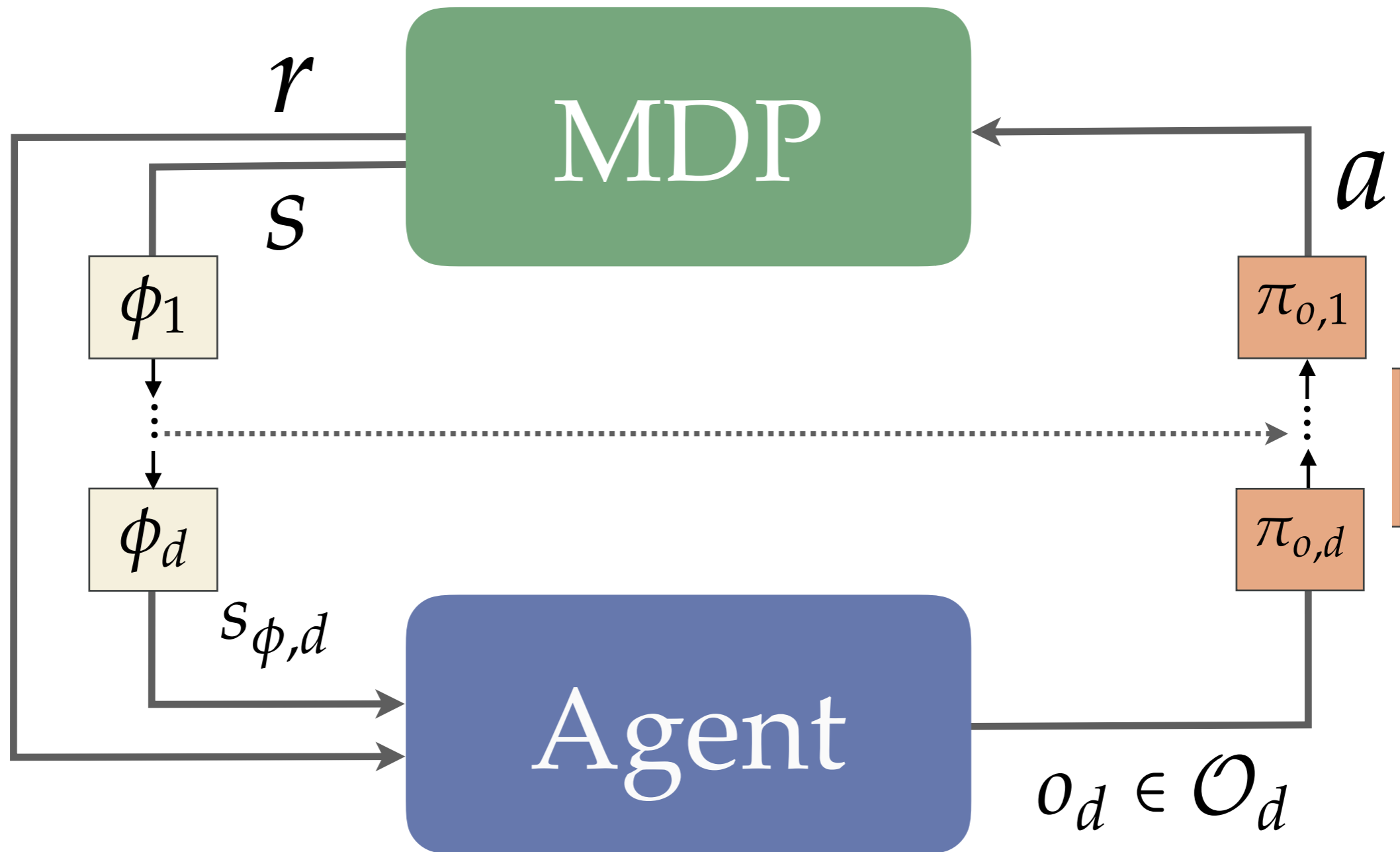
Doina  
Precup



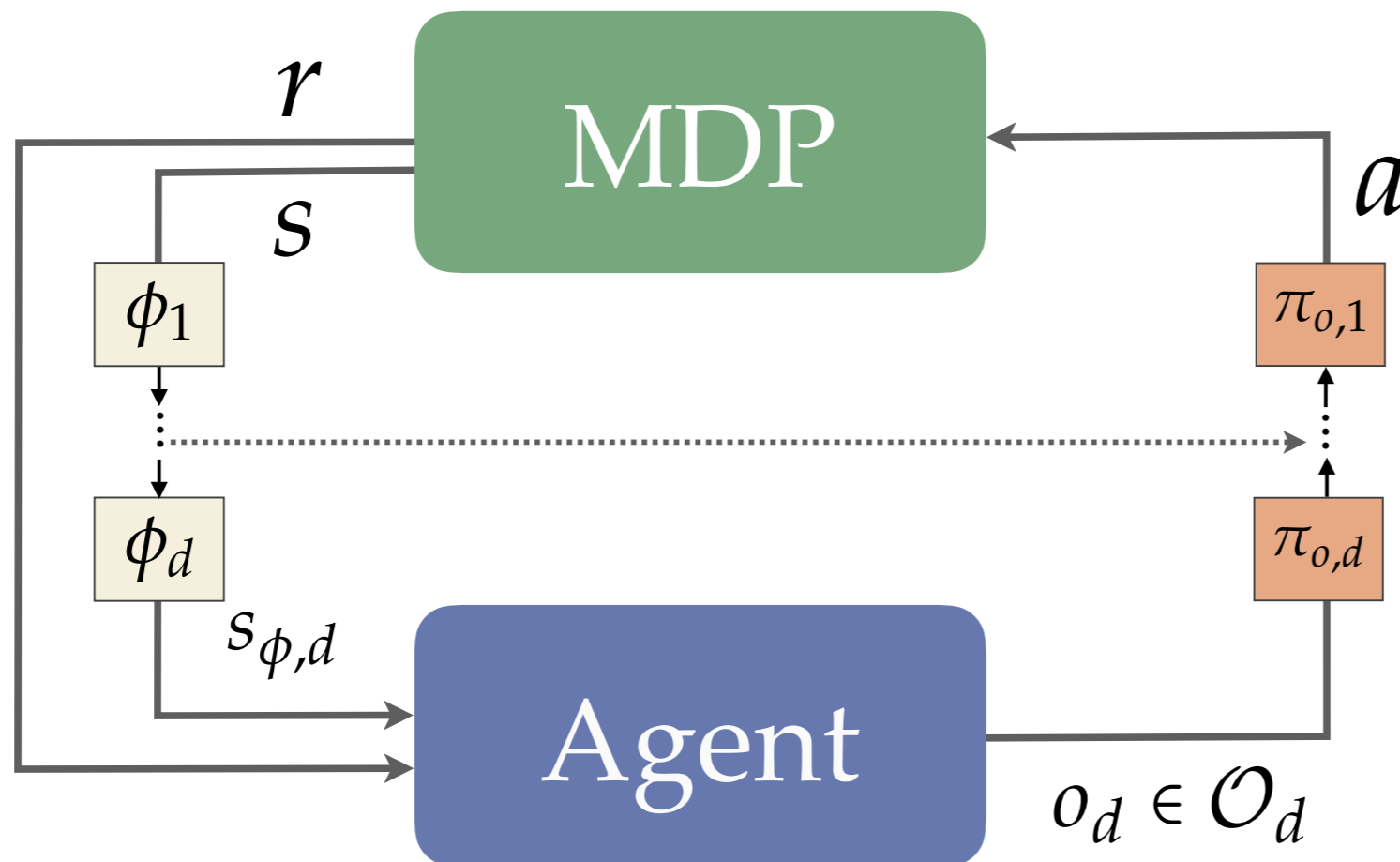
Michael L.  
Littman



# Hierarchical RL



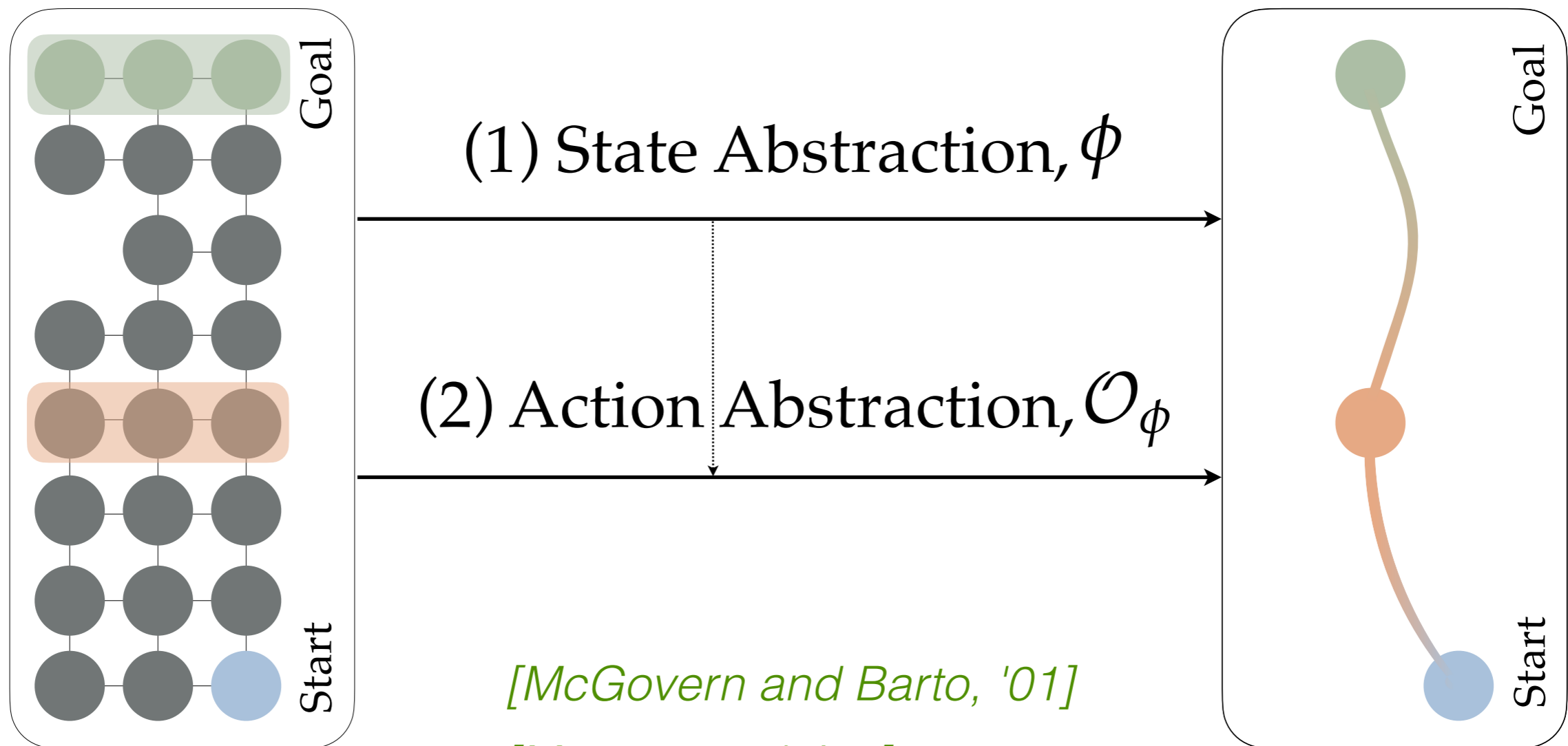
# Value-Preserving Hierarchical RL



[Ravindran, Barto '03, '04]  
[Majeed & Hutter '19]

**Question:** Which combinations of state abstractions and options preserve representation of good behavior?

# $\phi$ -Relative Options

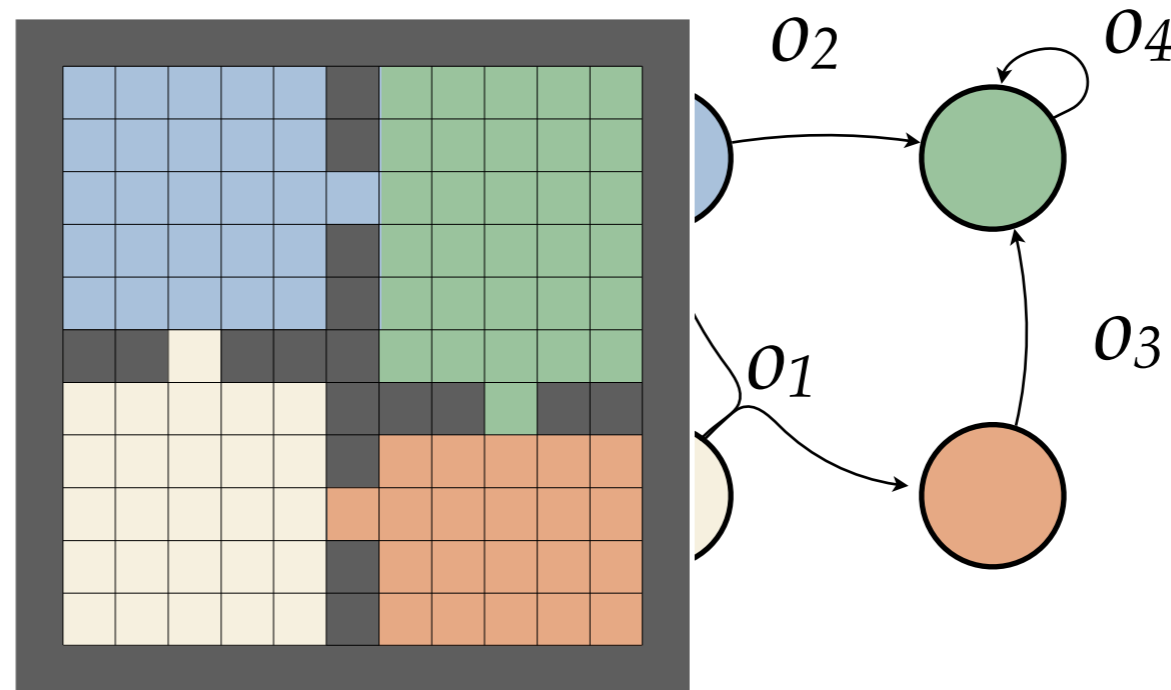


*[McGovern and Barto, '01]*

*[Mannor et al. '04]*

*[Provost et al. '06]*

# $\phi$ -Relative Options



Options must respect the abstract state boundaries.  
Given  $\phi$ .

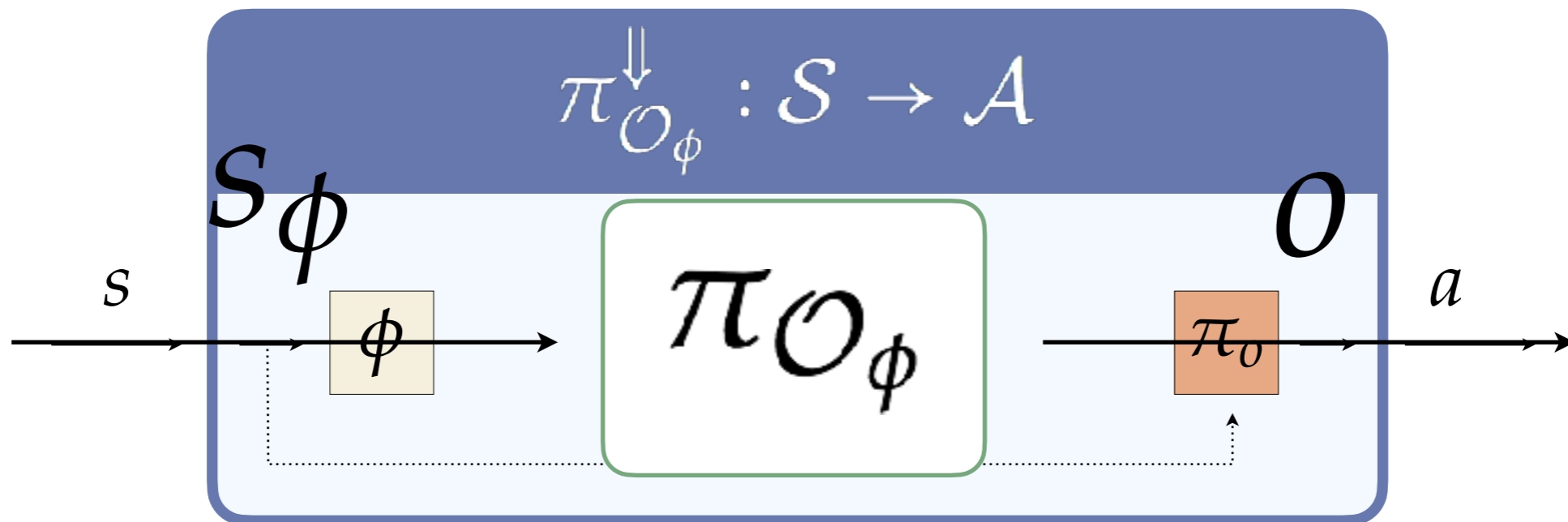
**Definition.** A set of options is said to be  $\phi$ -relative, denoted  $\mathcal{O}_\phi$ , if:

1. Each  $o \in \mathcal{O}_\phi$  initiates in some  $s_\phi$ , terminates when  $s \notin s_\phi$ .
2. For each abstract state, there is at least one  $o \in \mathcal{O}$  that initiates in that state.

# $\phi$ -Relative Options

(1) State Abstraction,  $\phi$

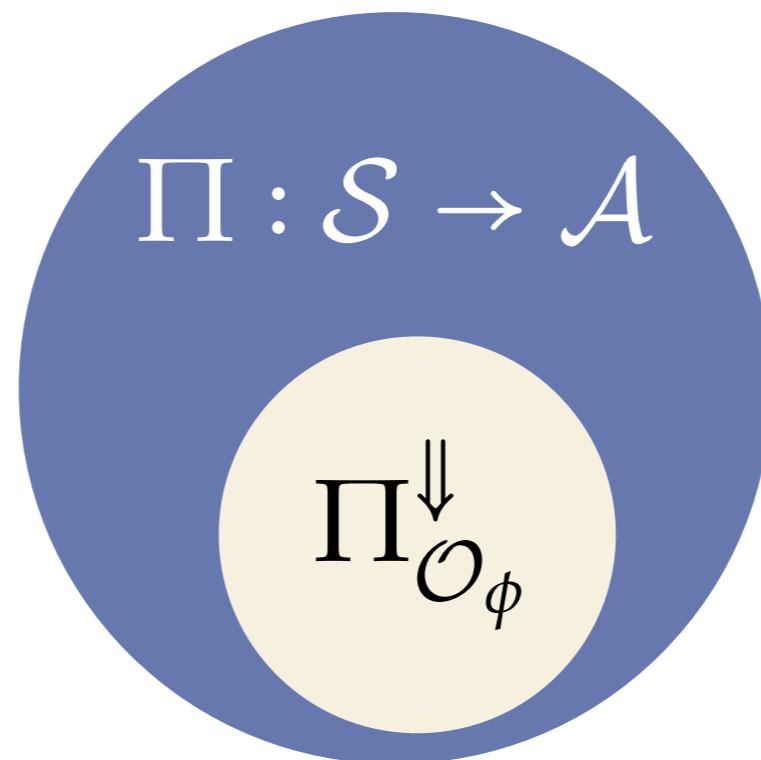
(2) Action Abstraction,  $\mathcal{O}_\phi$



# $\phi$ -Relative Options

(1) State Abstraction,  $\phi$

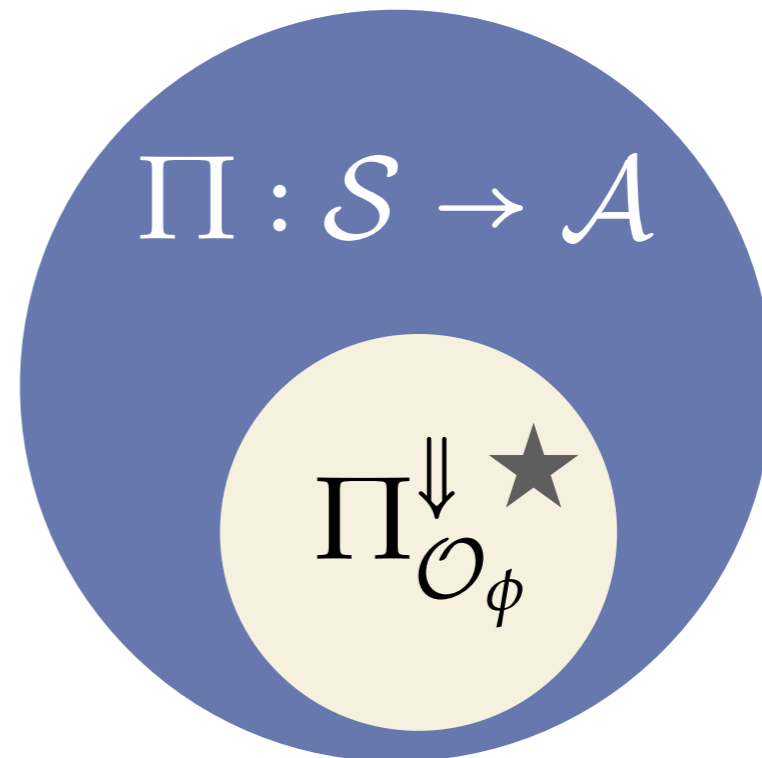
(2) Action Abstraction,  $\mathcal{O}_\phi$



# $\phi$ -Relative Options

(1) State Abstraction,  $\phi$

(2) Action Abstraction,  $\mathcal{O}_\phi$



**Question:** Which  $\phi, \mathcal{O}_\phi$  pairs induce a policy class  $\Pi \downarrow_{\mathcal{O}_\phi}$  such that the best abstract policy is still pretty good?

# Value-Preserving Abstractions

**Theorem.** There exist at least four classes of  $\phi, \mathcal{O}_\phi$  with bounded value loss:

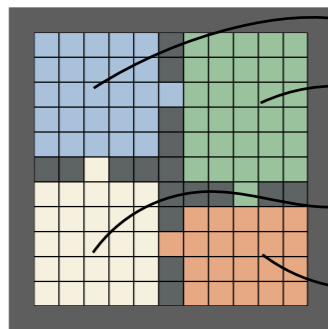
$$\min_{\pi_{\mathcal{O}_\phi}^\downarrow \in \Pi_{\mathcal{O}_\phi}^\downarrow} \max_{s \in \mathcal{S}} \left( V^*(s) - V^{\pi_{\mathcal{O}_\phi}^\downarrow}(s) \right) \leq \eta_p,$$

where  $\eta_p$  varies depending on the class.

**Question:** Which  $\phi, \mathcal{O}_\phi$  pairs induce a policy class  $\Pi_{\mathcal{O}_\phi}^\downarrow$  such that the best abstract policy is still pretty good?



# $\phi$ -Relative Option Classes



$$\forall s_\phi \in \mathcal{S}_\phi \exists o \in \Omega(s_\phi) :$$

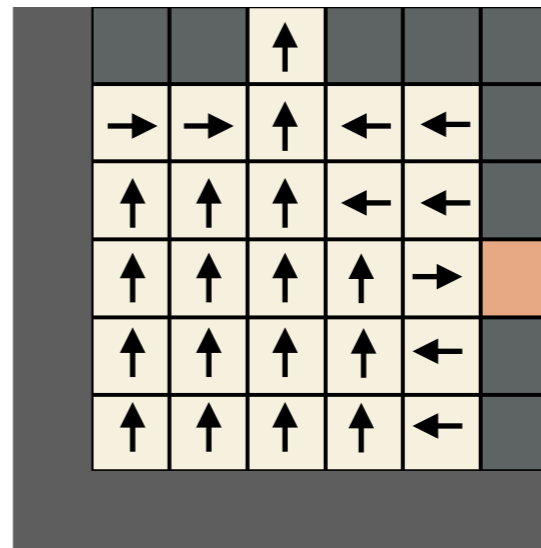
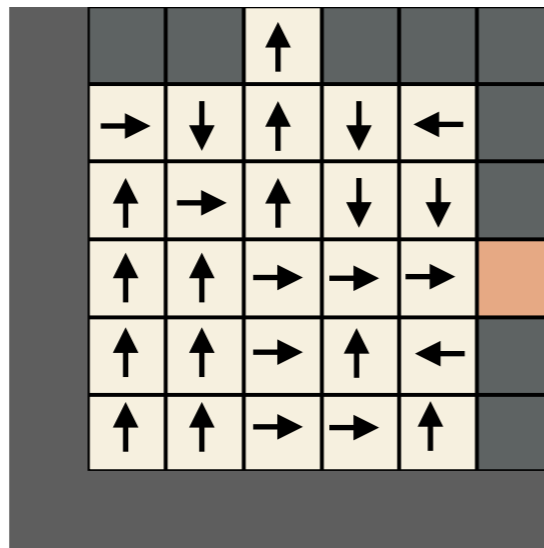
All options that initiate in  $S_\phi$

$O$

$\approx$

$O_{S_\phi}^*$

Determines option class



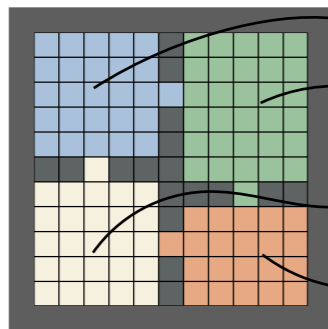
$$O_{S_\phi}^* = (s \in S_\phi, s \notin S_\phi, \pi^*)$$

initiate

terminate

policy

# $\phi$ -Relative Option Classes



All options that initiate in  $S_\phi$

$$\forall s_\phi \in \mathcal{S}_\phi \exists o \in \Omega(s_\phi) :$$

*Expressive  $Q^*$  Options*

$$|Q^*(s_\phi, o) - Q^*(s_\phi, o_{s_\phi}^*)| \leq \epsilon_Q$$

*Expressive Model Options*

$$|R_\gamma(s_\phi, o^*) - R_\gamma(s_\phi, o)| \leq \epsilon_R$$

and

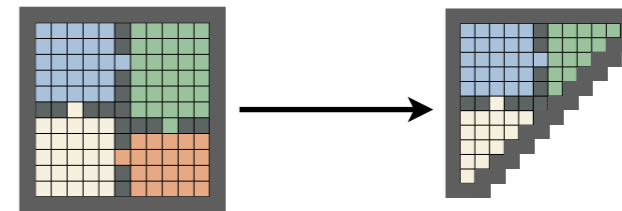
$$\|T_\gamma(\cdot | s_\phi, o^*) - T_\gamma(\cdot | s_\phi, o)\|_2 \leq \epsilon_T$$

*Expressive  $k$ -Step Options*

$$\max_{s \in \mathcal{S}_\phi, s' \in \mathcal{S}} |\mathbb{P}(s', k | s, o_{s_\phi}^*) - \mathbb{P}(s', k | s, o)| \leq \tau$$

[Nachum et al. 2019]

*Homomorphism Options*



[Ravindran and Barto '02, '03, '04]

# Value-Preserving Abstractions

**Theorem.**  $\min_{\pi_{\mathcal{O}_\phi}^\downarrow \in \Pi_{\mathcal{O}_\phi}^\downarrow} \max_{s \in \mathcal{S}} \left( V^*(s) - V^{\pi_{\mathcal{O}_\phi}^\downarrow}(s) \right) \leq \boxed{\eta_p}$

*Expressive  $Q^*$  Options*

$$\frac{\varepsilon_Q}{1 - \gamma}$$

*Expressive Model Options*

$$\frac{\varepsilon_R + |\mathcal{S}| \varepsilon_T R_{\text{MAX}}}{(1 - \gamma)^2}$$

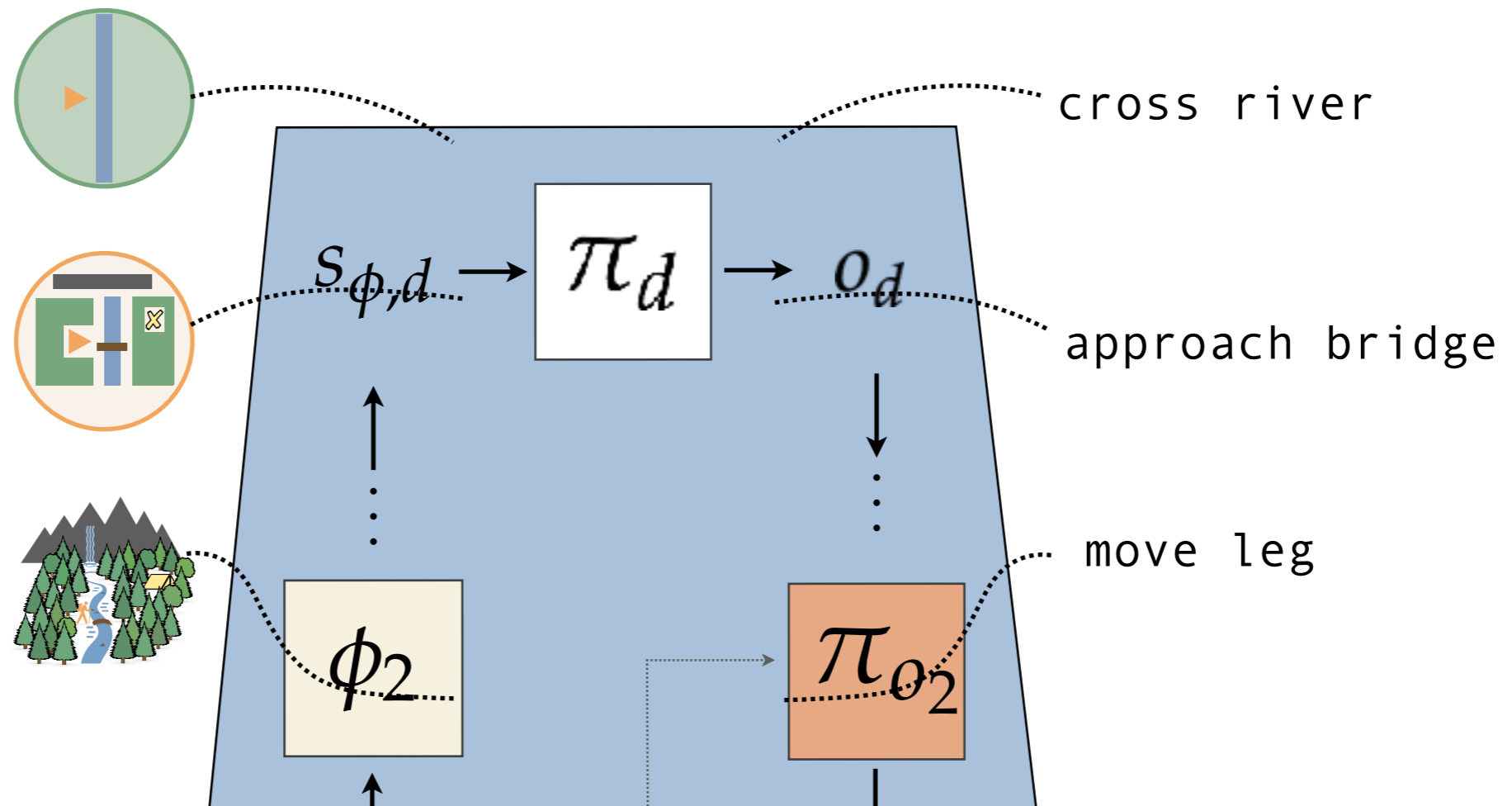
*Expressive  $k$ -Step Options*

$$\frac{\tau \gamma |\mathcal{S}|}{(1 - \gamma)^2}$$

*Homomorphism Options*

$$\frac{2}{1 - \gamma} \left( \varepsilon_r + \frac{\gamma R_{\text{MAX}} \varepsilon_p}{1 - \gamma} \frac{1}{2} \right)$$

# Value-Preserving Hierarchies

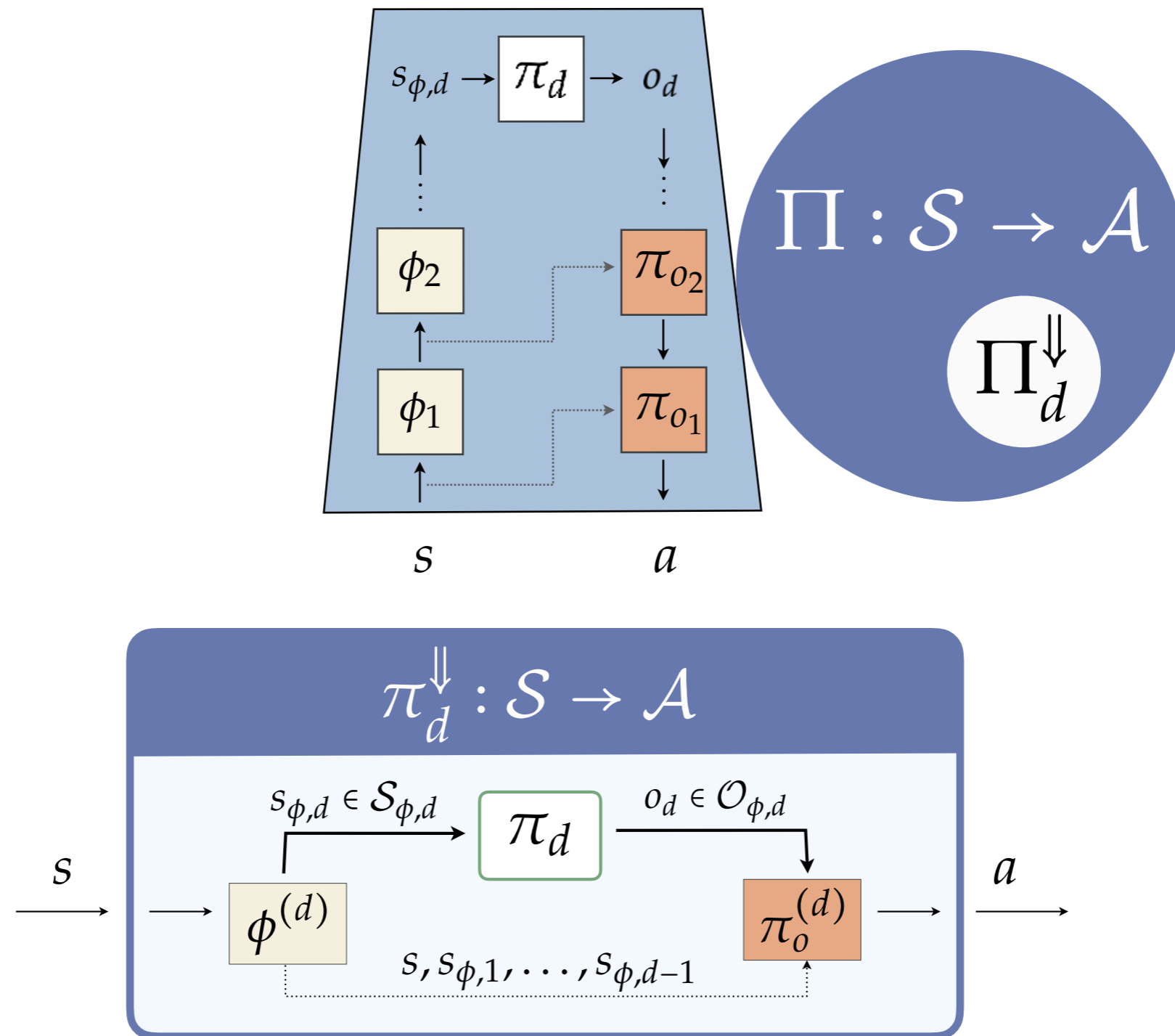


**Definition.** A depth  $d$  hierarchy  $H_d$  is defined by the pair

$$\phi^{(d)} = (\phi_1, \phi_2, \dots, \phi_d),$$

$$\mathcal{O}^{(d)} = (\mathcal{O}_1, \mathcal{O}_2, \dots, \mathcal{O}_d).$$

# Value-Preserving Hierarchies



# Value-Preserving Hierarchies

**Assumption 1.** *The value function is consistent throughout the hierarchy.*

 **value  
expressivity**

**Assumption 2.** *Subsequent levels of the hierarchy can represent policies similar in value to the best policy at the previous level.*

 **policy  
expressivity**

# Value-Preserving Hierarchies

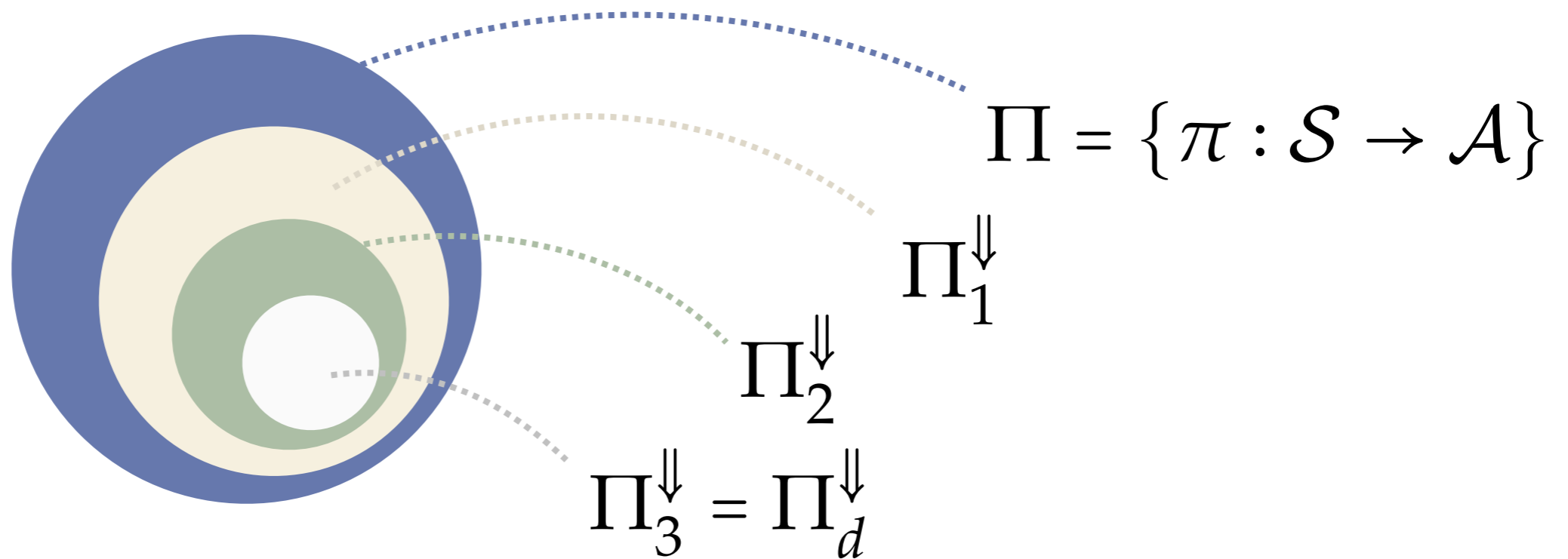
**Assumption 1.** *The value function is consistent throughout the hierarchy.*

**Assumption 2.** *Subsequent levels of the hierarchy can represent policies similar in value to the best policy at the previous level.*

**Theorem.** Any hierarchy  $H_d$  that satisfies Assumptions 1 and 2 has bounded value loss:

$$\min_{\pi_d^\downarrow \in \Pi_d^\downarrow} \max_{s \in \mathcal{S}} \left( V^*(s) - V^{\pi_d^\downarrow}(s) \right) \leq \underbrace{d}_{\text{depth}} \left( \underbrace{\kappa}_{\text{value expressivity}} + \underbrace{\ell}_{\text{policy expressivity}} \right)$$

# Value-Preserving Hierarchies

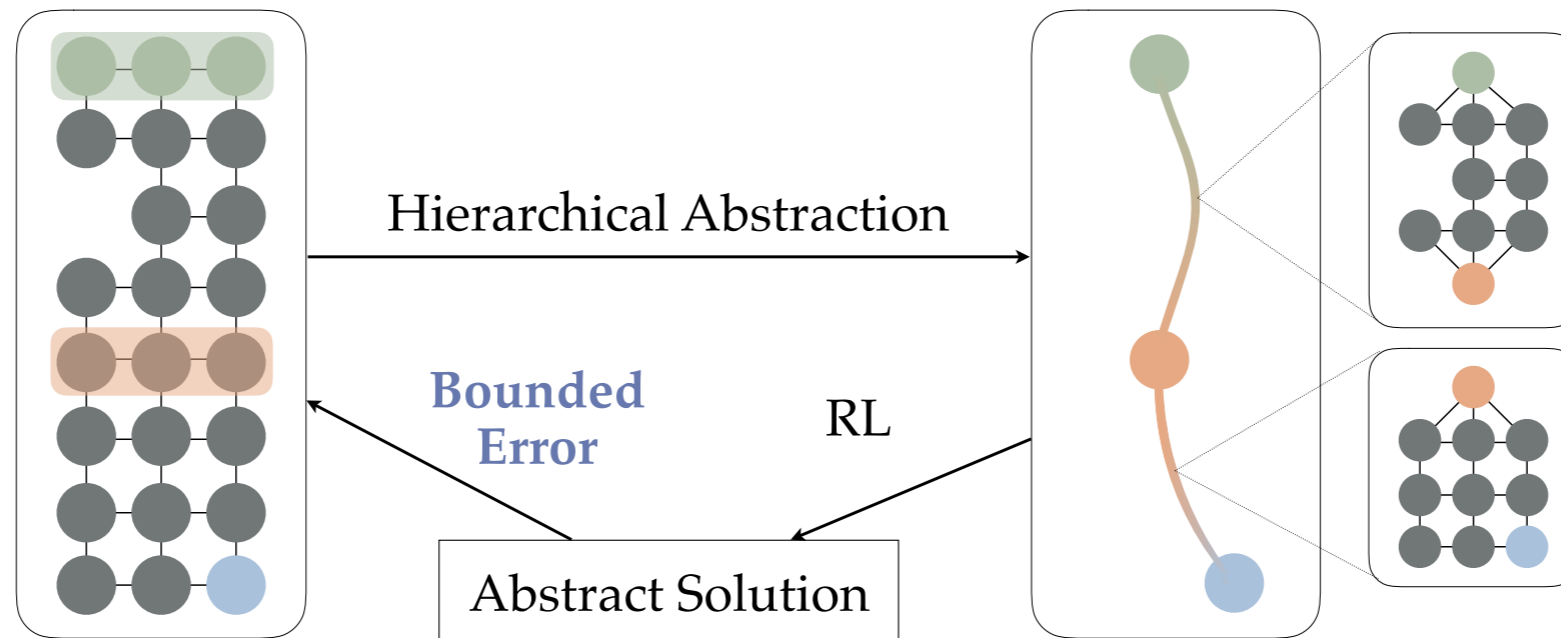


**Theorem.** Any hierarchy  $H_d$  that satisfies Assumptions 1 and 2 has bounded value loss:

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# Value-Preserving Hierarchies



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$$\min_{\pi_d^\downarrow \in \Pi_d^\downarrow} \max_{s \in \mathcal{S}} \left( V^*(s) - V^{\pi_d^\downarrow}(s) \right) \leq \underbrace{d}_{\text{depth}} \left( \underbrace{\kappa}_{\text{policy expressivity}} + \underbrace{\ell}_{\text{value expressivity}} \right)$$

# Thanks to Mentors!

## *Masters*



Joshua  
Schechter



Stefanie  
Tellex

## *Ph.D*



Michael L.  
Littman

## *Undergrad*



David  
Liben-Nowell



Ana  
Moltchanova



George  
Konidaris



Peter  
Stone



Will  
Dabney



Fernando  
Diaz



Owain  
Evans

## *Committee*

## *Internships*

# Thanks to Collaborators!



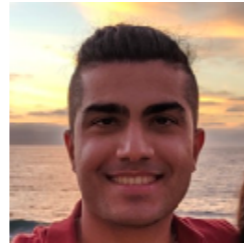
Alekh  
Agarwal



Cam  
Allen



Dilip  
Arumugam



Kavosh  
Asadi



Gabriel  
Barth-Marón



Stephen  
Brawner



Jonathon  
Cohen



Marie  
desJardins



Tom  
Griffiths



Yue  
Guo



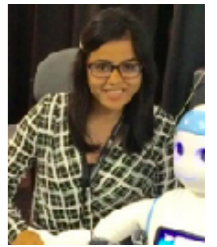
D. Ellis  
Hershkowitz



Mark  
Ho



Yuu  
Jinnai



Khimya  
Khetarpal



Akshay  
Krishnamurthy



Lucas  
Lehnert



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Park



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Emily  
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Salvatier



Robert  
Schapire



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Nathan  
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Edward  
Williams

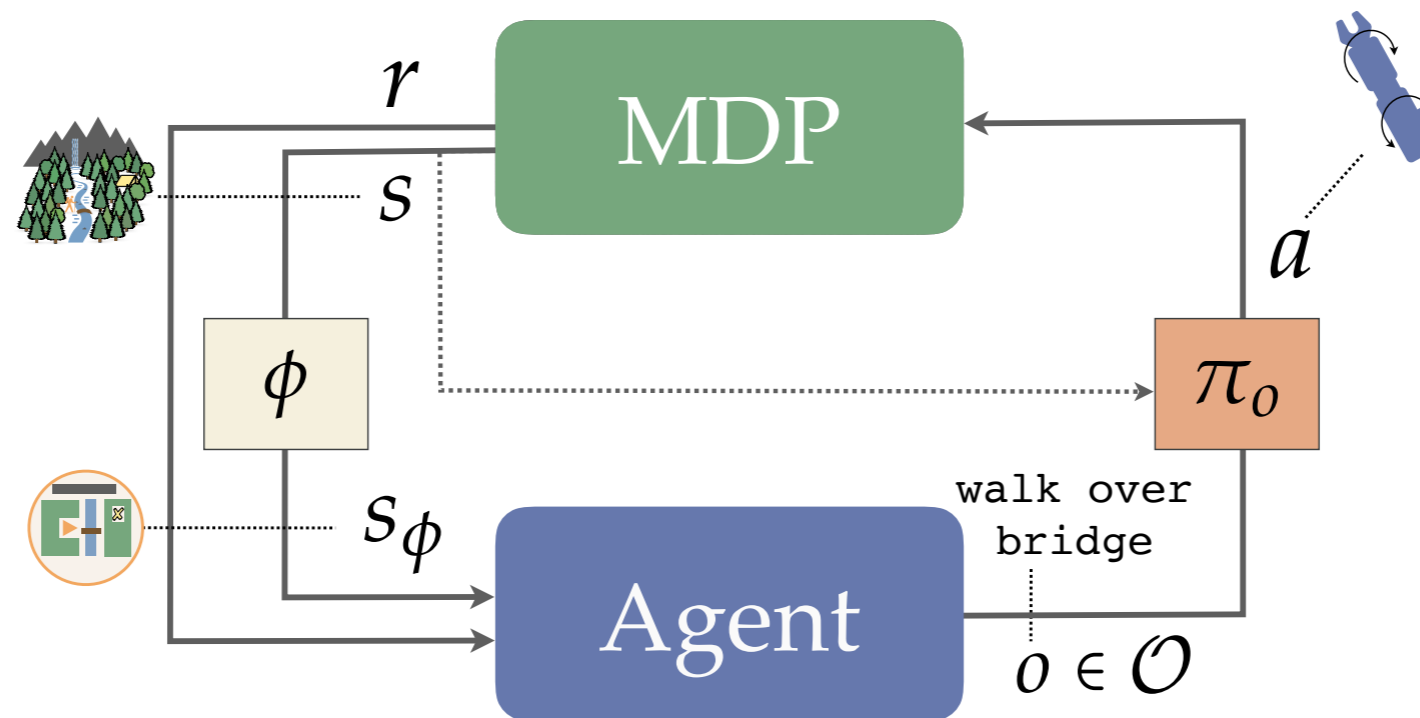


John  
Winder



Lawson  
Wong

# Summary



**Question:** How do effective RL **agents** come up with the right **state** and **action** abstractions of the **MDPs** they inhabit?

**Dissertation:** [david-abel.github.io/thesis.pdf](http://david-abel.github.io/thesis.pdf)

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