Omnipredictors[1]: One Predictor to Rule Them All

Heavily adapted from P. Gopalan's Talk at IAS

J. Setpal

April 18, 2024

[1]Gopalan, Kalai, Reingold, Sharan, Wieder

ERM Synopsis

We'll start with an *overview* of supervised learning paradigm:

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L is minimized over $\mathcal{D}$, not over the real world. This is **empirical risk**:

$$\min_{\boldsymbol{\theta}} \frac{1}{N} \sum_{i=1}^{N} L(f_{\boldsymbol{\theta}}(x_i), y_i) \tag{1}$$

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Boosting a weak agnostic learner is a critical aspect of the Omnipredictors approach to learning "multicalibrated partitions" (we'll get to this soon).

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Let's evaluate this empirically on ℓ_1 and ℓ_2 losses, which optimize for median and mean respectively:

$$\ell_1 = |y - \hat{y}|,\ \ell_2 = (y - \hat{y})^2 \tag{2}$$

$$x \sim f(\epsilon \sim \mathcal{U}[0,1]) := \begin{cases} 0 & \epsilon \leq 0.4 \\ \mathcal{U}[0.8, 1] & \text{otherwise} \end{cases} \tag{3}$$

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Omnipredictors provides a framework for rigorous guarantees, deriving $\tilde{p} \approx p^*$: a predictor that is able to *simultaneously minimize* a family of convex loss functions.

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"If you posit a more complex view of the world, I will subject you to a more rigorous test." – P. Gopalan.

Multigroup Fairness

We can split $\mathcal{D}$ into various *subgroups* based on **shared characteristics**. These can be explicit or implicit (i.e. subgroups we don't know of):

	Group-1	**Group-2**	**Group-3**	**Group-4**
Accuracy	0.9593	0.6249	0.3157	0.2664
Loss	0.0021	0.4102	1.3457	1.7664
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One notion of fairness stipulates equal risk for every subgroup. However, finding subgroups is hard for <u>high-dimensional data</u>.

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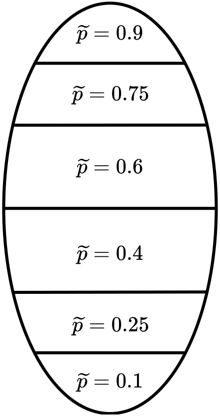

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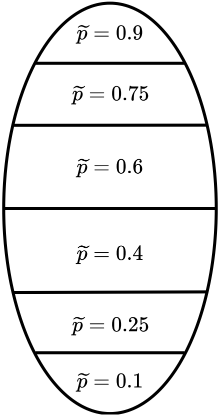

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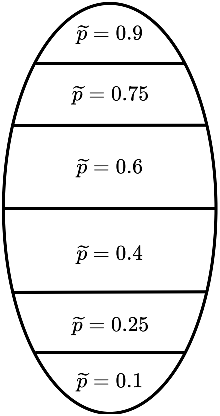

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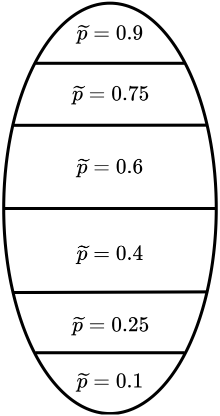

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If we can find correlations with the error, there's some advantage to be gained. We enforce multicalibration to train the **weak agnostic learner**.

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Multicalibration implies omniprediction for all convex loss functions.

Proof of Optimality

As proof for optimality, we evaluate binary classification. Assumptions:

1. p^* is boolean.
2. Perfect Mutlicalibration:
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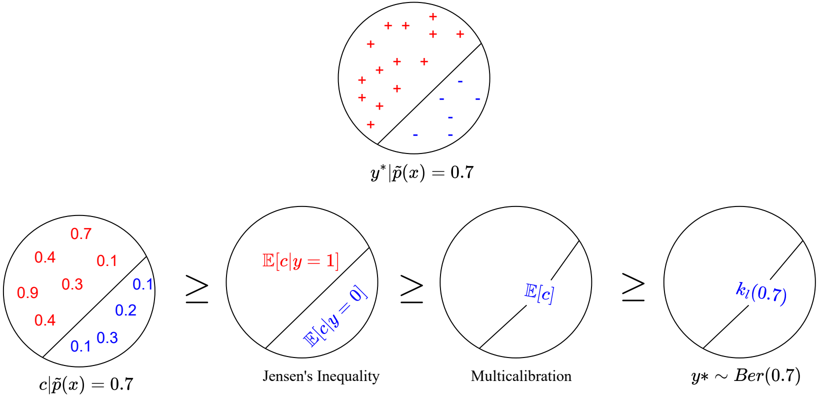

Training Agnostic Learners

We can train an agnostic learner using the following setup:

$$\mathcal{H} : \mathcal{X} \to \{0, 1\} \tag{8}$$

$$\mathcal{D} : \mathcal{X} \times \{0, 1\} \tag{9}$$

$$\ell(h \in \mathcal{H}) = \Pr_{(x,y)\sim\mathcal{D}}[h(x) \neq y] = \mathbb{E}_{(x,y)\sim\mathcal{D}}[\ell_1(y, h(x))] \tag{10}$$

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An agnostic learner for $\mathcal{H}$ produces f s.t. $\ell(f) \leq \text{OPT}(\mathcal{H}) + \epsilon$.

We (ϵ, W)-approximate $\mathcal{H}$ by $\mathcal{C}$ if $\forall h \in \mathcal{H}, \epsilon > 0$, $g_w \in \text{Lin}_{\mathcal{C}}(W)$:

$$\mathbb{E}_{(x,y)\sim\mathcal{D}}[|g_w(x) - h(x)|] \leq \epsilon \tag{12}$$

If partition $\mathcal{S}$ is $\frac{\epsilon}{2W}$-approximately multicalibrated for $\mathcal{C}, \mathcal{D}$, $\ell(h^{\mathcal{S}}_{\ell_1}) \leq \text{OPT}(\mathcal{H}) + \epsilon$ and can identify multicalibrated partitions.

Thank you!

Have an awesome rest of your day!

Slides:

https://cs.purdue.edu/homes/jsetpal/slides/omnipredictors.pdf