How do Machines Learn?

COM 100π – Advanced Presentational Speaking

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It quantifies the essence of the data *without memorizing it*; so, is **machine learning**.

Loss Function

Now, what do we do if our data is weird[1]?

x	y
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So; how would we find this line? We create a differentiable function we can use as the **score** for the line.

Whichever line has the best score, is the line of best fit. For Linear Regression, we can use **Mean Squared Error**:

$$L(y_{\text{pred}}, y_{\text{actual}}) = (y_{\text{actual}} - y_{\text{pred}})^2$$

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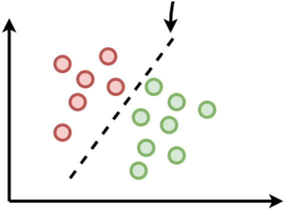

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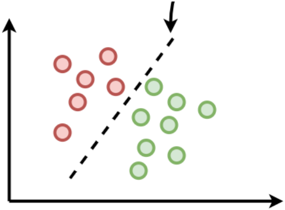

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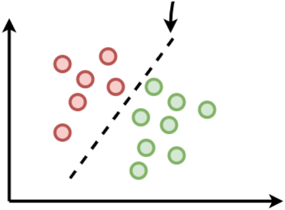

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This map called an **activation function**. Our new line function is:

$$y = \sigma_{\text{sigmoid}}(mx + b)$$

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We can solve this by adding **nonlinearity**, using the ReLU activation:

$$\sigma_{\text{relu}}(x) = max(0,\ x)$$

Multi-Layer Perceptron #2

Finally, we add the sigmoid activation to make the predictions class probabilities. Our new equation looks like:

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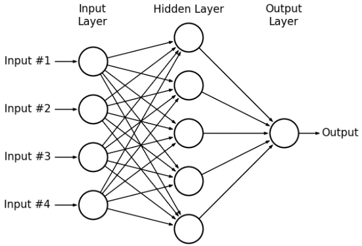
Graphically, this is what it looks like:



Let's have some fun: `https://playground.tensorflow.org/`

Thank you!

Have an awesome rest of your day!

Slides:

https://www.cs.purdue.edu/homes/jsetpal/slides/how-ml.pdf