Modelling of temperature distribution within a dental profile on account of laser irradiation

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Abstract: Thermal-induced destruction of pulpal material on account of prolonged exposure to laser irradiation frequently offers serious challenges for application of lasers in dentistry. Here, we develop a mathematical model of heat transfer within the dental profile due to laser irradiation and attempt to locate the probable paths of heat flow by identifying the pertinent thermal resistances. Corresponding to a maximum allowable rise of temperature, we obtain suitable ranges of laser power, so that a negligible heat flux prevails at tooth locations farthest from the spot of laser treatment, thereby obtaining safe operating parameter regimes and avoiding any undesirable pulpal damage.

Keywords: laser irradiation; pulpal damage; thermal model; dentistry.

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1 Introduction

Laser treatment is preferred for several dental applications such as cavity preparation, drilling, laser-induced analgesia of teeth and removal of black caries. Dentists are currently able to use lasers, instead of conventional drills, to remove decayed portions of diseased teeth. The use of laser in such applications reduces vibration during drilling and, therefore, little or no anaesthesia is needed when lasers are being used. Laser treatment is also preferred by the patients, since it is virtually painless. Such excellent promises offered by lasers for precise and modernised dental treatments have stimulated continuing research into their use for the removal of hard dental tissues. However, in practice, slow material removal rates and unacceptable local tissue damage have inhibited the clinical use of lasers in many hard tissue applications. Studies on ablation of enamel and dentine, using pulsed lasers such as Nd : YAG or Ho : YAG laser (Goldman et al., 1965; Adrian et al., 1971; Picroux and Pope, 1984; Launay et al., 1987; Serebro et al., 1987; Tjan et al., 1989; Midda and Renton-Harper, 1991; Niernz, 1996; Yon Fraunhofer et al., 1993; Miserendino et al., 1993; Whitters et al., 1994, 1995; Shoji et al., 1985; White et al., 1994; Yon Fraunhofer and Allen, 1993; Arcoria et al., 1994), have shown the evidence of cracking, melting, charring, fissuring of tooth surfaces and insufficient material removal, resulting in poor surface preparation. One of the major problems associated with laser treatment has been identified to be the large amount of heat generated due to laser irradiation and the consequent localised surface heating problems. Challenges in this regard are many, and a fundamental scientific understanding of the entire process is probably far behind the corresponding technological advancements at this moment.

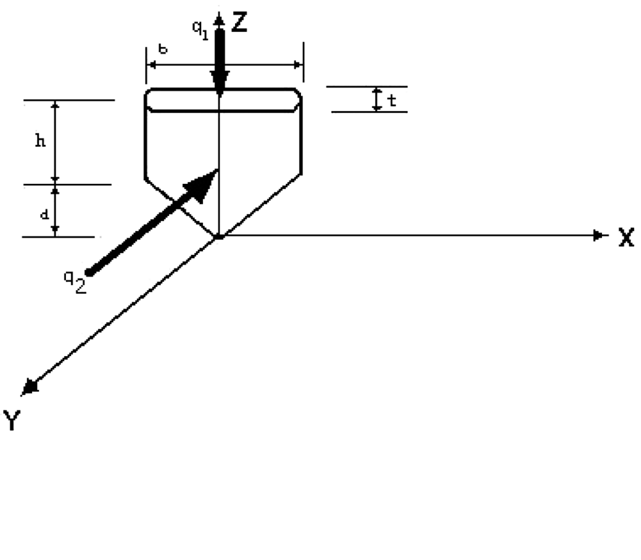
Of various types of industrial and medical lasers available, Carbon-dioxide (CO_2) lasers have attracted wide interest in recent years for use in various dental hard-tissue procedures. CO_2 laser radiation is within the infrared region of the electromagnetic spectrum, with a wavelength of 10.6 mm. This laser wavelength is absorbed strongly in dental enamel, dentin and cementum (Walsh, 1996). Consequently, useful working temperatures of hundreds to thousands of degrees Celsius may be generated at the laser impact site, with a very low rate of transmission of thermal energy into the pulp (Frame, 2000). Indeed, the temperature rise in the pulp chamber/root canal has been shown to be rather minimal, for the continuous wave mode of emission (Vinod, 2002). Parameters for thermal injury to dental pulp have been defined by Zach and Cohen (1965), who examined histological changes in pulps of monkeys exposed to a non-laser heat source (soldering iron). Their work defined a ‘critical’ threshold for a temperature rise of 5.5°C, above which an unacceptably high incidence of pulpal necrosis occurred. Below 5.5°C, reversible and mild pulpit took place. Below 2.2°C, no histological changes were discernible. Based on these threshold values, subsequent experimental studies, employing continuous wave CO_2 lasers have concluded that exposures should not exceed 10 Joules (J), to ensure that the ‘critical’ temperature rise of 5.5°C does not occur (Vinod, 2002; http://www.agnr.umd.edu/users/bioreng/chaptr5.pdf). These studies have also revealed that, to avoid pulpal necrosis, continuous wave exposures should not exceed 30 J. However, there are no data available on the thermal effects of CO_2 lasers operated in pulsed (‘chopped’) modes. On the other hand, from a purely theoretical standpoint, pulsed modes of operation are likely to be safer in the purview of tissue damage, in a sense that cooling can occur between laser pulses (Biomaterials Properties Database, 1996).

The present study, accordingly, aims to quantify temperature changes within the dental profile, on account of laser treatments executed in pulsed modes. First, we formulate the equivalent thermal circuitry, showing various resistances to heat transfer. Then, we determine temperature distributions along the dental profile, utilising our mathematical model, under prevailing operating conditions. Next, we determine the possible ranges of laser power to ensure that temperature rises are within the constraints of a safe operating limit, from a theoretical perspective. Such a kind of mathematical modelling, in case of laser-dental tissue interactions, is yet to be attempted in the literature, to the best of our knowledge. However, such information can act as invaluable inputs to dental practitioners for proper planning and control of dental treatments under different conditions of laser irradiation (Rode et al., 2003).

2 Mathematical model

Laser irradiation on a tooth can be affected in many ways, from different angles. Here, for the sake of simplicity and fundamental understanding, we consider two mutually perpendicular directions of laser irradiation, as shown in Figure 1. Figure 1 shows two lasers 1 and 2, with respective heat fluxes, q_1 and q_2. Energy corresponding to q_1 is incident on the top surface of the corresponding tooth along the negative Z direction and that corresponding to q_2 is incident on the lateral surface of the tooth along negative$Y$-direction. We treat these two cases separately. In the first case, only irradiation of laser one is considered and in the second case, only irradiation of laser two is considered. It can be noted here that since the cross-sectional area for the inner portion of a tooth varies significantly in a manner which is very different in different human beings, we employ a 'mean area' to convert the respective heat fluxes into representative heat transfer rates.

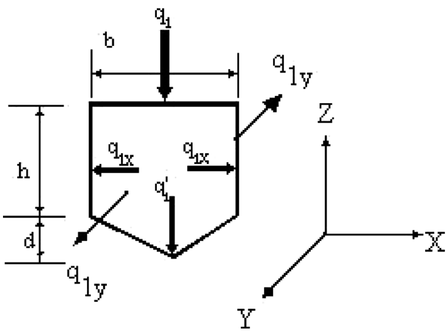
Figure 1 Two lasers 1 and 2, with corresponding irradiation fluxes of q_1 and q_2, respectively. The heat flux q_1 is incident on the top face (face parallel to the dental profile) of the tooth, i.e., along negative Z-direction, and the heat flux q_2 is incident on the front face (face normal to the dental profile) of tooth, i.e., along negative Y-direction



2.1 Case 1: Effects of irradiation of laser 1

Figure 2 shows laser one with an irradiation heatflux of q_1. Some part of the heat (q_1') propagates through the irradiated tooth and then through soft tissues in negative Z-direction. The heat flux q_1' must be a minimum, since it may cause damage of soft tissues. A part of the heat flows along the Y-direction, and accounts for a heat loss $(q_y = 2q_{1y})$ from a tooth to its surroundings. The remaining part of the heat, q_{1x}, propagates through the tooth profile along positive and negative X-directions.

Figure 2 Details of heat flow along the treated tooth, for laser 1. Some part of the heat, q_1', propagates through the irradiated tooth and then through soft tissues in negative Z direction. Another part of heat flows along positive and negative Y-directions. The remaining part of the heat, q_{1x}, propagates through the tooth profile in positive and negative X-directions



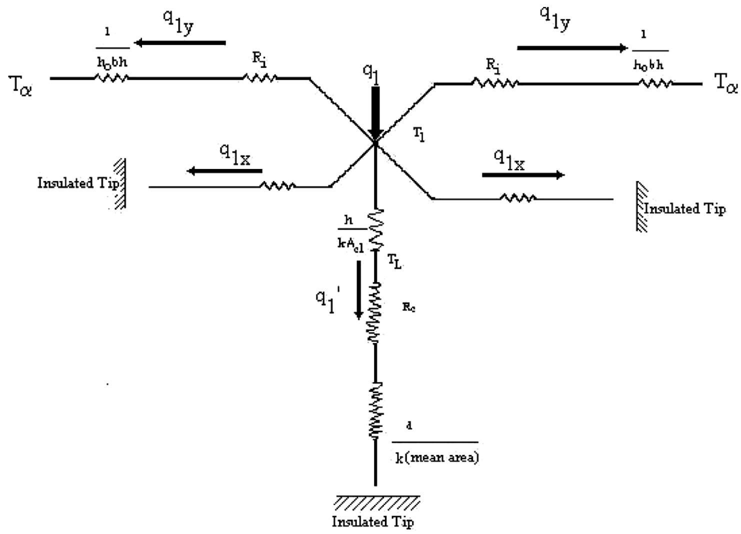
Propagation of thermal disturbance in this manner encounters various thermal resistances. The resistance in conduction for the exposed part of a tooth (through which q_1' must pass) is numerically equal to $[h/(kA_{cl})]$. Also, there is a contact resistance (R_c) and conductive resistance based on mean area available for the heat flux (q_1') to enter into soft tissues. To prevent damage of soft tissues, an insulated tip condition must be obtained, i.e., q_1' must be substantially low (virtually negligible). The resistance to propagation of heat flux q_{1y} from one side of exposed tooth to its surroundings is basically a convective resistance, which is numerically equal to $[1/(h_o bh)]$. Also, there are some internal resistances (denoted by R_i) in the positive and negative Y-directions, primarily attributable to the thickness of the respective tooth. Such resistances, however, are much smaller in magnitude in comparison to other thermal resistances acting on the system and can, therefore, be neglected. The heat lost from one side of an exposed tooth to its surroundings (q_{1y}) can be described as:

$$q_{1y} = \frac{T_1 - T_\alpha}{1/h_o bh}. \tag{1}$$

Hence, the total heat lost by tooth to its surroundings becomes $q_y = 2q_{1y}$. The remaining part of the heat flux, q_{1x}, propagates through the tooth profile in positive and negative X-direction, upto a maximum length of L. We can consider here that, beyond a length L, there is an insulated tip through which no heat flux can propagate, i.e., there will be no perceptible changes in tooth temperature beyond that location. The entire thermal circuitry, depicting an interplay of all the thermal resistances mentioned as above, is represented in Figure 3. From an overall heat balance applied on the thermal system, we get,

Energy absorbed = Energy transfer in *X*-direction + heat going to the soft tissues + energy transfer to surrounding due to convection + internal energy increase of the tooth.

Figure 3 Thermal circuit for irradiation of laser 1



At quasi-steady state (laser power switch-off mode), there is no further internal energy change of the tooth, and accordingly, the above simplifies to:

$$q_1 = (2q_{1x}) + q_1' + q_y. \tag{2a}$$

$$\text{i.e.,} \quad q_{1x} = 0.5(q_1 - q_1' - q_y). \tag{2b}$$

To determine the maximum possible temperature rise along the dental profile (to arrive at conservative estimates for dental treatments), we need to assume that there is a negligible heat transfer to the surroundings due to convection. Also, q_1' must be negligibly small, as discussed earlier, so as to prevent any unwanted damage of soft tissues. Under these circumstances, we obtain

$$q_{1x} = 0.5q_1 \tag{2c}$$

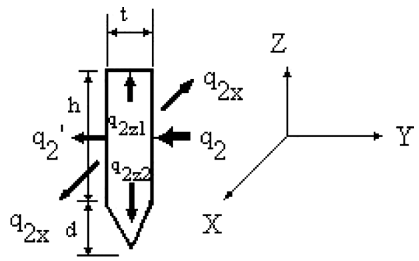
where, q_1 is the heat flux irradiation into the tooth out of lasing action.

2.2 *Case 2: Effects of irradiation of laser 2*

Figure 4 shows a situation in which a heatflux of q_2 is irradiated on account of lasing action. Some part of the heat flux, q_{2z2}, propagates through the irradiated tooth and then through the soft tissues in the negative *Z*-direction. The heat flux, q_{2z2}, must be a minimum, since it may cause undesirable thermal damages to soft tissues. Another part of the heat flux, q_{2z1}, propagates through the irradiated tooth and is subsequently dissipated to its surroundings along the positive Z-direction. A part of the heat that flows along negative Y-direction enters into the oral cavity, and the corresponding heat flux is taken to be q_2'. The remaining part of the heat flux, q_{2x}, propagates through the dental profile along the positive and negative X-directions. Similar to Case 1, each heat flux encounters some thermal resistances under these conditions as well. The resistance in conduction for the exposed part of the tooth (through which the heat flux q_2' must pass) is numerically equal to the sum of a conductive resistance, $t/(kA_{cl})$ and a convective resistance, $1/(h_o bh)$. Accordingly, the heat flux that leaves the exposed tooth and enters the oral cavity, i.e., q_2', can be described as:

$$q_2' = \frac{T_1 - T_\alpha}{(t/kA_{cl} + 1/h_o bh)}. \tag{3}$$

Figure 4 Details of heat flow along the treated tooth, for laser 2. Some part of the heat flux, q_{2z2}, propagates through the irradiated tooth and then through soft tissues in negative Z-direction. Another part of the heat flux, q_{2z1}, propagates through the irradiated tooth, and eventually gets dissipated from the tooth to its surroundings along positive Z-direction. A part of the heat flux, q_2', which flows along the negative Y-direction, gets dissipated into the oral cavity. The remaining part of the heat flux, q_{2x}, propagates through the tooth profile along X-direction



The conductive resistance for the exposed part of tooth through which q_{2z2} must pass is equal to the sum of a conductive resistance of magnitude $(h/2)/(kbt)$ and a contact resistance (R_c). Beyond these resistances, an insulated tip condition must be obtained, since the heat flux q_{2z2} must not enter the soft tissues, in order to prevent any associated thermal damage. Further, the heat flux (q_{2z1}) propagating through the upper portion of an exposed tooth needs to overcome a net thermal resistance that equals the sum of a conductive resistance for the upper portion of exposed part of tooth, namely, $(h/2)/(kbt)$ and a convective resistance, namely, $1/(h_o bt)$. Also, there are some internal resistances (denoted by $R_{t'}$) acting in both the positive and negative Z-directions, offered by the thickness of the concerned tooth, which can virtually be neglected for reasons mentioned earlier. The heat lost from upper portion of an exposed tooth to its surroundings, can accordingly be described in terms of the corresponding heat flux, q_{2z1}, as:

$$\text{So,} \quad q_{2z1} = \frac{T_1 - T_\alpha}{(h/2/kbt) + (1/h_o bt)}. \tag{4}$$

The remaining part of the heat flux, q_{1x}, propagates through the tooth profile in the positive and negative X-directions up to a maximum length of L. We can consider that, beyond a length L, there is an insulated tip through which no heat flux can propagate.

The resultant thermal circuitry is depicted in Figure 5. From an overall energy balance at steady state, as applicable for this case, we get:

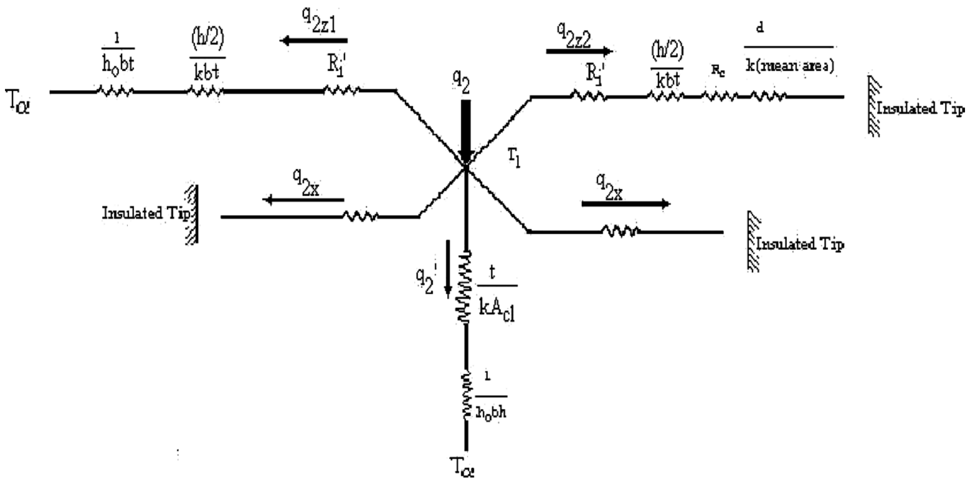
Energy absorbed = Energy transfer in *X*-direction + heat going into the soft tissues + energy transfer to surrounding due to convection, i.e.,

$$q_2 = (2q_{2x}) + q_{2z2} + (q_2' + q_{2z1}) \tag{5a}$$

$$\text{or,} \quad q_{2x} = 0.5[q_2 - q_{2z2} - (q_2' + q_{2z1})]. \tag{5b}$$

To determine the maximum possible temperature rise along the dental profile, we need to assume that there is negligible heat transfer to the surroundings due to convection. Also, q_{2z1} must be negligibly small in order to minimise unwanted damage to soft tissues. Hence, for determining the maximum possible temperature rise along the dental profile, we take the heat flux in *X*-direction(q_{2x}) as $0.5q_2$, where q_2 is the heat flux absorbed due to laser irradiation.

Figure 5 Thermal circuit for irradiation of laser 2

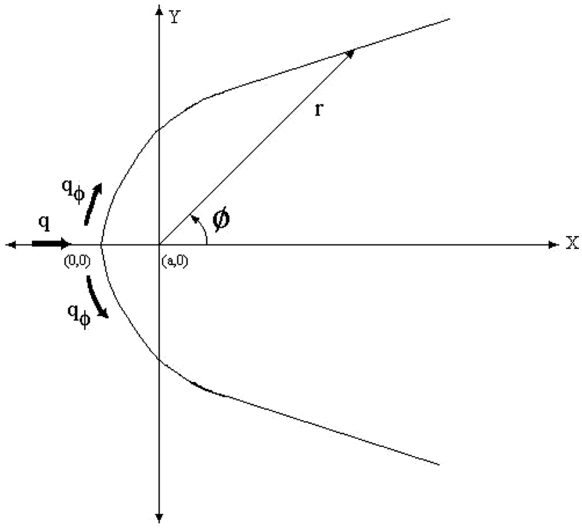


It can be noted here that the net effect of any general laser treatment can be thought of as a superposition of the individual consequences of Cases 1 and 2, described as above. The resultant heat flux along the dental profile, q_ϕ, which can be represented as a vectorial combination of both q_{1x} and q_{2x}, must be such that the temperature rise along the dental profile is constrained within safe limits of 5.5°C, so that irreversible pulpal changes can be avoided, and at the same time any damage of the adjoining soft tissues can be prevented.

With an overall estimation of the nature of heat flux distribution on account of the lazing action mentioned as above, it may be possible to formulate differential equations for temperature distributions in the teeth on the bases of well postulated boundary conditions. It is important to mention here that the directions in which heat fluxes traverse along a dental profile offer rather complicated geometrical paths, on account of the typical tooth arrangement along jaw-lines. For the sake of mathematical analysis, without loss of any generality, we assume the dental profile of the shape of a generic polar curve, expressed in a curvilinear form of $r = f(\phi)$, where r is the radial location and

ϕ is the polar angle, as depicted in Figure 6. With such considerations, an integrated heat transfer analysis for the dental profile can be performed as follows.

Figure 6 A schematic diagram showing the coordinate system and the directions of heat transfer through the teeth arranged in a polar dental profile ($r = f(\theta)$)



Heat flow in circumferential direction can be estimated as:

$$q_\phi = q_\phi + \mathrm{d}_\phi + \mathrm{d}q_{\mathrm{conv}}. \tag{6}$$

where, dq_{conv} represents a convective heat transfer rate. Heat flux in the circumferential direction can be obtained as:

$$q'_\phi = \left(-\frac{k}{r}\right)\frac{\mathrm{d}T}{\mathrm{d}\varphi}. \tag{7}$$

Hence, we can write:

$$q_\phi = Aq'_\phi = \left(-\frac{kA}{r}\right)\frac{\mathrm{d}T}{\mathrm{d}\varphi} \tag{8}$$

where A is an averaged area of cross-section, taken to be ht.

Accordingly,

$$q_{\phi+d\phi} = q_\phi + \frac{\mathrm{d}q_\phi}{\mathrm{d}\phi}\mathrm{d}\phi = \left(-\frac{kA}{r}\right)\frac{\mathrm{d}T}{\mathrm{d}\phi} + \frac{\mathrm{d}}{\mathrm{d}\phi}\left[\left(-\frac{kA}{r}\right)\frac{\mathrm{d}T}{\mathrm{d}\phi}\right]\mathrm{d}\phi. \tag{9}$$

$$\Rightarrow \mathrm{d}q_{\mathrm{conv}} = q_\phi - q_{\phi+d\phi} = \frac{\mathrm{d}}{\mathrm{d}\phi}\left[\left(\frac{kA}{r}\right)\frac{\mathrm{d}T}{\mathrm{d}\phi}\right]\mathrm{d}\phi. \tag{10}$$

Now, since the rate of convective heat transfer, i.e., dq_{conv}, can also be quantified as $h_o dA_s(T - T_\alpha)$, we can write from equation (10):

$$h_o dA_s(T - T_\alpha) = \frac{\mathrm{d}}{\mathrm{d}\phi}\left[\left(\frac{kA}{r}\right)\frac{\mathrm{d}T}{\mathrm{d}\phi}\right]\mathrm{d}\phi \tag{11}$$

$$\text{i.e.,} \quad \left(\frac{kA}{r}\right)\frac{\mathrm{d}^2T}{\mathrm{d}\phi^2} + (kA)\frac{\mathrm{d}T}{\mathrm{d}\phi}\left[\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{r}\right)\right]\frac{\mathrm{d}r}{\mathrm{d}\phi} - h_o\frac{\mathrm{d}A_s}{\mathrm{d}\phi}(T - T_\alpha) = 0. \tag{12}$$

Here, $\frac{\mathrm{d}A_s}{\mathrm{d}\phi} = pr$, where, p = exposed surface perimeter = $(2h + t)$.

Hence

$$\frac{\mathrm{d}^2T}{\mathrm{d}\phi^2} - \left(\frac{1}{r}\right)\frac{\mathrm{d}T}{\mathrm{d}\phi}\frac{\mathrm{d}r}{\mathrm{d}\phi} - \frac{h_o r^2 p}{kA}(T - T_\alpha) = 0 \tag{13}$$

$$\text{i.e.,} \quad \frac{\mathrm{d}^2T}{\mathrm{d}\phi^2} - \left(\frac{1}{r}\right)\frac{\mathrm{d}T}{\mathrm{d}\phi}\frac{\mathrm{d}r}{\mathrm{d}\phi} - \frac{h_o r^2(2h+t)}{kht}(T - T_\alpha) = 0. \tag{14}$$

Now, assuming temperature rise $(\theta) = T - T_\alpha$, we can write

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}\phi^2} - \left(\frac{1}{r}\right)\frac{\mathrm{d}\theta}{\mathrm{d}\phi}\frac{\mathrm{d}r}{\mathrm{d}\phi} - \frac{h_o r^2(2h+t)}{kht}\theta = 0. \tag{15}$$

To obtain a solution of the above equation, we assume the dental profile to be of a parabolic shape, i.e.,

$$r = \frac{2a}{(1 - \cos\phi)} \tag{16}$$

$$\Rightarrow r\frac{\mathrm{d}\phi}{\mathrm{d}r} = -\frac{(1 - \cos\phi)}{\sin\phi}. \tag{17}$$

Substituting the expressions from equations (16) and (17), equation (15) can be rewritten as:

$$(1 - \cos\phi)\frac{\mathrm{d}^2\theta}{\mathrm{d}\phi^2} - \sin\phi\frac{\mathrm{d}\theta}{\mathrm{d}\phi} - \frac{4a^2h_o(2h+t)}{kht(1 - \cos\phi)}\theta = 0. \tag{18}$$

Introducing another variable angle $(\phi_1) = \phi - \phi_r$, where, ϕ_r = irradiation angle, equation (18) takes the form:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}\phi_1^2} - \frac{\sin(\phi_1 + \phi_r)}{(1 - \cos(\phi_1 + \phi_r))}\frac{\mathrm{d}\theta}{\mathrm{d}\phi_1} - \frac{4a^2h_o(2h+t)}{kht(1 - \cos(\phi_1 + \phi_r))^2}\theta = 0. \tag{19}$$

The above equations are consistent with the following boundary conditions:

- At the point of irradiation i.e.,

$$\phi_1 = 0,\ \frac{\mathrm{d}\theta}{\mathrm{d}\phi_1} = -\left(\frac{r}{k}\right) q'_{\phi}\ \text{(specified)}$$

- At $\phi_1 = \phi_{1\max}$, $\theta = \theta_\alpha$ (*ambient condition inside the mouth*).

3 Results and discussion

Equation (19), coupled with the pertinent boundary conditions, is solved using a 'shooting method' that converts the corresponding second order boundary value problem into two coupled first order initial value problems. Each initial value problem is solved using a 4th order Runge Kutta method (Scheid, 2006). The pertinent variables and problem data used for the present simulation are summarised in Table 1. Cases with various laser powers are studied, within a range of 1.5–5 W. As a representative case, the maximum possible temperature rise along the dental profile due to laser irradiation is plotted against the modified polar angle (ϕ_1), for a laser power of 2 W and 1% duty cycle. The corresponding temperature rise at the point of irradiation is approximately 2.25°C, as depicted in Figure 7, which is well within the safe limits of operation. The corresponding temperature gradients are plotted in Figure 8. It can be observed from Figure 8 that temperature gradients decay non-linearly, as one moves away from the spot of laser irradiation, along the dental profile. Such non-linear variations can be attributed to the curvilinear shape of the dental profile arranged along the jaw lines. In Figure 9, we show the variation of the maximum temperature rise with laser power, corresponding to a duty cycle of 1%. It can be observed from the figure that the characteristic maximum temperature rise is almost a linear function of the laser power, which can be utilised as important information for efficient control of laser-based dental treatments. Further, beyond a laser of power 5 W (at 1% duty cycle), the characteristic temperature rise tends to become slightly more than the prescribed limit of 5.5°C. Hence, under the operating conditions and the prescribed safety limits, a higher laser power cannot be recommended.

Table 1 Table of simulation parameters

Problem data	*Numerical value*
Laser power	1.5–5 W
Duty cycle	1%
Irradiation angle	180º
Laser efficiency	20%
Convective heat transfer coefficient of surrounding (h_o)	3.1 $W/m^2 \cdot K$
Thermal conductivity of tooth material (k)	0.6 W/m·K
Length of exposed portion of tooth (h)	8 mm
Thickness of tooth (t)	3 mm
Focal length of the parabolic dental profile (a)	6.45 mm

Figure 7 Temperature variation along the dental profile, corresponding to a laser power of 2 W with 1% duty cycle

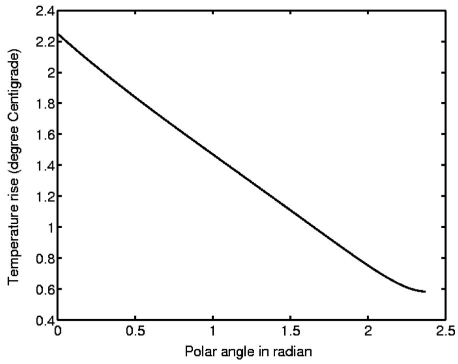


Figure 8 Variation of temperature gradient along the dental profile, corresponding to a laser power of 2 W with 1% duty cycle

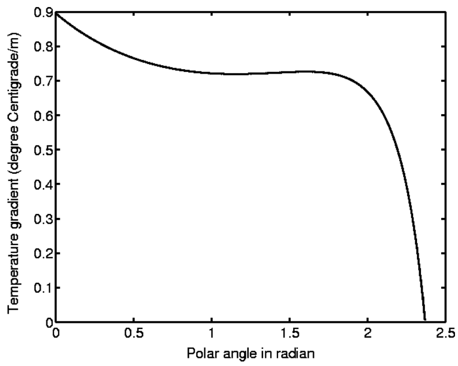
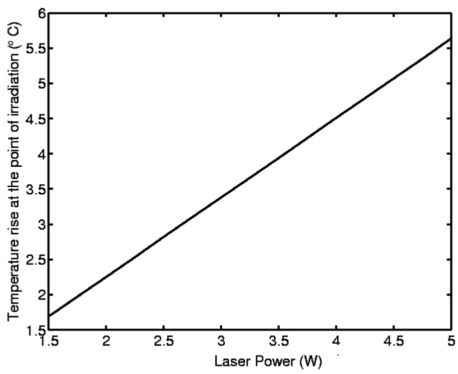


Figure 9 Maximum temperature rise during laser irradiation, as a function of laser power, corresponding to 1% duty cycle



4 Conclusions

In the present study, we obtain a range of laser powers that can be safely allowed so as to ensure that no pulpal necrosis can occur, through a novel mathematical modelling.

The present model neglects heat transfer to immediate surroundings on account of convection as well as any heat transfer into the soft tissues (Dua and Chakraborty, 2005), and hence, yields a conservative estimate for design of a hard-tissue dental treatment procedures. Therefore, in practice, temperature rise along the dental profile must be less than the values obtained in this study, corresponding to the same lasing conditions. Thus, safety of the use of a particular laser can be assured. Future efforts may also be directed towards validating the theoretical model through thermocouple-based temperature sensing protocols. For more details on such design of experiments, the studies conducted by Pike et al. (2007) may be referred.

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Website

http://www.agnr.umd.edu/users/bioreng/chaptr5.pdf

Nomenclature

q_1	Heat flux along negative *Z* direction
q_2	Heat flux along negative *Y* direction
α	Thermal diffusivity of tooth material
h_o	Convective heat transfer coefficient of surrounding
A_{cl}	Cross sectional area of the laser tip
T_α	Absolute temperature of the surrounding
T_1	Absolute temperature of tooth due to irradiation
c	Specific heat of tooth material
k	Thermal conductivity of tooth material
R_c	Contact resistance
ρ	Density of tooth material
θ	Temperature rise
ϕ_r	Irradiation angle
T	Absolute temperature of tooth
t	Thickness of a tooth
b	Breadth of a tooth
h	Length of exposed portion of a tooth
d	Length of portion of a tooth portruding into the gum