Homework 2 Released

- Due: Tuesday, October 2^{nd} at 3PM (beginning of class)
- Please Typeset Your Solutions (LaTeX, Word etc…)
- You may collaborate, but must write up your own solutions in your own words

Recap

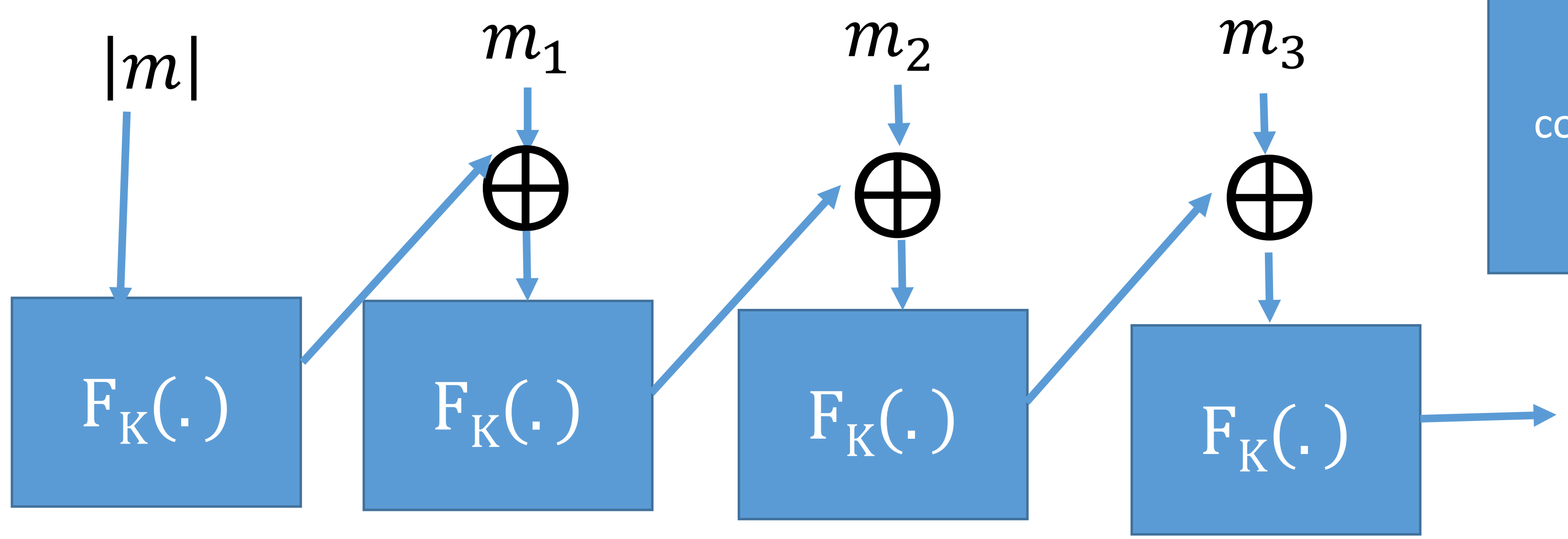
- Message Authentication Codes
  - Integrity vs Confidentiality
  - **Example:** $\mathrm{Mac}_{\mathrm{k}}(m) = \mathrm{F}_{\mathrm{K}}(m)$
  - Extension to unbounded messages and pitfalls (block re-ordering, truncation)
  - ~~CBC-MAC~~
- Authenticated Encryption + CCA-Security
  - ~~Encrypt and Authenticate [SSL]~~
  - Authenticate then Encrypt [TLS] (Caution Required)
  - Encrypt **then** Authenticate!

$$Enc_K(m) = \langle \mathrm{c}, \mathrm{Mac}'_{K_M}(\mathrm{c}) \rangle \text{ where } \mathrm{c} = \mathrm{Enc}'_{K_E}(m)$$

CBC-MAC

Caveat: Tricky Padding Issues arise if |m| is not a multiple of the block-length. See textbook.

We will see a simpler MAC construction using hash functions soon.



$$\tau = \mathrm{Mac}_K(m)$$

Advantages over Previous Solution

- Both MACs are secure
- Works for unbounded length messages
- Canonical Verification
- Short Authentication tag
- ~~Parallelizable~~

for i=1,…,d

$t_i = \mathrm{Mac}'_K(r \parallel \ell \parallel i \parallel m_i)$

(encode i and ℓ as n/4 bit strings)

Output $\langle r, t_1, \dots, t_d \rangle$

Recap: Authenticated Encryption

- Authenticated Encryption → CCA-Security (by definition)

- Conceptual Distinction
  - CCA-Security the goal is secrecy (hide message from active adversary)
  - Authenticated Encryption: the goal is integrity + secrecy

- CCA-Security does not necessarily imply Authenticate Encryption
  - But most natural CCA-Secure constructions are also Authenticated Encryption Schemes
  - Some constructions are CCA-Secure, but do not provide Authenticated Encryptions, but they are less efficient.

Secure Communication Session

- Solution Protocol? Alice transmits $c_1 = Enc_K(m_1)$ to Bob, who decrypts and sends Alice $c_2 = Enc_K(m_2)$ etc…
- Authenticated Encryption scheme is
  - Stateless
  - For fixed length-messages
- We still need to worry about
  - Re-ordering attacks
    - Alice sends 2n-bit message to Bob as $c_1 = Enc_K(m_1)$, $c_2 = Enc_K(m_2)$
  - Replay Attacks
    - Attacker who intercepts message $c_1 = Enc_K(m_1)$ can replay this message later in the conversation
  - Reflection Attack
    - Attacker intercepts message $c_1 = Enc_K(m_1)$ sent from Alice to Bob and replays to c_1 Alice only

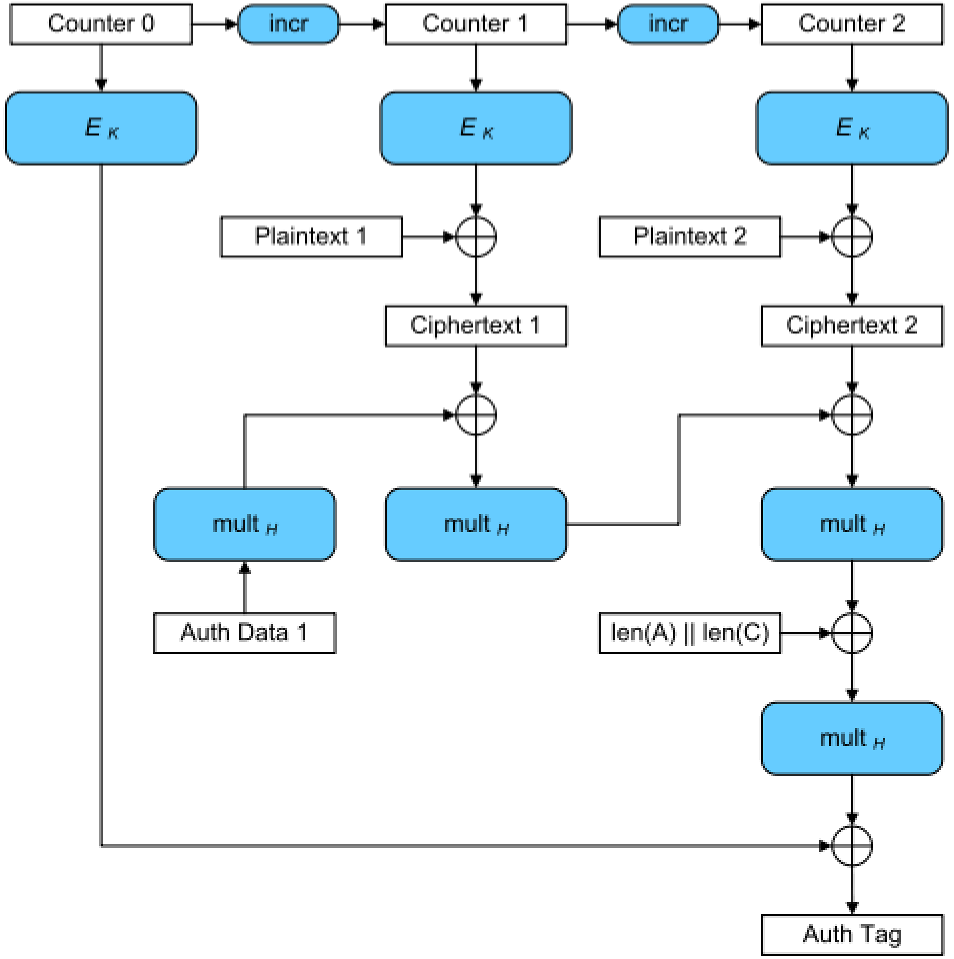
Secure Communication Session

- Defense
  - Counters ($CTR_{A,B}$,$CTR_{B,A}$)
    - Number of messages sent from Alice to Bob ($CTR_{A,B}$) --- initially 0
    - Number of messages sent from Bob to Alice ($CTR_{B,A}$) --- initially 0
    - Protects against Re-ordering and Replay attacks
  - Directionality Bit
    - $b_{A,B} = 0$ and $b_{B,A} = 1$ (e.g., since A < B)

- Alice: To send m to Bob, set $c=Enc_K(b_{A,B} \parallel CTR_{A,B} \parallel m)$, send c and increment $CTR_{A,B}$
- Bob: Decrypts c, (if $\perp$ then reject), obtain $b \parallel CTR \parallel m$
  - If $CTR \neq CTR_{A,B}$ or $b \neq b_{A,B}$ then reject
  - Otherwise, output m and increment $CTR_{A,B}$

Galois Counter Mode (GCM)

- AES-GCM is an Authenticated Encryption Scheme
- **Bonus:** Authentication Encryption with Associated Data
  - Ensure integrity of ciphertext
  - Attacker cannot even generate new/valid ciphertext!
  - Ensures attacker cannot tamper with associated packet data
    - Source IP
    - Destination IP
    - Why can’t these values be encrypted?
- Encryption is largely parallelizable!



Cryptography
CS 555

Week 5:
- Cryptographic Hash Functions
- HMACs
- Generic Attacks
- Random Oracle Model
- Applications of Hashing

Readings: Katz and Lindell Chapter 5, Appendix A.4

Week 5: Topic 1: Cryptographic Hash Functions

Hash Functions

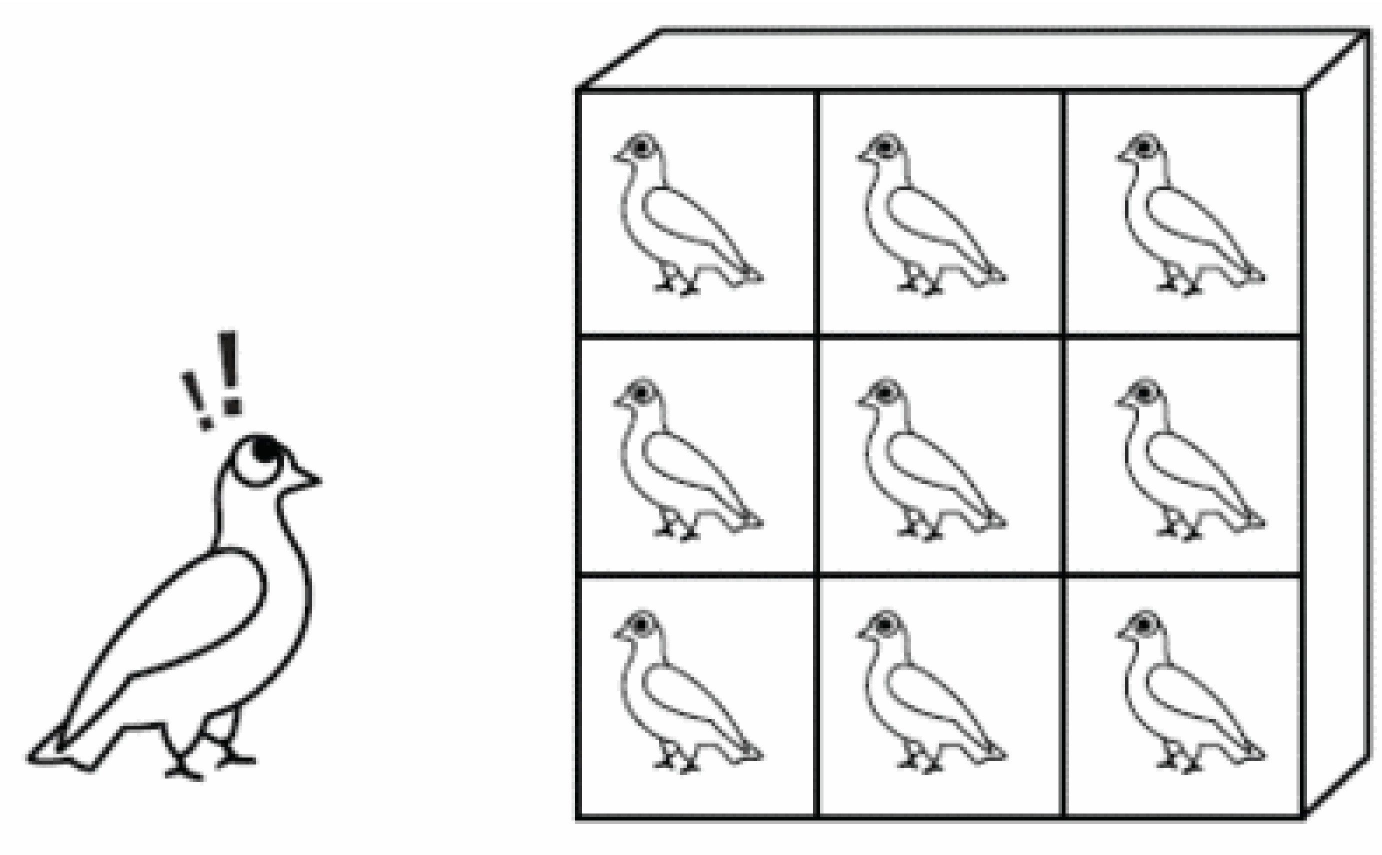
$$H(x)=y$$

Long Input: $\mathbf{x}$

Short Output: **y** s.t.
$|\mathbf{y}| \ll |\mathbf{x}|$

Pigeonhole Principle

"You cannot fit 10 pigeons into 9 pigeonholes"



Hash Collisions

By Pigeonhole Principle there must exist x and y s.t.

$$H(x) = H(y)$$

Classical Hash Function Applications

- Hash Tables
  - O(1) lookup*

- “Good hash function” should yield “few collisions”

* Certain terms and conditions apply

Collision-Resistant Hash Function

Intuition: Hard for computationally bounded attacker to find *any pair* x, x' s.t.

$$H(x) = H(x')$$

How to formalize this intuition?

- **Attempt 1:** For all PPT A,

$$\Pr[A(1^n) = (x, x') \; s.t \; H(x) = H(x')] \leq negl(n)$$

- **The Problem:** Let x, x' be given s.t. $H(x) = H(x')$

$$A_{x,x'}(1^n) = (x, x')$$

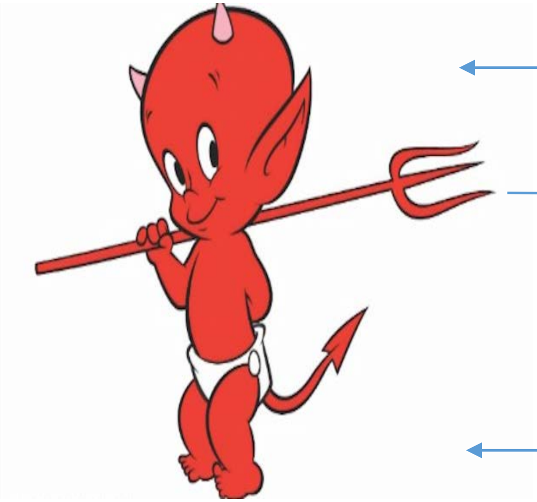
- We are assuming that |x| > |H(x)|. Why?
  - H(x)=x is perfectly collision resistant! (but with no compression)

Keyed Hash Function Syntax

- Two Algorithms
  - $\mathrm{Gen}(1^n; R)$ (Key-generation algorithm)
    - **Input:** Random Bits R
    - **Output:** ~~Secret~~ key s
  - $H^s(m)$ (Hashing Algorithm)
    - **Input:** key s and message $\mathrm{m} \in \{0,1\}^*$ (unbounded length)
    - **Output:** hash value $H^s(m) \in \{0,1\}^{\ell(n)}$

- Fixed length hash function
  - $m \in \{0,1\}^{\ell'(n)}$ with $\ell'(n) > \ell(n)$

Collision Experiment ($HashColl_{A,\Pi}(n)$)



s

x_1, x_2

$$HashColl_{A,\Pi}(n) = \begin{cases} 1 & \text{if } H^s(x_1) = H^s(x_2) \\ 0 & \text{otherwise} \end{cases}$$

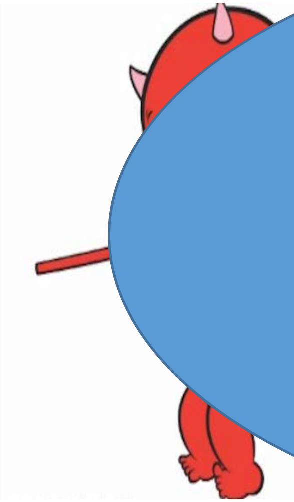
$s = \mathrm{Gen}(1^n; R)$

Definition: (Gen,H) is a collision resistant hash function if

$$\forall PPT\ A\ \exists \mu\ (\text{negligible})\ \text{s.t}$$

$$\Pr\left[HashColl_{A,\Pi}(n) = 1\right] \leq \mu(n)$$

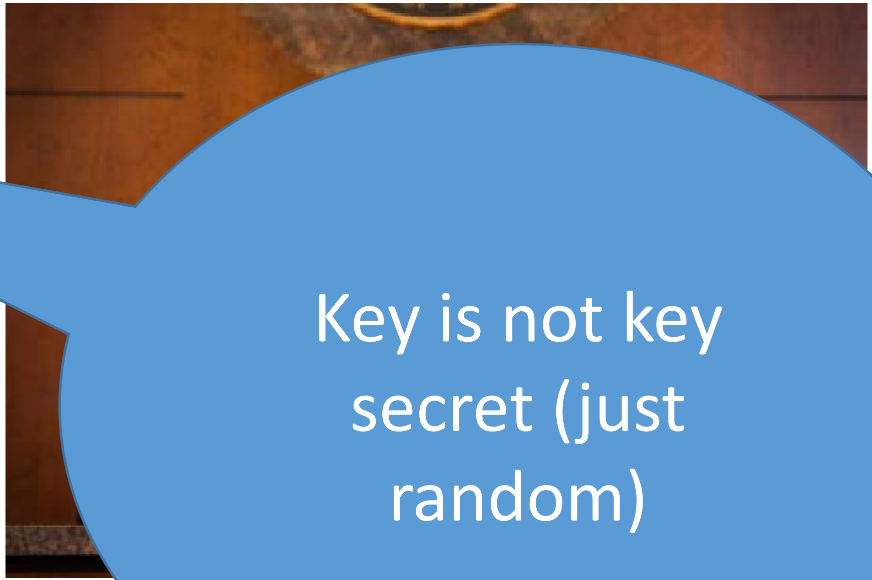
Collision Experiment ($HashColl_{A,\Pi}(n)$)



For simplicity we will sometimes just say that H (or H^s) is a collision resistant hash function

$= H^s(x_2)$

ise



Key is not key secret (just random)

Definition: (Gen,H) is a collision resistant hash function if

$$\forall PPT\ A\ \exists \mu \text{ (negligible) s.t}$$

$$\Pr\left[HashColl_{A,\Pi}(n)=1\right] \le \mu(n)$$

Theory vs Practice

- Most cryptographic hash functions used in practice are un-keyed
  - Examples: MD5, SHA1, SHA2, SHA3
- Tricky to formally define collision resistance for keyless hash function
  - There is a PPT algorithm to find collisions
  - We just usually can't find this algorithm ☺

Formalizing Human Ignorance:
Collision-Resistant Hashing without the Keys

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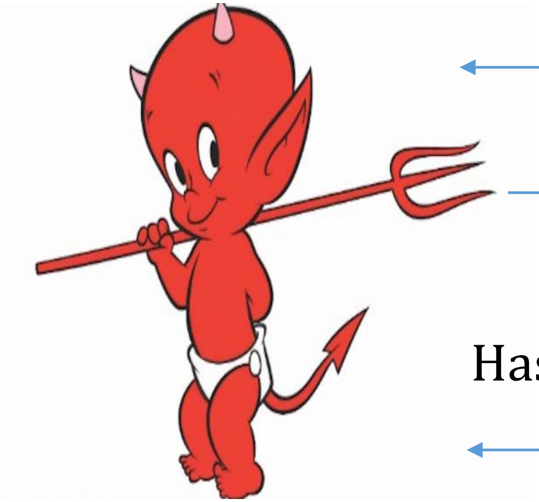
31 January 2007

Abstract. There is a foundational problem involving collision-resistant hash-functions: com-

Weaker Requirements for Cryptographic Hash

- Target-Collision Resistance



s,x

x’

$$\mathrm{HashTgtColl}_{A,\Pi}(n) = \begin{cases} 1 & \text{if } H^s(x') = H^s(x) \\ 0 & \text{otherwise} \end{cases}$$

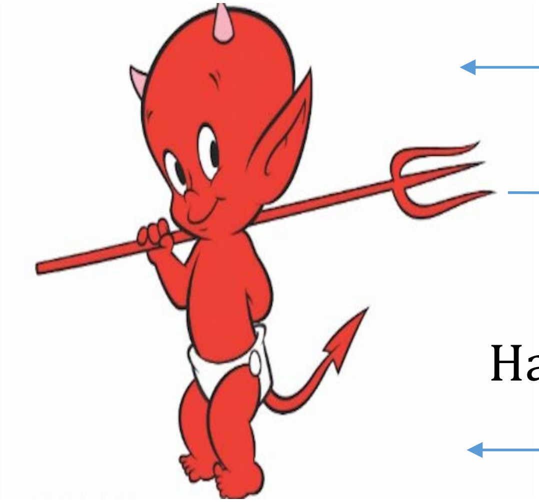
$s = \mathrm{Gen}(1^n; R)$

$x \in \{0,1\}^n$

Question: Why is collision resistance stronger?

Weaker Requirements for Cryptographic Hash

- Preimage Resistance (One-Wayness)



s, y

x

$$\mathrm{HashPreImgRes}_{A,\Pi}(n) = \begin{cases} 1 & \text{if } H^{s}(x) = y \\ 0 & \text{otherwise} \end{cases}$$

$\mathrm{s} = \mathrm{Gen}(1^n; R)$

$y \in \{0,1\}^{\ell(n)}$

Question: Why is collision resistance stronger?

Merkle-Damgård Transform

- Most cryptographic hash functions accept fixed length inputs
- What if we want to hash arbitrary length strings?

Construction: (Gen,h) fixed length hash function from 2n bits to n bits

$$H^s(x_1, \ldots, x_d) = h^s\Big(h^s\Big(h^s\Big(h^s\Big(\ldots h^s(0^n \parallel x_1)\Big) \parallel x_{d-1}\Big) \parallel x_d\Big) \parallel |x|\Big)$$

Merkle-Damgård Transform

Construction: (Gen,h) fixed length hash function from 2n bits to n bits

$H^s(x)$ =

1. Break x into n bit segments $x_1,..,x_d$ (pad last block by 0's)
2. $z_0 = 0^n$ (initialization)
3. For i = 1 to d
   1. $z_i = h^s(z_{i-1} \parallel x_i)$
4. Output $z_{d+1} = h^s(z_d \parallel L)$ where L encodes $|x|$ as an n-bit string

Merkle-Damgård Transform

Theorem: If (Gen,h) is collision resistant then so is (Gen,H)

Proof: Show that any collision in H^s yields a collision in h^s. Thus a PPT attacker for (Gen,H) can be transformed into PPT attacker for (Gen,h).

Suppose that

$$H^s(x) = H^s(x')$$

(note x and x' may have different lengths)

Merkle-Damgård Transform

Theorem: If (Gen,h) is collision resistant then so is (Gen,H)

Proof: Suppose that

$$H^s(x) = H^s(x')$$

Case 1: |x|=|x'| (proof for case two is similar)

$$H^s(x) = z_d = h^s(z_{d-1} \parallel x_d) = H^s(x') = z'_d = h^s(z'_{d-1} \parallel x'_d)$$

$$z_{d-1} \parallel x_d =? \, z'_{d-1} \parallel x'_d$$

No → Found collision

Yes?

$$z_{d-1} = h^s(z_{d-2} \parallel x_{d-1}) = h^s(z'_{d-2} \parallel x'_{d-1}) = z'_{d-1}$$

Merkle-Damgård Transform

Theorem: If (Gen,h) is collision resistant then so is (Gen,H)

Proof: Suppose that

$$H^s(x) = H^s(x')$$

Case 1: |x|=|x'| (proof for case two is similar)

If for some i we have $z_{i-1} \parallel x_i \neq z'_{i-1} \parallel x'_i$ then we will find a collision

But x and x' are different!

Week 5: Topic 2: HMACs and Generic Attacks

Keyed Hash Function Syntax

- Two Algorithms
  - $\mathrm{Gen}(1^n; R)$ (Key-generation algorithm)
    - **Input:** Random Bits R
    - **Output:** ~~Secret~~ key s
  - $H^s(m)$ (Hashing Algorithm)
    - **Input:** key s and message $\mathrm{m} \in \{0,1\}^*$
    - **Output:** hash value $H^s(m) \in \{0,1\}^{\ell(n)}$

MACs for Arbitrary Length Messages

$\text{Mac}_K(m)$=

- Select random n/4 bit string r
- Let $t_i = \text{Mac}'_K(r \parallel \ell \parallel i \parallel m_i)$ for i=1,…,d
  - (Note: encode i and ℓ as n/4 bit strings)
- **Output** $\langle r, t_1, \ldots, t_d \rangle$

Theorem 4.8: If Π' is a secure MAC for messages of fixed length n, above construction $\Pi = (\text{Mac}, \text{Vrfy})$ is secure MAC for arbitrary length messages.

MACs for Arbitrary Lengt

Disadvantage 1: Long output

Disadvantages: Lose Strong-MAC Guarantee (Multiple valid MACs of same message)

ri

$\| \; i \; \|$

- (Note: e… and ℓ as n/4 bi…

- **Output** $\langle r, t_1, \ldots, t_d \rangle$

Randomized Construction (no canonical verification). Disadvantage?

Theorem 4.8: If Π' i… above constructio… messages.

Hash and MAC Construction

Start with $\Pi = (\mathrm{Mac}, \mathrm{Vrfy})$, a secure MAC for messages of fixed length, and (Gen_H, H) a collision resistant hash function and define Π'

$$Mac'_{\langle K_M, S\rangle}(m) = Mac_{K_M}\left(H^s(m)\right)$$

$$Vrfy'_{\langle K_M, S\rangle}(m,t) = Vrfy_{K_M}(H^s(m), t)$$

Theorem 5.6: Π' is a secure MAC for arbitrary length message assuming that Π is a secure MAC and (Gen_H, H) is collision resistant.

Note: If $\mathrm{Vrfy}_{K_M}(m,t)$ is canonical then $\mathrm{Vrfy}'_{\langle K_M, S\rangle}(m,t)$ is canonical.

Hash and MAC Construction

Start with (Mac,Vrfy) a MAC for messages of fixed length and (Gen_H,H) a collision resistant hash function

$$Mac'_{\langle K_M,S\rangle}(m) = Mac_{K_M}\left(H^s(m)\right)$$

Theorem 5.6: Above construction is a secure MAC.

Proof Intuition: If attacker successfully forges a valid MAC tag t' for unseen message m' then either

- **Case 1:** $H^s(m') = H^s(m_i)$ for some previously requested message m_i
- **Case 2:** $H^s(m') \neq H^s(m_i)$ for every previously requested message m_i

Hash and MAC Construction

Theorem 5.6: Above construction is a secure MAC.

Proof Intuition: If attacker successfully forges a valid MAC tag t' for unseen message m' then either

- **Case 1:** $H^s(m') = H^s(m_i)$ for some previously requested message m_i
  - **Attacker can find hash collisions!**
- **Case 2:** $H^s(m') \neq H^s(m_i)$ for every previously requested message m_i
  - **Attacker forged a valid new tag on the "new message" $\boldsymbol{H^s(m')}$**
  - **Violates security of the original fixed length MAC**

Recap

- Definition of Collision Resistant Hash Functions (Gen,H)
  - Definitional challenges
  - Gen(1^n) outputs a public seed.
- Merkle-Damgård construction to hash arbitrary length strings
  - Proof of correctness
- Hash and MAC construction
  - Proof of correctness

MAC from Collision Resistant Hash

- Failed Attempt:

$$Mac_{\langle k,S \rangle}(m) = H^s(k \parallel m)$$

Broken if H^s uses Merkle-Damgård Transform. Let m_3 encode length of $m_1 \parallel m_2$

$$Mac_{\langle k,S \rangle}(m_1 \parallel m_2 \parallel m_3) = h^s(h^s(h^s(h^s(h^s(0^n \parallel k) \parallel m_1) \parallel m_2) \parallel m_3) \parallel L_3)$$
$$= h^s\left(Mac_{\langle k,S \rangle}(m_1 \parallel m_2) \parallel L_3\right)$$

Why does this mean $\boldsymbol{Mac_{\langle k,S \rangle}}$ is broken?

HMAC

$$Mac_{\langle k,S\rangle}(m) = H^s\Big((k \oplus \text{opad}) \parallel H^s\big((k \oplus \text{ipad}) \parallel m\big)\Big)$$

ipad?

HMAC

$$Mac_{\langle k,S\rangle}(m) = H^s\Big((k \oplus \text{opad}) \parallel H^s\big((k \oplus \text{ipad}) \parallel m\big)\Big)$$

$$\text{ipad} = \text{inner pad}$$
$$\text{opad} = \text{outer pad}$$

Both ipad and opad are fixed constants.

Why use key twice?

Allows us to prove security from *weak collision resistance* of H^s

HMAC Security

$$Mac_{\langle k,S\rangle}(m) = H^s\left((k \oplus \mathrm{opad}) \parallel H^s\big((k \oplus \mathrm{ipad}) \parallel m\big)\right)$$

Theorem (Informal): Assuming that H^s is weakly collision resistant and that (certain other plausible assumptions hold) this is a secure MAC.

Weak Collision Resistance: Give attacker oracle access to $f(m) = \ H^s(k \parallel m)$ (secret key k remains hidden).

Attacker Goal: Find distinct m,m' such that $f(m) = f(m')$

HMAC in Practice

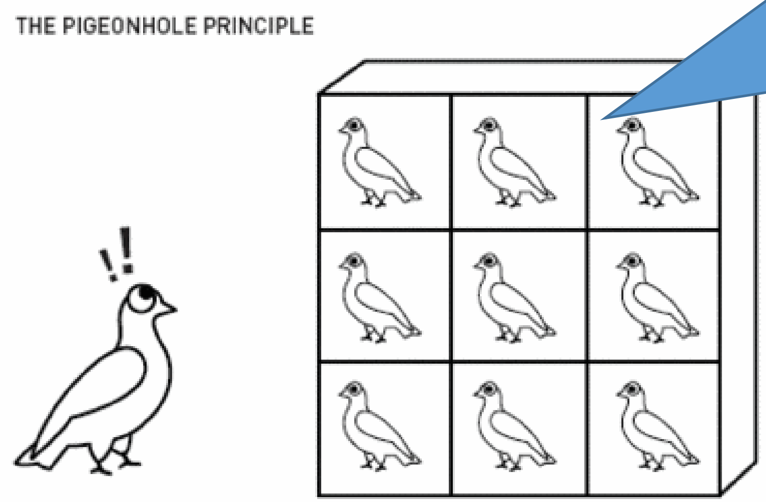
- MD5 can no longer be viewed as collision resistant

- However, HMAC-MD5 remained unbroken after MD5 was broken
  - Gave developers time to replace HMAC-MD5
  - Nevertheless, don't use HMAC-MD5!
- HMAC-SHA1 still seems to be okay (temporarily), despite collision
- HMAC is efficient and unbroken
  - CBC-MAC was not widely deployed because it is "too slow"
  - Instead practitioners often used heuristic constructions (which were breakable)

Finding Collisions

- Ideal Hashing Algorithm
  - Random function H from $\{0,1\}^*$ to $\{0,1\}^\ell$
  - Suppose attacker has oracle access to H(.)

- **Attack 1:** Evaluate H(.) on $2^\ell+1$ distinct inputs.



Can we do better?

Birthday Attack for Finding Collisions

- Ideal Hashing Algorithm
  - Random function H from {0,1}* to $\{0,1\}^{\ell}$
  - Suppose attacker has oracle access to H(.)

- **Attack 2:** Evaluate H(.) on $q = 2^{(\ell/2)+1} + 1$ distinct inputs $x_1,\ldots,x_q$.

$$\Pr[\text{No Collision}] = \Pr[\forall i < j.\, H(\mathrm{x}_i) \neq H(\mathrm{x}_j)]$$

$$= \Pr[\boldsymbol{D_2}] \prod_{i=3}^{q} Pr\left[\boldsymbol{D_i} \middle| \boldsymbol{D_{i-1}}, \ldots, \boldsymbol{D_2}\right]$$

$$\boldsymbol{D_i = event\ that}\ H(x_i) \neq H(x_{i-11}), \ldots, H(x_1)$$

Birthday Attack for Finding Collisions

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$$\Pr[\forall i < j.\, H(x_i) \neq H(x_j)] =$$

$$1 \times \overbrace{\underbrace{\left(1 - \frac{1}{2^\ell}\right)}_{}}^{\overbrace{\Pr[H(x_2) \neq H(x_1)]}^{\boldsymbol{D_2}}} \times \overbrace{\left(1 - \frac{2}{2^\ell}\right)}^{\Pr[\boldsymbol{D_3} \mid \boldsymbol{D_2}]} \times \cdots \times \overbrace{\left(1 - \frac{2^{(\ell/2)+1}}{2^\ell}\right)}^{\Pr\left[\boldsymbol{D_q} \middle| \boldsymbol{D_{q-1}},\ldots,\boldsymbol{D_2}\right]}$$

Birthday Attack for Finding Collisions

- Ideal Hashing Algorithm
  - Random function H from $\{0,1\}^*$ to $\{0,1\}^\ell$
  - Suppose attacker has oracle access to H(.)

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$$\Pr[\forall i < j.\, H(x_i) \neq H(x_j)] = 1\left(1 - \frac{1}{2^\ell}\right)\left(1 - \frac{2}{2^\ell}\right)\left(1 - \frac{3}{2^\ell}\right)\ldots\left(1 - \frac{2^{(\ell/2)+1}}{2^\ell}\right)$$

$$\approx \exp\left(\frac{-q(q-1)}{2^{\ell+1}}\right)$$

Birthday Attack for Finding Collisions

- Ideal Hashing Algorithm
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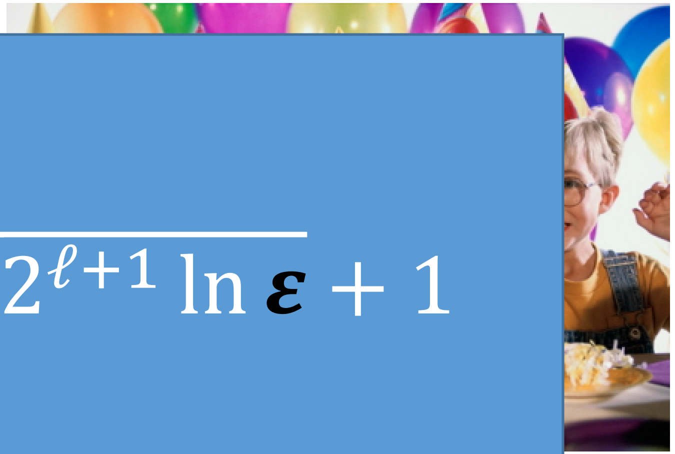
$$\approx \exp\left(\frac{-q(q-1)}{2^{\ell+1}}\right) < \exp\left(\frac{-4 2^{\ell}}{2^{\ell+1}}\right) = e^{-2} < \frac{1}{2}$$

Birthday Attack for Finding Collisions



- Ideal Hashing Algorit
  - Random function H f
  - Suppose attacker has

$$\exp\left(\frac{-q(q-1)}{2^{\ell+1}}\right) < \boldsymbol{\varepsilon} \text{ for } q > \sqrt{2^{\ell+1}\ln\boldsymbol{\varepsilon}} + 1$$

- **Attack 2:** Evaluate H(

$$\Pr[\forall i < j.\, H(\mathrm{x_i}) \neq H(\mathrm{x_j})] = \left(1 - \frac{1}{2^{\ell}}\right)\left(1 - \frac{2}{2^{\ell}}\right)\left(1 - \frac{3}{2^{\ell}}\right)\ldots\left(1 - \frac{2^{(\ell/2)+1}}{2^{\ell}}\right)$$

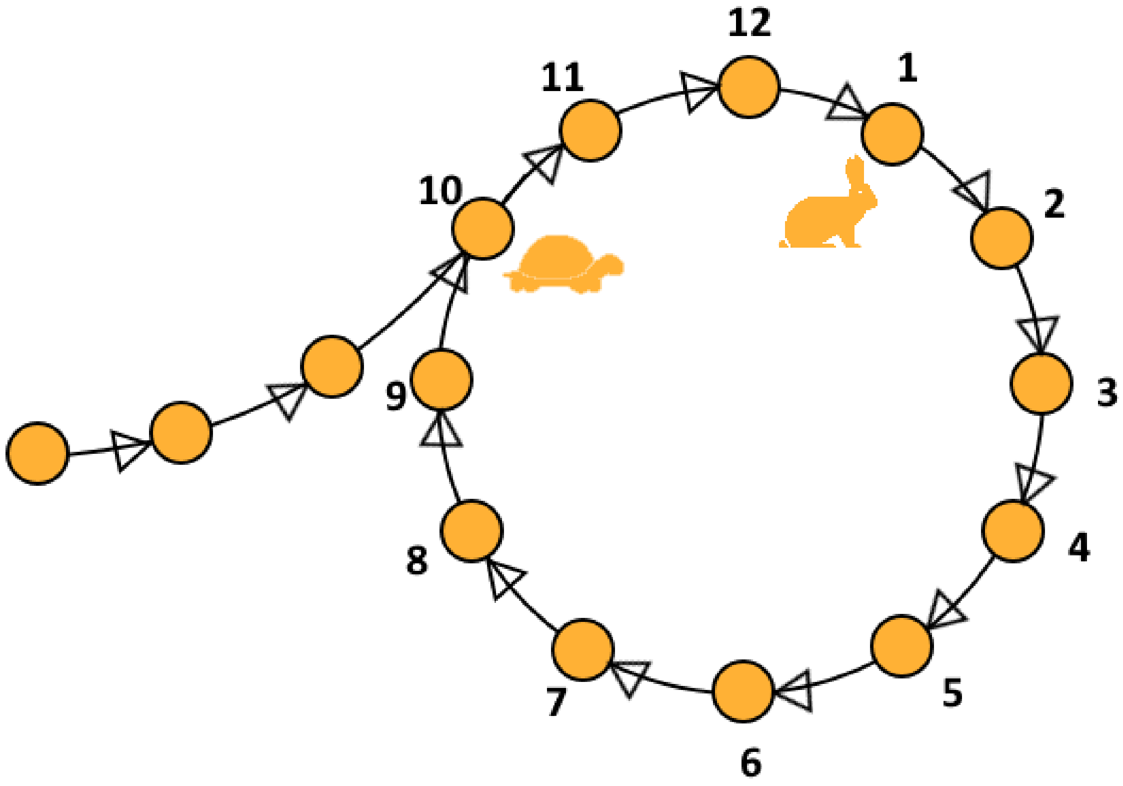
$$\approx \exp\left(\frac{-q(q-1)}{2^{\ell+1}}\right) < \exp\left(\frac{-4 2^{\ell}}{2^{\ell+1}}\right) = e^{-2} < \frac{1}{2}$$

Birthday Attack for Finding Collisions

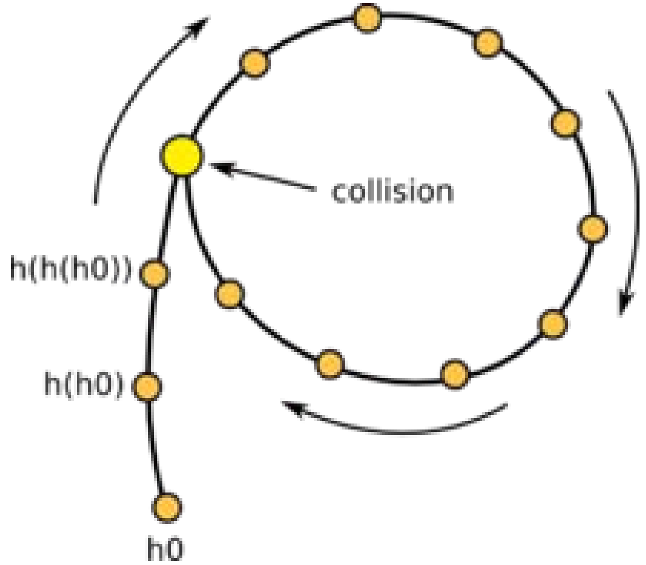
- Ideal Hashing Algorithm
  - Random function H from $\{0,1\}^*$ to $\{0,1\}^{\ell}$
  - Suppose attacker has oracle access to H(.)

- **Attack 2:** Evaluate H(.) on $q = 2^{(\ell/2)+1} + 1$ distinct inputs $x_1,\ldots,x_q$.
- Store values $(x_i, H(x_i))$ in a hash table of size q
  - Requires time/space $O(q) = O\left(\sqrt{2^{\ell}}\right)$
  - Can we do better?

Floyd's Cycle Finding Algorithm



- A cycle denotes a hash collision
- Occurs after $O\left(2^{\ell/2}\right)$ steps by birthday paradox
- First attack phase detects cycle
- Second phase identifies collision

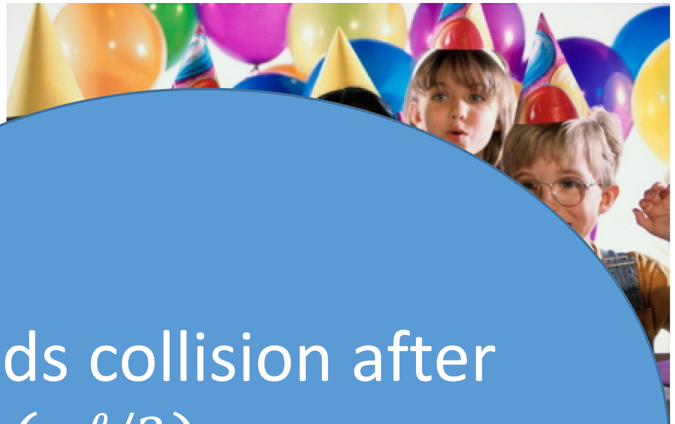


- Analogy: Cycle detection in linked list
- Can traverse "linked list" by computing H

Small Space Birthday Attack

- **Attack 2:** Select random x_0, define $x_i = H(x_{i-1})$
  - Initialize: $x=x_0$ and $x'=x_0$
  - Repeat for i=1,2,…
    - x:=H(x) now $x = x_i$
    - x':=H(H(x')) now $x' = x_{2i}$
    - If x=x' then break
  - Reset $x=x_0$ and set x'=x
  - Repeat for j=1 to i
    - If H(x) = H(x') then output x,x'
    - Else x:= H(x), x' = H(x) Now $x=x_j$ AND $x' = x_{i+j}$

Small Space Birthday Attack



- **Attack 2:** Select random x_0, define $x_i = H(x_{i-1})$
  - Initialize: $x=x_0$ and $x'=x_0$
  - Repeat for i=1,2,…
    - $x:=H(x)$ now $x = x_i$
    - $x':=H(H(x'))$ now $x' = x_{2i}$
    - If $x=x'$ then break
  - Reset $x=x_0$ and set $x'=x$
  - Repeat for j=1 to i
    - If $H(x) = H(x')$ then output x,x'
    - Else $x:= H(x)$, $x' = H(x)$ Now $x=x_j$ AND $x' = x_{i+j}$

Finds collision after $O\left(2^{\ell/2}\right)$ steps in expectation

Small Space Birthday Attack

- Can be adapted to find "meaningful collisions" if we have a large message space $O(2^{\ell})$

- **Example**: $S = S_1 \cup S_2$ with $|S_1| = |S_2| = 2^{\ell-1}$
  - S_1 = Set of positive recommendation letters
  - S_2 = Set of negative recommendation letters

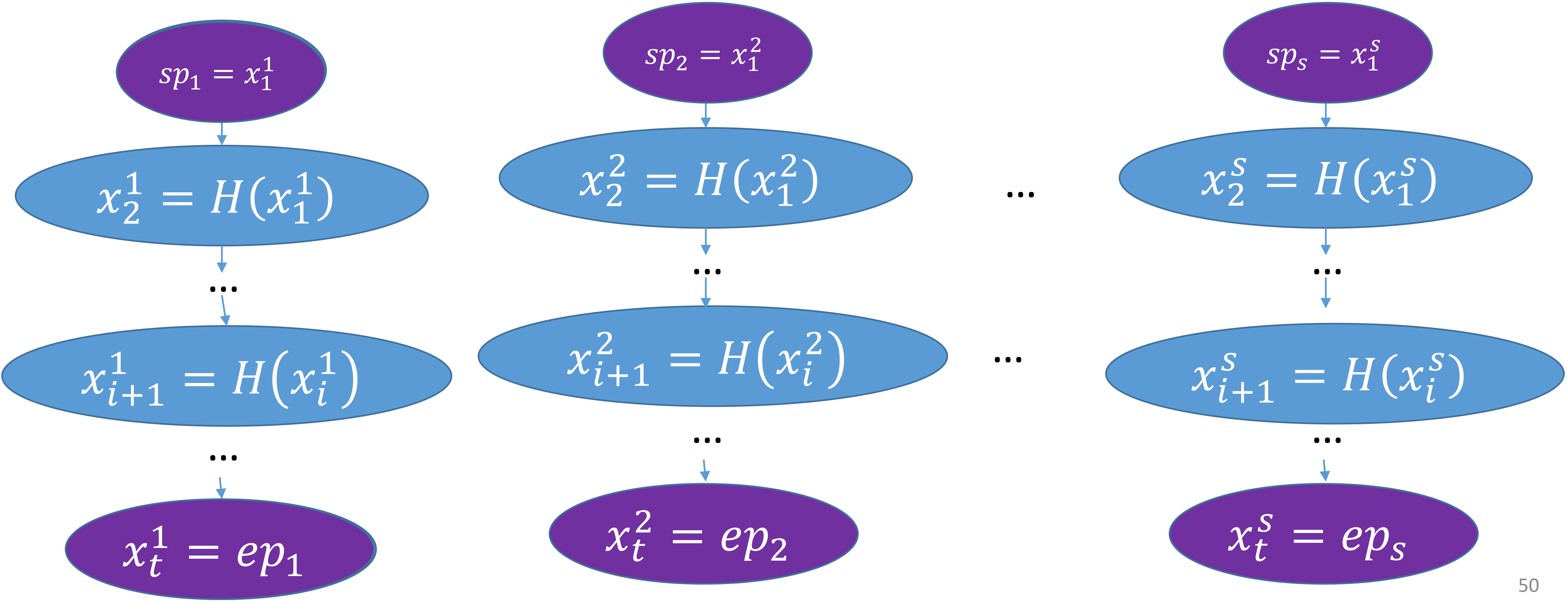
- **Goal**: find $z_1 \in S_1$, $z_2 \in S_2$, such that $H(z_1) = H(z_2)$

- Can adapt previous attack by assigning unique binary string $b(x) \in \{0,1\}^{\ell}$ of length to each $x \in S$

$$x_i = H(b(x_{i-1}))$$

Targeted Collision Attacks

- Precomputation ($t \times s$ steps, $2s$ memory)



Targeted Collision Attacks

- Precomputation ($t \times s$ steps, $2s \times \ell$ memory)

$sp_j = x_1^j$

… $x_2^j = H(x_1^j)$ …

…

… $x_{i+1}^j = H(x_i^j)$ …

…

$x_t^j = ep_j$

- **Goal:** Find collision for target $y = H(x)$

$y_0 = y$

$y_1 = H(y_0)$

…

$y_i = H(y_{i-1})$

…

$y_k = ep_j$

Targeted Collision Attacks

- Precomputation ($t \times s$ steps, $2s \times \ell$ memor

Suppose $y = x_i^j$ for some $i \le t, j \le s$
→
$$y = H\left(x_{i-1}^j\right) = H^{i-1}\left(sp_j\right)$$
(takes t steps to recover x_{i-1}^j from sp_j)

- **Goal:** Find col… n for target $y = H(x)$

$sp_j = x_1^j$

$x_2^j =$

…

$t \times s > 2^{\ell+2}$ → good chance that
$y = x_i^j$ for some $i \le t, j \le s$

…

x_{i+1}^j

… Not quite true…chains can intersect and may not represent $t \times s$ *distinct* points

x

$y_0 = y$

$y_1 = H(y_0)$

…

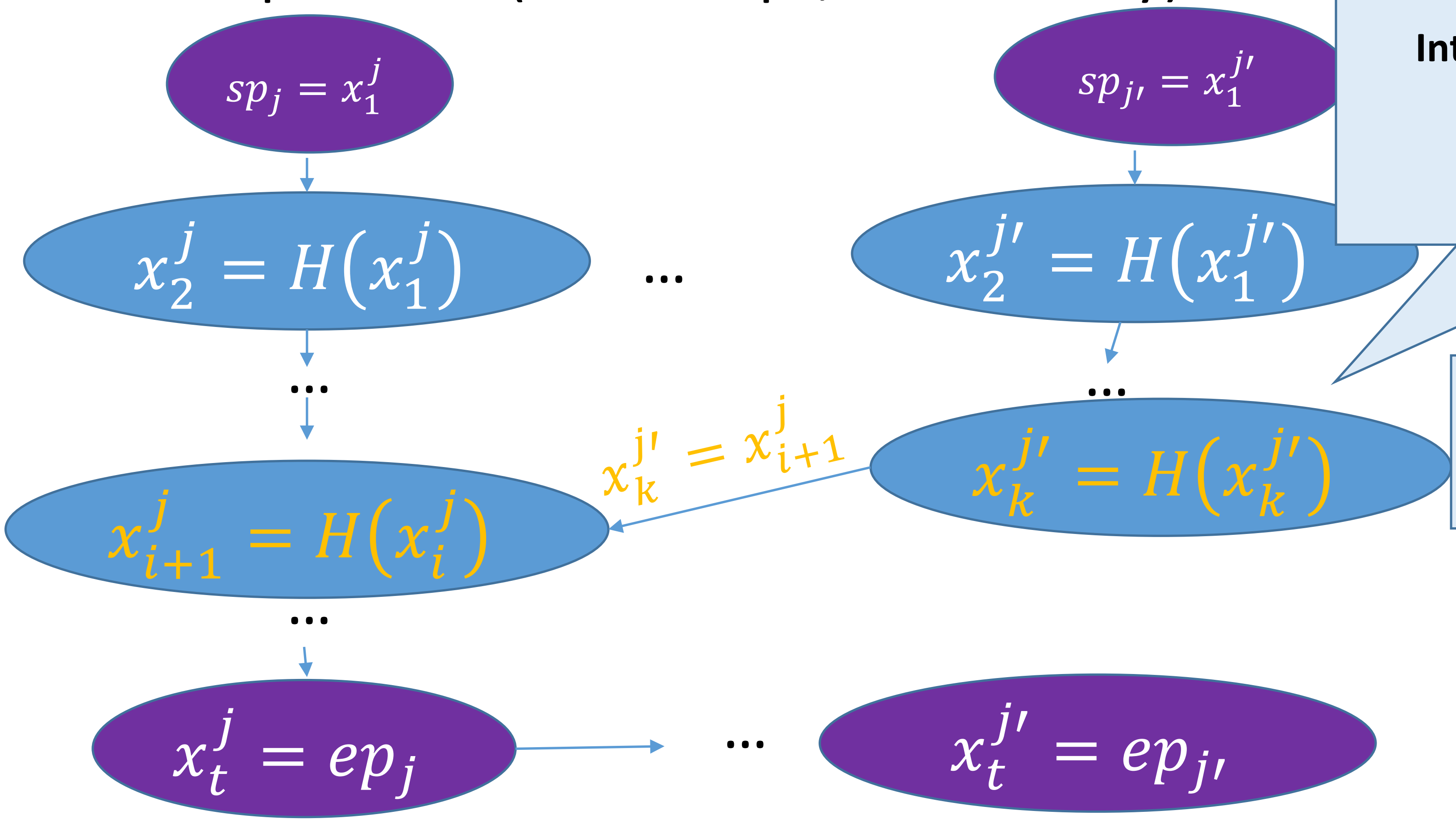
$y_i = H(y_{i-1})$

…

$y_k = ep_j$

Intersecting Chains

- Precomputation ($t \times s$ steps, $2s$ memory)

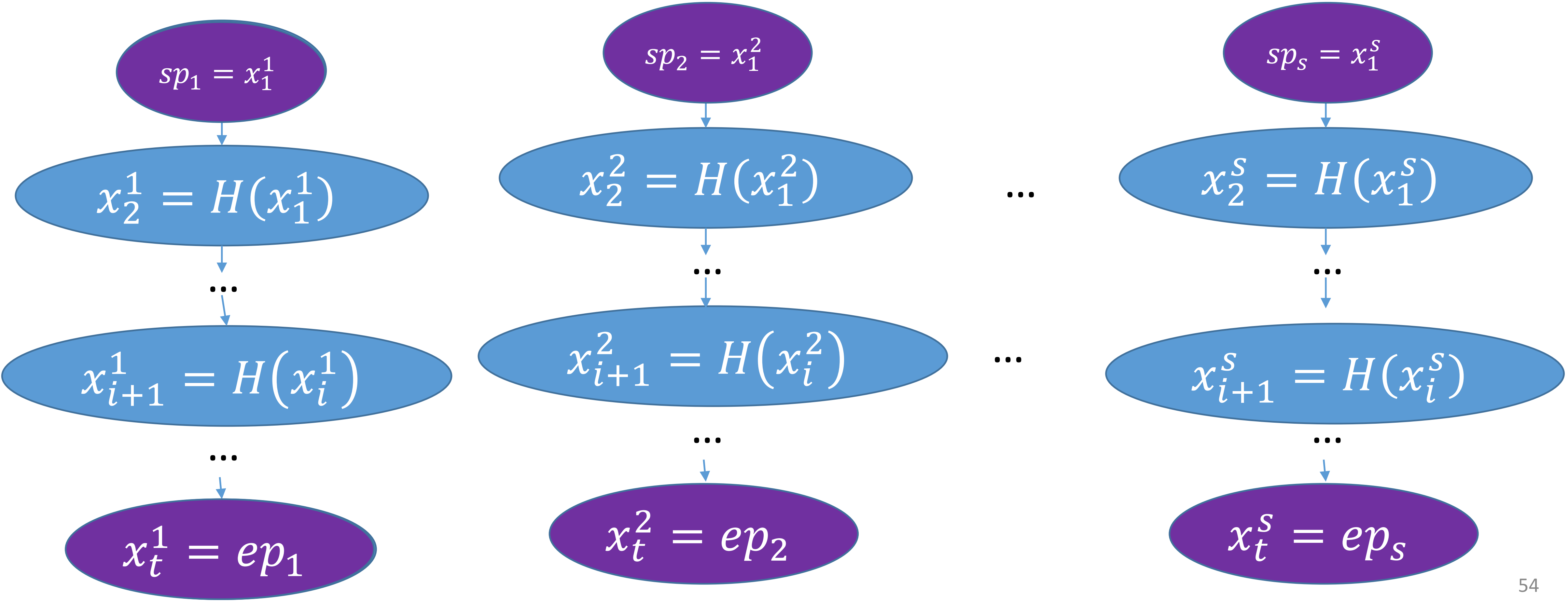


Intersecting chains contain $\ll st$ distinct points.

After initial intersection the chains merge together ☹

Targeted Collision Attacks

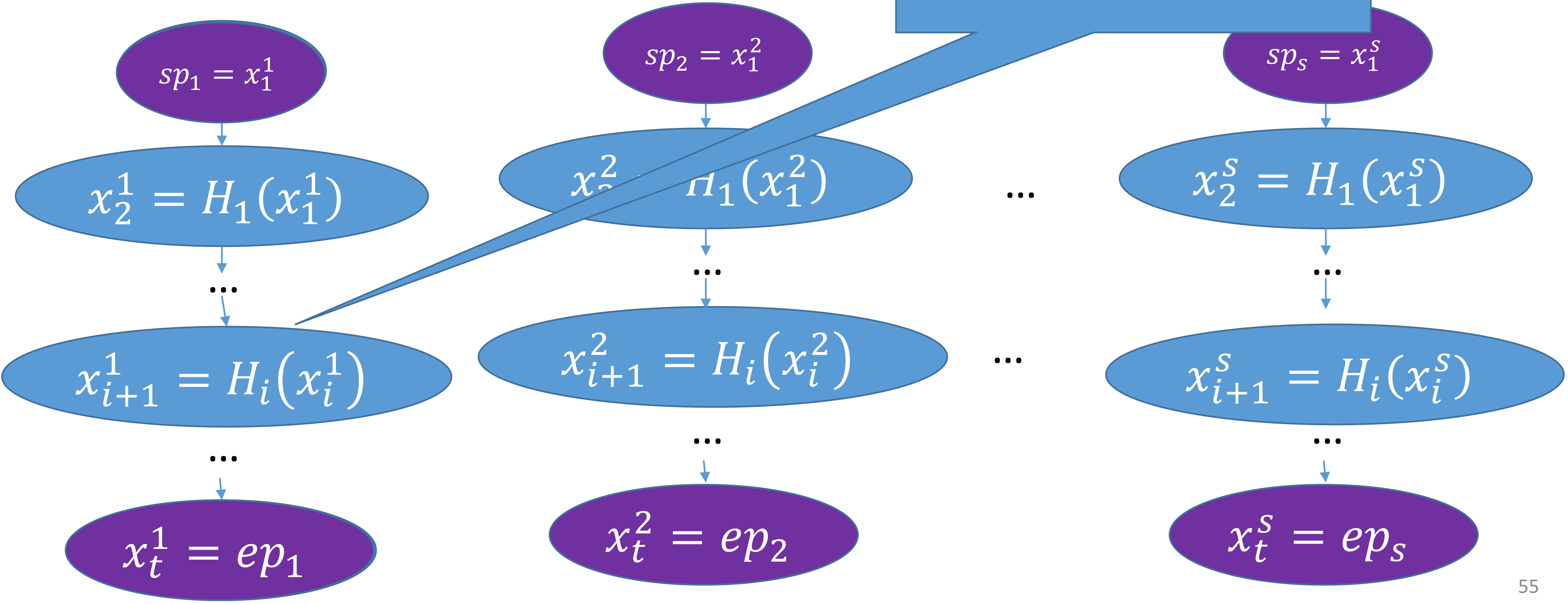
- Precomputation ($t \times s$ steps, $2s$ memory)



Targeted Collision Attacks

- Precomputation ($t \times s$ steps, $2s$ mem

$$H_i(x) = H\left(F_{K_i}(x)\right)$$



Targeted Collision Attacks

- Precomputation ($t \times s$ steps, $2s$ mem

$H_i(x) = H\left(F_{K_i}(x)\right)$

$sp_1 = x_1^1$

$x_2^1 = H_1(x_1^1)$

…

$x_{i+1}^1 = H_i(x_i^1)$

…

$x_t^1 = ep_1$

$sp_2 = x_1^2$

$sp_s = x_1^s$

Ensures Chains Contain: $\Omega(st)$ distinct points
small overlap between chains

Untangling Chains: If $x_i^1 = x_j^2$ with $i \neq j$
then (whp) $H_i(x_i^1) \neq H_j(x_j^2)$

Targeted Collision Attacks

- Precomputation ($t \times s$ steps, $2s \times \ell$ memor

Suppose $y = x_i^j$ for some $i \le t, j \le s$
→
$$y = H_{i-1}\left(F_{K_{i-1}}\left(x_{i-1}^j\right)\right)$$
(takes t steps to recover x_{i-1}^j from sp_j)

- **Goal:** Find col n for target $y = H(x)$

$sp_j = x_1^j$

$t \times s > 2^{\ell+2}$ → good chance that
$y = x_i^j$ for some $i \le t, j \le s$

…

$x_2^j =$

…

…

$x_{i+1}^j =$

***False Positive*:**
$y \ne x_i^j$ for any $i \le t, j \le s$
(expect about $O(st^2/2^\ell)$)
→
Running Time: $O(st^3/2^\ell)$

$x_t^j = ep_j$

$y_0 = y$

$y_1 = H_1(y_0)$

…

$y_i = H_{i-1}(y_{i-1})$

…

$y_k = ep_j$

Targeted Collision Attacks

- Precomputation ($t \times s$ steps, $2s \times \ell$ memory)

…l: Find collision for target $y = H(x)$

…

…

Set $s = 2^{\frac{2\ell}{3}+1}, t = 2^{\frac{\ell}{3}+1}$

Precomputation: $\mathbf{O}(2^{\ell})$

Space: $\mathbf{O}\left(2^{\frac{2\ell}{3}} \times \ell\right)$

Collision Search: $\boldsymbol{O}\left(2^{\frac{2\ell}{3}}\right)$

$y_0 = y$

Amortized cost to find $2^{\frac{\ell}{3}}$ targeted collisions

$y_i = H_{i-1}(y_{i-1})$

…

$y_k = ep_j$

Applications

- Key-Recovery Attacks on Block Cipher $E: \mathcal{K} \times \{0,1\}^n \rightarrow \{0,1\}^n$
  - Pre-Computation: $O(|\mathcal{K}|)$
  - **Crack** $2^{\frac{n}{3}}$ **secret keys in total time** $\boldsymbol{O}(|\mathcal{K}|)$ with space $\mathrm{s} = \boldsymbol{O}\left(2^{\frac{2n}{3}}\right)$
  - Run prior attack with "hash function" $\mathrm{H}: \{0,1\}^n \rightarrow \{0,1\}^n$
    - $\mathrm{H}(K) = E_K(r)$ for some random (fixed) $r \in \{0,1\}^n$
- Password Cracking
  - Attacker is given $H'(x_1)$,…, $H'(x_k)$ for passwords $x_1, \ldots, x_k \in \mathcal{PWDs}$ with $|\mathcal{PWDs}| \ll |\mathcal{K}|$
  - **Goal:** Recover passwords $x_1, \ldots, x_k$
  - Can crack **all** $k = |\mathcal{PWDs}|^{1/3}$ passwords in total time $\boldsymbol{O}(|\mathcal{PWDs}|)$ with space $s = \boldsymbol{O}\left(|\mathcal{PWDs}|^{2/3}\right)$
  - Domain Challenge: $\mathrm{H}': |\mathcal{PWDs}| \rightarrow \{0,1\}^n$ with $|\mathcal{PWDs}| \ll 2^n$
    - Define (pseudo)random mapping $\mu: \{0,1\}^n \rightarrow \mathcal{PWDs}$
    - Run prior attack with "hash function" $\mathrm{H}: \mathcal{PWDs} \rightarrow \mathcal{PWDs}$ as $\mathrm{H}(x) = \mu\left(\mathrm{H}'(x)\right)$

Week 5: Topic 3:
Random Oracle Model +
Hashing Applications

(Recap) Collision-Resistant Hash Function

Intuition: Hard for computationally bounded attacker to find x,y s.t. H(x) = H(y)

How to formalize this intuition?

- **Attempt 1:** For all PPT A,

$$\Pr\left[A_{x,y}(1^n) = (x, y) \; s.t \; H(x) = H(y)\right] \leq negl(n)$$

- **The Problem:** Let x,y be given s.t. H(x)=H(y)

$$A_{x,y}(1^n) = (x, y)$$

- We are assuming that |x| > |H(x)|. Why?
  - H(x)=x is perfectly collision resistant! (but with no compression)

(Recap) Keyed Hash Function Syntax

- Two Algorithms
  - $\mathrm{Gen}(1^n; R)$ (Key-generation algorithm)
    - Input: Random Bits R
    - Output: ~~Secret~~ key s
  - $H^s(m)$ (Hashing Algorithm)
    - Input: key s and message $\mathrm{m} \in \{0,1\}^*$ (unbounded length)
    - Output: hash value $H^s(m) \in \{0,1\}^{\ell(n)}$

- Fixed length hash function
  - $m \in \{0,1\}^{\ell'(n)}$ with $\ell'(n) > \ell(n)$

When Collision Resistance Isn't Enough

- **Example**: Message Commitment
  - Alice sends Bob: $\mathrm{H}^s(r \parallel m)$ (e.g., predicted winner of NCAA Tournament)
  - Alice can later reveal message (e.g., after the tournament is over)
    - Just send r and m (note: r has fixed length)
    - Why can Alice not change her message?
  - In the meantime Bob shouldn't learn *anything* about m

- **Problem**: Let (Gen,H') be collision resistant then so is (Gen,H)

$$H^s(x_1, \dots, xd) = H'^s(x_1, \dots, xd) \parallel x_d$$

When Collision Resistance Isn't Enough

- **Problem**: Let (Gen,H') be collision resistant then so is (Gen,H)

$$H^s(x_1, \ldots, x_d) = H'^s(x_1, \ldots, x_d) \parallel x_d$$

- (Gen,H) definitely does not hide all information about input $(x_1, \ldots, x_d)$

- **Conclusion**: Collision resistance is not sufficient for message commitment

The Tension

- **Example**: Message Commitment
  - Alice sends Bob: $\mathrm{H}^s(r \parallel m)$ (e.g., predicted winners of NCAA Final Four)
  - Alice can later reveal message (e.g., after the Final Four is decided)
  - In the meantime Bob shouldn't learn anything about m

This is still a reasonable approach in practice!

- No attacks when instantiated with any reasonable candidate (e.g., SHA3)
- Cryptographic hash functions seem to provide "something" beyond collision resistance, but how do we model this capability?

Random Oracle Model

- Model hash function H as a truly random function
- Algorithms can only interact with H as an oracle
  - **Query**: x
  - **Response**: H(x)
- If we submit the same query you see the same response
- If x has not been queried, then the value of H(x) is uniform

- **Real World:** H instantiated as cryptographic hash function (e.g., SHA3) of fixed length (no Merkle-Damgård)

Back to Message Commitment

- **Example**: Message Commitment
  - Alice sends Bob: $\mathrm{H}(r \parallel m)$ (e.g., predicted winners of NCAA Final Four)
  - Alice can later reveal message (e.g., after the Final Four is decided)
    - Just send r and m (note: r has fixed length)
    - Why can Alice not change her message?
  - In the meantime Bob shouldn't learn *anything* about m

- **Random Oracle Model**: Above message commitment scheme is secure (Alice cannot change m + Bob learns nothing about m)
- **Information Theoretic Guarantee** against any attacker with q queries to H

Random Oracle Model: Pros

- It is easier to prove security in Random Oracle Model
- Suppose we are simulating attacker A in a reduction
  - **Extractability**: When A queries H at x we **see this query** and learn x (and can easily find H(x))
  - **Programmability**: We can set the value of H(x) to a value of our choice
    - As long as the value is correctly distribute i.e., close to uniform
- Both **Extractability** and **Programmability** are useful tools for a security reduction!

Random Oracle Model: Pros

- It is easier to prove security in Random Oracle Model
- Provably secure constructions in random oracle model are often much more efficient (compared to provably secure construction is “standard model”
- Sometimes we only know how to design provably secure protocol in random oracle model

Random Oracle Model: Cons

- Lack of formal justification
- Why should security guarantees translate when we instantiate random oracle with a real cryptographic hash function?

- We can construct (contrived) examples of protocols which are
  - Secure in random oracle model…
  - But broken in the real world

Random Oracle Model: Justification

"A proof of security in the random-oracle model is significantly better than no proof at all."

- **Evidence of sound design** (any weakness involves the hash function used to instantiate the random oracle)
- **Empirical Evidence for Security**

"there have been no successful real-world attacks on schemes proven secure in the random oracle model"

Hash Function Application: Fingerprinting

- The hash h(x) of a file x is a unique identifier for the file
  - Collision Resistance → No need to worry about another file y with H(y)=H(y)

- Application 1: Virus Fingerprinting

- Application 2: P2P File Sharing

- Application 3: Data deduplication

Tamper Resistant Storage

$H(m_1)$

m_1

m_1'

Dropbox

Tamper Resistant Storage

File Index	Hash
1	$H(m_1)$
2	$H(m_2)$
3	$H(m_3)$

Disadvantage: Too many hashes to store

Dropbox

m_1, m_2, m_3

Send file 1

m_1'

Tamper Resistant Storage

Disadvantage: Need all files to compute hash m_1,m_2,m_3

$H(m_1,m_2,m_3)$

m_1,m_2,m_3

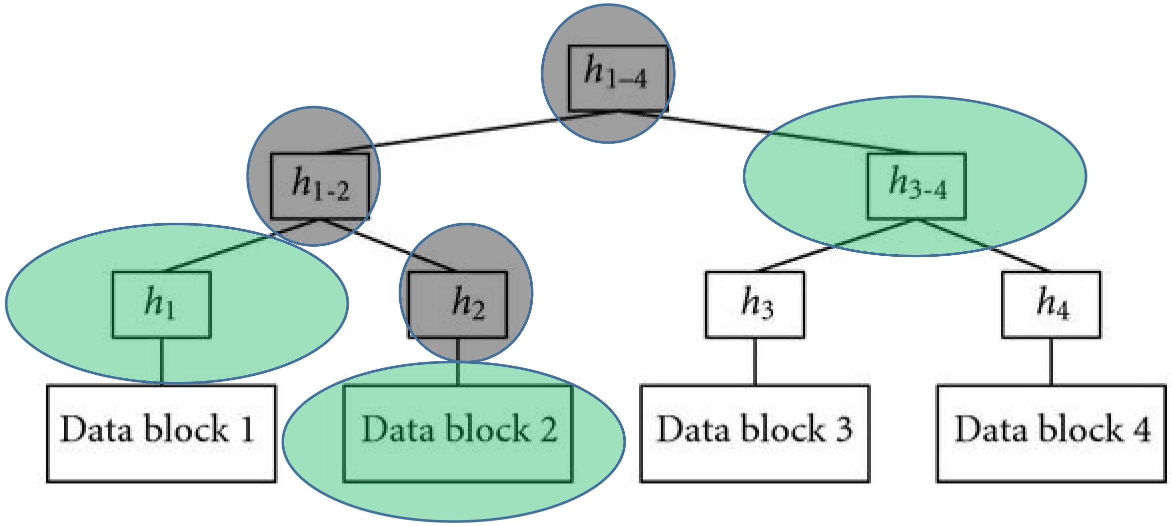
Send file 1

m_1'

Dropbox

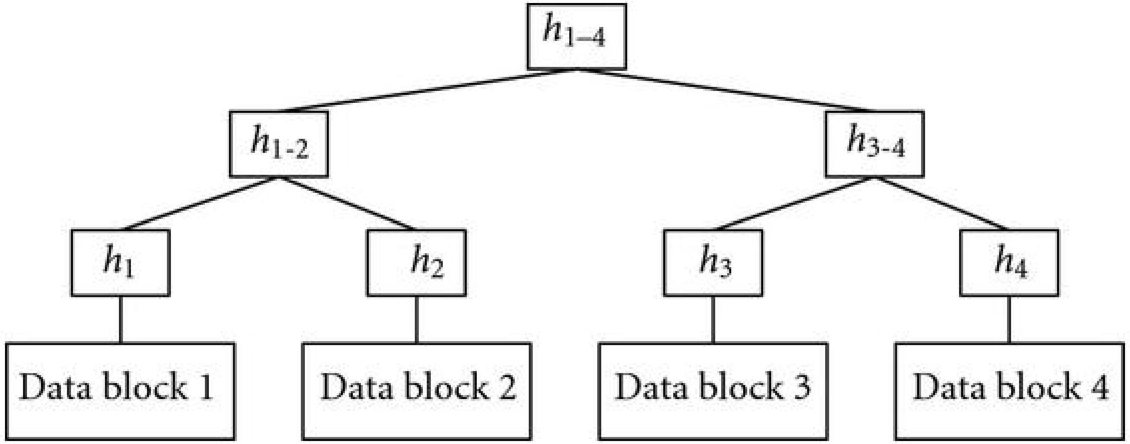
Merkle Trees

- **Proof of Correctness for data block 2**



- **Verify that root matches**
- **Proof consists of just log(n) hashes**
  - **Verifier only needs to permanently store only one hash value**

Merkle Trees



Theorem: Let (Gen, h^s) be a collision resistant hash function and let $H^s(m)$ return the root hash in a Merkle Tree. Then H^s is collision resistant.

Tamper Resistant Storage

Root: H_{1-4}

m_1, m_2, m_3, m_4

Send file 2

m_2', h_1, h_{3-4}

Dropbox

Commitment Schemes

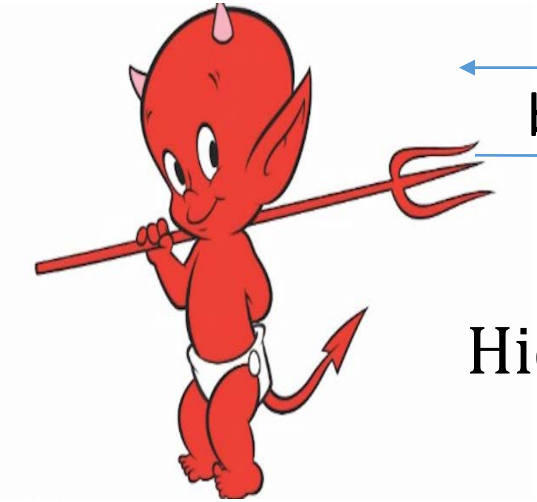
- Alice wants to commit a message m to Bob
  - And possibly reveal it later at a time of her choosing
- Properties
  - Hiding: commitment reveals nothing about m to Bob
  - Binding: it is infeasible for Alice to alter message

Commitment Hiding $\left(\text{Hiding}_{A,Com}(n)\right)$

m_0, m_1

commit(r, m_b)

b'



$$\text{Hiding}_{A,Com}(n) = \begin{cases} 1 & \text{if } b = b' \\ 0 & otherwise \end{cases}$$

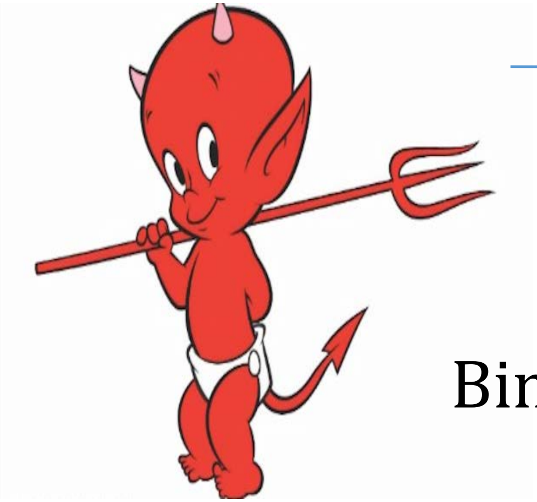
r = Gen(.)

Bit b

$$\forall PPT\ A\ \exists \mu\ (\text{negligible})\ \text{s.t}$$

$$\Pr\left[\text{Hiding}_{A,Com}(n) = 1\right] \leq \frac{1}{2} + \mu(n)$$

Commitment Binding ($\mathrm{Binding}_{A,Com}(n)$)



r_0, r_1, m_0, m_1 $\longrightarrow$

$$\mathrm{Binding}_{A,Com}(n) = \begin{cases} 1 & \text{if } \textbf{commit}(\mathbf{r_0,m_0}) = \textbf{commit}(\mathbf{r_1,m_1}) \\ 0 & otherwise \end{cases}$$

$$\forall PPT\ A\ \exists \mu \text{ (negligible) s.t}$$
$$\Pr\left[\mathrm{Binding}_{A,Com}(n) = 1\right] \leq \mu(n)$$

Secure Commitment Scheme

- **Definition:** A secure commitment scheme is **hiding** and **binding**
- **Hiding**

$$\forall PPT\ A\ \exists \mu\ (\text{negligible})\ \text{s.t}$$

$$\Pr\left[\text{Hiding}_{A,Com}(n) = 1\right] \leq \frac{1}{2} + \mu(n)$$

- **Binding**

$$\forall PPT\ A\ \exists \mu\ (\text{negligible})\ \text{s.t}$$

$$\Pr\left[\text{Binding}_{A,Com}(n) = 1\right] \leq \mu(n)$$

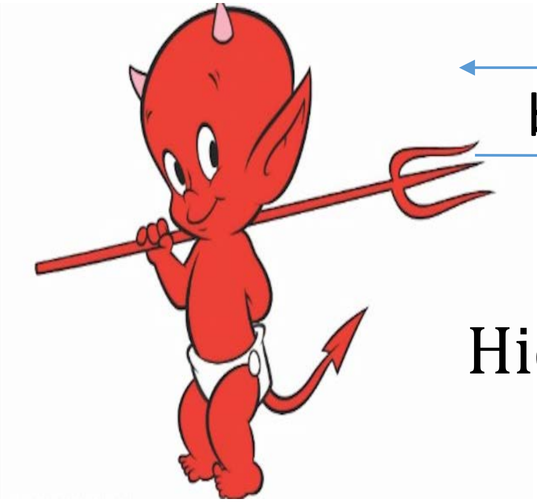
Commitment Scheme in Random Oracle Model

- **Commit**(r,m):=H(m|r)

- **Reveal**(c):= (m,r)

Theorem: In the random oracle model this is a secure commitment scheme.

Commitment Hiding $\left(\mathrm{Hiding}_{A,Com}(n)\right)$



m_0,m_1 →

$H(r,m_b)$ ←

b' →

$$\mathrm{Hiding}_{A,Com}(n) = \begin{cases} 1 & if\, b = b' \\ 0 & otherwise \end{cases}$$

r = Gen(.)

Bit b

$$\forall \cancel{PPT}\ A \text{ making at most } q(n) \text{ queries}$$

$$\Pr\left[\mathrm{Hiding}_{A,Com}(n) = 1\right] \le \frac{1}{2} + \frac{q(n)}{2^{|r|}}$$

Other Applications

- Password Hashing

- Key Derivation

Next Week

- Stream Ciphers
- Block Ciphers
- Feistel Networks
- DES, 3DES
- Read Katz and Lindell 6.1-6.2