Homework 2 Released

- Due: Thursday, September 28th at 9 AM (beginning of class)
- Please Typeset Your Solutions (LaTeX, Word etc…)
- You may collaborate, but must write up your own solutions in your own words

Cryptography
CS 555

Week 5:

- Loose Ends
- Cryptographic Hash Functions
- HMACs
- Generic Attacks
- Random Oracle Model
- Applications of Hashing

Readings: Katz and Lindell Chapter 5, Appendix A.4

Fall 2017

Recap

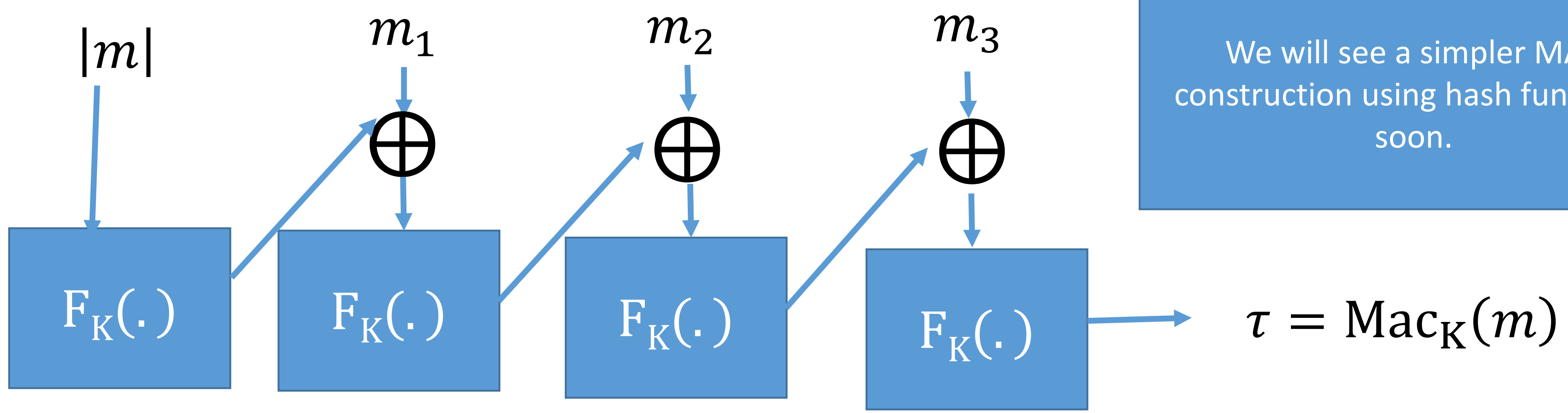
- Message Authentication Codes
  - Integrity vs Confidentiality

$$\mathrm{Mac_k}(m) = \mathrm{F_K}(m)$$

  - Extension to unbounded messages and pitfalls (block re-ordering, truncation)
  - ~~CBC-MAC~~
- Authenticated Encryption + CCA-Security
  - ~~Encrypt and Authenticate [SSH]~~
  - Authenticate then Encrypt [TLS] (Caution Required)
  - Encrypt **then** Authenticate!

$$Enc_K(m) = \langle \mathrm{c}, \mathrm{Mac}'_{K_M}(\mathrm{c}) \rangle \text{ where } \mathrm{c} = \mathrm{Enc}'_{K_E}(m)$$

CBC-MAC



Caveat: Tricky Padding Issues arise if m is not a multiple of the block-length. See textbook.

We will see a simpler MAC construction using hash functions soon.

Advantages over Previous Solution

- Both MACs are secure
- Works for unbounded length messages
- Canonical Verification
- Short Authentication tag
- ~~Parallelizable~~

Let $t_i = \mathrm{Mac}'_K(r \parallel \ell \parallel i \parallel m_i)$
for i=1,…,d
(Note: encode i and ℓ as n/4 bit strings)
Output $\langle r, t_1, \ldots, t_d \rangle$

Building Authenticated Encryption

Theorem: (Encrypt-then-authenticate) Let $\mathrm{Enc}'_{K_E}(m)$ be a CPA-Secure encryption scheme and let $\mathrm{Mac}'_{K_M}(m)$ be a secure MAC. Then the following construction is an authenticated encryption scheme.

$$Enc_K(m) = \langle \mathrm{c}, \mathrm{Mac}'_{K_M}(\mathrm{c}) \rangle \text{ where } \mathrm{c} = \mathrm{Enc}'_{K_E}(m)$$

Proof Intuition: Suppose that we have already shown that any PPT attacker wins $\mathrm{Encforge}_{A,\Pi}$ with negligible probability.

Why does CCA-Security now follow from CPA-Security?

CCA-Attacker has decryption oracle, but cannot exploit it! Why?

Always sees ⊥ “invalid ciphertext” when he query with unseen ciphertext

Proof Sketch

1. Let **ValidDecQuery** be event that attacker submits new/valid ciphertext to decryption oracle

2. Show Pr[**ValidDecQuery**] = negl(n) for any PPT attacker
   - **Hint:** Follows from strong security of MAC since
   $$Enc_K(m) = \langle \mathrm{c}, \mathrm{Mac}'_{K_M}(\mathrm{c}) \rangle$$
   - This also implies unforgeability.

3. Show that attacker who does not issue valid decryption query wins CCA-security game with probability ½ + negl(n)
   - Hint: otherwise we can use A to break CPA-security
   - Hint 2: simulate decryption oracle by always returning ⊥ when given new ciphertext

Secure Communication Session

- Solution? Alice transmits $c_1 = Enc_K(m_1)$ to Bob, who decrypts and sends Alice $c_2 = Enc_K(m_2)$ etc…
- Authenticated Encryption scheme is
  - Stateless
  - For fixed length-messages
- We still need to worry about
  - Re-ordering attacks
    - Alice sends 2n-bit message to Bob as $c_1 = Enc_K(m_1)$, $c_2 = Enc_K(m_2)$
  - Replay Attacks
    - Attacker who intercepts message $c_1 = Enc_K(m_1)$ can replay this message later in the conversation
  - Reflection Attack
    - Attacker intercepts message $c_1 = Enc_K(m_1)$ sent from Alice to Bob and replays to c_1 Alice only

Secure Communication Session

- Defense
  - Counters ($CTR_{A,B}$,$CTR_{B,A}$)
    - Number of messages sent from Alice to Bob ($CTR_{A,B}$) --- initially 0
    - Number of messages sent from Bob to Alice ($CTR_{B,A}$) --- initially 0
    - Protects against Re-ordering and Replay attacks
  - Directionality Bit
    - $b_{A,B} = 0$ and $b_{B,A} = 1$ (e.g., since $A < B$)

- Alice: To send m to Bob, set $c = Enc_K(b_{A,B} \parallel CTR_{A,B} \parallel m)$, send c and increment $CTR_{A,B}$
- Bob: Decrypts c, (if $\perp$ then reject), obtain $b \parallel CTR \parallel m$
  - If $CTR \neq CTR_{A,B}$ or $b \neq b_{A,B}$ then reject
  - Otherwise, output m and increment $CTR_{A,B}$

Authenticated Security vs CCA-Security

- Authenticated Encryption → CCA-Security (by definition)

- CCA-Security does not necessarily imply Authenticate Encryption
  - But most natural CCA-Secure constructions are also Authenticated Encryption Schemes
  - Some constructions are CCA-Secure, but do not provide Authenticated Encryptions, but they are less efficient.
- Conceptual Distinction
  - CCA-Security the goal is secrecy (hide message from active adversary)
  - Authenticated Encryption: the goal is integrity + secrecy

Week 5: Topic 1: Cryptographic Hash Functions

Hash Functions

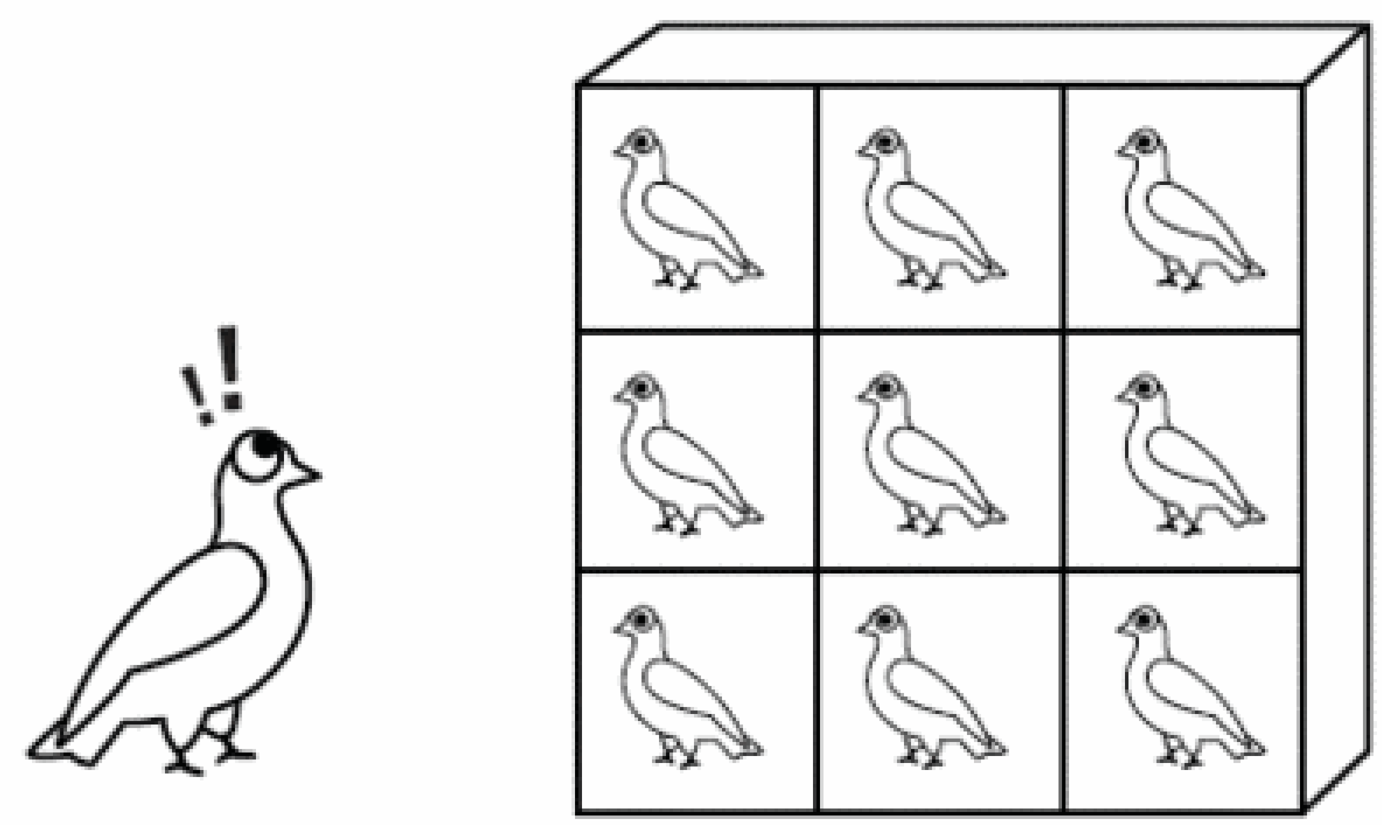
$$H(x)=y$$

Long Input: x

Short Output: y

Pigeonhole Principle

“You cannot fit 10 pigeons into 9 pigeonholes”



Hash Collisions

By Pigeonhole Principle there must exist x and y s.t.

$$H(x) = H(y)$$

Classical Hash Function Applications

- Hash Tables
  - O(1) lookup*
- “Good hash function” should yield “few collisions”

* Certain terms and conditions apply

Collision-Resistant Hash Function

Intuition: Hard for computationally bounded attacker to find x,y s.t. H(x) = H(y)

How to formalize this intuition?

- **Attempt 1:** For all PPT A,

$$\Pr\left[A_{x,y}(1^n) = (x, y)\ s.t\ H(x) = H(y)\right] \leq negl(n)$$

- **The Problem:** Let x,y be given s.t. H(x)=H(y)

$$A_{x,y}(1^n) = (x, y)$$

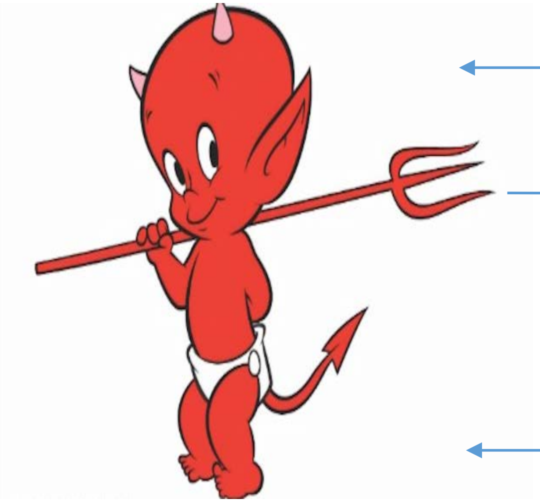
- We are assuming that |x| > |H(x)|. Why?
  - H(x)=x is perfectly collision resistant! (but with no compression)

Keyed Hash Function Syntax

- Two Algorithms
  - $\mathrm{Gen}(1^n; R)$ (Key-generation algorithm)
    - Input: Random Bits R
    - Output: ~~Secret~~ key s
  - $H^s(m)$ (Hashing Algorithm)
    - Input: key s and message $\mathrm{m} \in \{0,1\}^*$ (unbounded length)
    - Output: hash value $H^s(m) \in \{0,1\}^{\ell(n)}$

- Fixed length hash function
  - $m \in \{0,1\}^{\ell'(n)}$ with $\ell'(n) > \ell(n)$

Collision Experiment ($HashColl_{A,\Pi}(n)$)



s

x_1, x_2

$$HashColl_{A,\Pi}(n) = \begin{cases} 1 & if\ H^s(x_1) = H^s(x_2) \\ 0 & otherwise \end{cases}$$

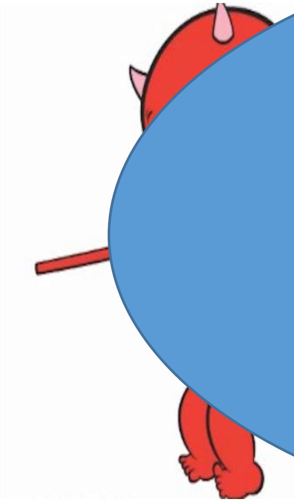
$s = \text{Gen}(1^n; R)$

Definition: (Gen,H) is a collision resistant hash function if

$$\forall PPT\ A\ \exists \mu\ \text{(negligible) s.t}$$

$$\Pr\left[HashColl_{A,\Pi}(n) = 1\right] \leq \mu(n)$$

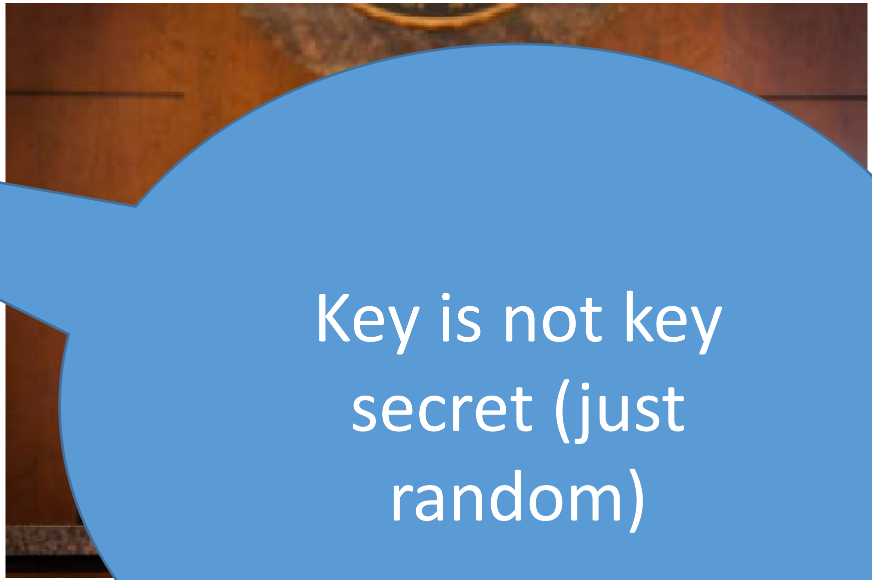
Collision Experiment ($HashColl_{A,\Pi}(n)$)



For simplicity we will sometimes just say that H (or H^s) is a collision resistant hash function

$= H^s(x_2)$

ise



Key is not key secret (just random)

Definition: (Gen,H) is a collision resistant hash function if

$$\forall PPT\ A\ \exists \mu \text{ (negligible) s.t}$$

$$\Pr\left[HashColl_{A,\Pi}(n)=1\right] \leq \mu(n)$$

Theory vs Practice

- Most cryptographic hash functions used in practice are un-keyed
  - Examples: MD5, SHA1, SHA2, SHA3
- Tricky to formally define collision resistance for keyless hash function
  - There is a PPT algorithm to find collisions
  - We just usually can't find this algorithm ☺

Formalizing Human Ignorance:
Collision-Resistant Hashing without the Keys

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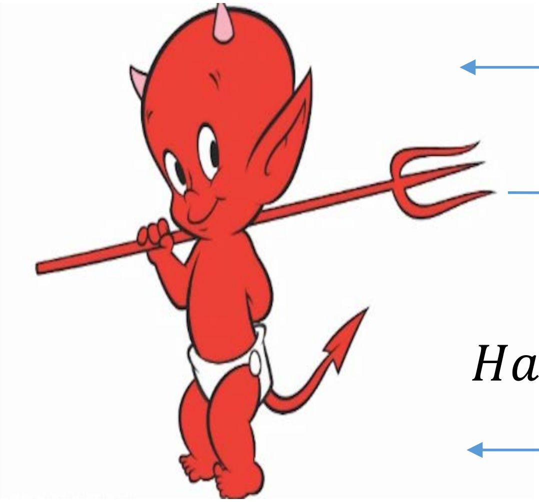
31 January 2007

Abstract. There is a foundational problem involving collision-resistant hash-functions: com-

Weaker Requirements for Cryptographic Hash

- Target-Collision Resistance



s,x

x’

$$HashTgtColl_{A,\Pi}(n) = \begin{cases} 1 & if\ H^s(x') = H^s(x) \\ 0 & otherwise \end{cases}$$

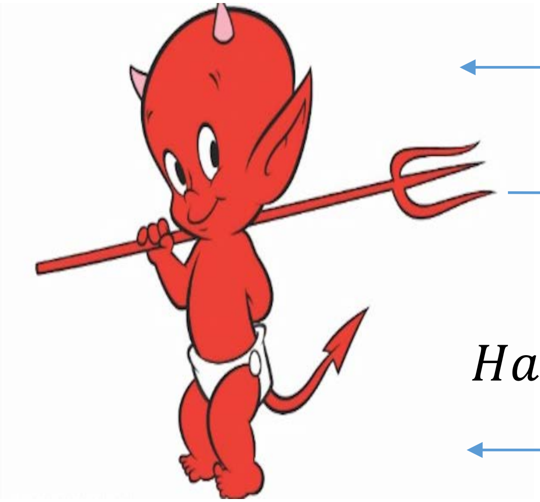
$s = \mathrm{Gen}(1^n; R)$

$x \in \{0,1\}^n$

Question: Why is collision resistance stronger?

Weaker Requirements for Cryptographic Hash

- Preimage Resistance (One-Wayness)



s, y

x

$$HashPreImgRes_{A,\Pi}(n) = \begin{cases} 1 & if\ H^s(x) = y \\ 0 & otherwise \end{cases}$$

$s = \mathrm{Gen}(1^n; R)$

$y \in \{0,1\}^{\ell(n)}$

Question: Why is collision resistance stronger?

Merkle-Damgård Transform

- Most cryptographic hash functions accept fixed length inputs
- What if we want to hash arbitrary length strings?

Construction: (Gen,h) fixed length hash function from 2n bits to n bits

$$H^s(x_1, \dots, xd) = h^s\big(h^s\big(h^s\big(\dots h^s(0^n \parallel x_1)\big) \parallel x_{d-1}\big) \parallel x_d\big)$$

Merkle-Damgård Transform

Construction: (Gen,h) fixed length hash function from 2n bits to n bits

$H^s(x)$ =

1. Break x into n bit segments $x_1,..,x_d$ (pad last block by zeros if needed)
2. $z_0 = 0^n$ (initialization)
3. For i = 1 to d+1
   1. $z_i = h^s(z_{i-1} \parallel x_i)$
4. Output z_{d+1}

Merkle-Damgård Transform

Theorem: If (Gen,h) is collision resistant then so is (Gen,H)

Proof: Show that any collision in H^s yields a collision in h^s. Thus a PPT attacker for (Gen,H) can be transformed into PPT attacker for (Gen,h).

Suppose that

$$H^s(x) = H^s(x')$$

(note x and x' may have different lengths)

Merkle-Damgård Transform

Theorem: If (Gen,h) is collision resistant then so is (Gen,H)

Proof: Suppose that

$$H^s(x) = H^s(x')$$

Case 1: |x|=|x'| (proof for case two is similar)

$$H^s(x) = z_{d+1} = h^s(z_d \parallel x_d) = H^s(x') = z'_{d+1} = h^s(z'_d \parallel x'_d)$$

$$z_d \parallel x_d =? \; z'_d \parallel x'_d$$

No → Found collision

Yes?

$$h^s(z_{d-1} \parallel x_{d-1}) = h^s(z'_{d-1} \parallel x'_{d-1})$$

Merkle-Damgård Transform

Theorem: If (Gen,h) is collision resistant then so is (Gen,H)

Proof: Suppose that

$$H^s(x) = H^s(x')$$

Case 1: |x|=|x'| (proof for case two is similar)

If for some i we have $z_i \parallel x_i \neq z_i' \parallel x_i'$ then we will find a collision

But x and x' are different!

Week 5: Topic 2: HMACs and Generic Attacks

MACs for Arbitrary Length Messages

$\text{Mac}_K(m)$=

- Select random n/4 bit string r
- Let $t_i = \text{Mac}'_K(r \parallel \ell \parallel i \parallel m_i)$ for i=1,…,d
  - (Note: encode i and ℓ as n/4 bit strings)
- **Output** $\langle r, t_1, \ldots, t_d \rangle$

Theorem 4.8: If Π' is a secure MAC for messages of fixed length n, above construction $\Pi = (\text{Mac}, \text{Vrfy})$ is secure MAC for arbitrary length messages.

MACs for Arbitrary Lengt

Disadvantage 1: Long output

Two Disadvantages:

1. Lose Strong-MAC Guarantee
2. Security game arguably should give attacker Vrfy(.) oracle

(CPA vs CCA security)

ri

|| i ||

and ℓ as n/4 bi

- **Output** $\langle r, t_1, \ldots, t_d \rangle$

Randomized Construction (no canonical verification). Disadvantage?

Theorem 4.8: If Π′ i
above constructio
messages.

Hash and MAC Construction

Start with (Mac,Vrfy) a MAC for messages of fixed length and (Gen_H,H) a collision resistant hash function

$$Mac'_{\langle K_M, S\rangle}(m) = Mac_{K_M}\left(H^s(m)\right)$$

Theorem 5.6: Above construction is a secure MAC.

Note: If $\mathrm{Vrfy}_{K_M}(m,t)$ is canonical then $\mathrm{Vrfy}'_{\langle K_M, S\rangle}(m,t)$ can be canonical.

Hash and MAC Construction

Start with (Mac,Vrfy) a MAC for messages of fixed length and (Gen_H,H) a collision resistant hash function

$$Mac'_{\langle K_M, S\rangle}(m) = Mac_{K_M}\left(H^s(m)\right)$$

Theorem 5.6: Above construction is a secure MAC.

Proof Intuition: If attacker successfully forges a valid MAC tag t’ for unseen message m’ then either

- **Case 1:** $H^s(m') = H^s(m_i)$ for some previously requested message m_i
- **Case 2:** $H^s(m') \neq H^s(m_i)$ for every previously requested message m_i

Hash and MAC Construction

Theorem 5.6: Above construction is a secure MAC.

Proof Intuition: If attacker successfully forges a valid MAC tag t' for unseen message m' then either

- **Case 1:** $H^s(m') = H^s(m_i)$ for some previously requested message m_i
  - **Attacker can find hash collisions!**
- **Case 2:** $H^s(m') \neq H^s(m_i)$ for every previously requested message m_i
  - **Attacker forged a valid new tag on the "new message" $\boldsymbol{H^s(m')}$**
  - **Violates security of the original fixed length MAC**

MAC from Collision Resistant Hash

- Failed Attempt:

$$Mac_{\langle k,S\rangle}(m) = H^s(k \parallel m)$$

Broken if H^s uses Merkle-Damgård Transform

$$Mac_{\langle k,S\rangle}(m_1 \parallel m_2 \parallel m_3) = h^s(h^s(h^s(h^s(0^n \parallel k) \parallel m_1) \parallel m_2) \parallel m_3)$$

$$= h^s\left(Mac_{\langle k,S\rangle}(m_1 \parallel m_2) \parallel m_3\right)$$

Why does this mean $Mac_{\langle k,S\rangle}$ is broken?

HMAC

$$Mac_{\langle k,S\rangle}(m) = H^s\Big((k \oplus \text{opad}) \parallel H^s\big((k \oplus \text{ipad}) \parallel m\big)\Big)$$

ipad?

HMAC

$$Mac_{\langle k,S\rangle}(m) = H^s\left((k \oplus \text{opad}) \parallel H^s\big((k \oplus \text{ipad}) \parallel m\big)\right)$$

$$\text{ipad} = \text{inner pad}$$
$$\text{opad} = \text{outer pad}$$

Both ipad and opad are fixed constants.

Why use key twice?

Allows us to prove security from *weak collision resistance* of H^s

HMAC Security

$$Mac_{\langle k,S\rangle}(m) = H^s\left((k \oplus \mathrm{opad}) \parallel H^s\big((k \oplus \mathrm{ipad}) \parallel m\big)\right)$$

Theorem (Informal): Assuming that H^s is weakly collision resistant and that (certain other plausible assumptions hold) this is a secure MAC.

Weak Collision Resistance: Give attacker oracle access to $f(m) = H^s(k \parallel m)$ (secret key k remains hidden).

Attacker Goal: Find distinct m,m’ such that $f(m) = f(m')$

HMAC in Practice

- MD5 can no longer be viewed as collision resistant

- However, HMAC-MD5 remained unbroken after MD5 was broken
  - Gave developers time to replace HMAC-MD5
  - Nevertheless, don’t use HMAC-MD5!

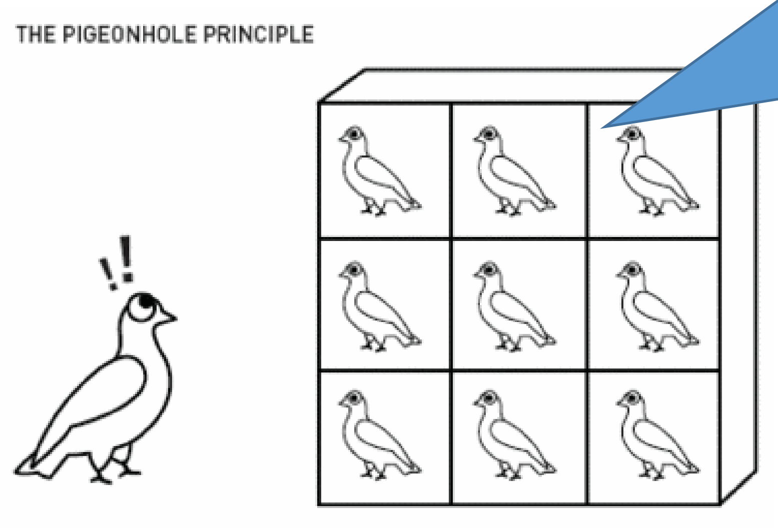
- HMAC is efficient and unbroken
  - CBC-MAC was not widely deployed because it as “too slow”
  - Instead practitioners often used heuristic constructions (which were breakable)

Finding Collisions

- Ideal Hashing Algorithm
  - Random function H from $\{0,1\}^*$ to $\{0,1\}^\ell$
  - Suppose attacker has oracle access to H(.)

- **Attack 1:** Evaluate H(.) on $2^\ell+1$ distinct inputs.

Can we do better?



Birthday Attack for Finding Collisions

- Ideal Hashing Algorithm
  - Random function H from $\{0,1\}^*$ to $\{0,1\}^\ell$
  - Suppose attacker has oracle access to H(.)

- **Attack 2:** Evaluate H(.) on $q = 2^{(\ell/2)+1} + 1$ distinct inputs $x_1,\ldots,x_q$.

$$\Pr[\forall i < j. H(x_i) \neq H(x_j)] = 1\left(1 - \frac{1}{2^\ell}\right)\left(1 - \frac{2}{2^\ell}\right)\left(1 - \frac{3}{2^\ell}\right)\ldots\left(1 - \frac{2^{(\ell/2)+1}}{2^\ell}\right) < \frac{1}{2}$$

Birthday Attack for Finding Collisions

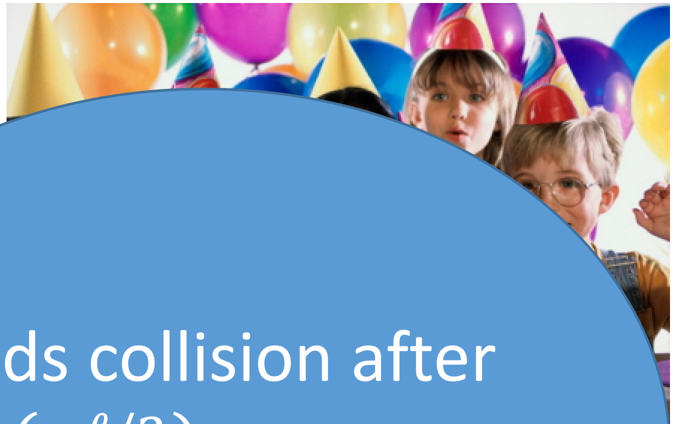
- Ideal Hashing Algorithm
  - Random function H from $\{0,1\}^*$ to $\{0,1\}^\ell$
  - Suppose attacker has oracle access to H(.)

- **Attack 2:** Evaluate H(.) on $q = 2^{(\ell/2)+1} + 1$ distinct inputs $x_1,\ldots,x_q$.
- Store values $(x_i, H(x_i))$ in a hash table of size q
  - Requires time/space $O(q) = O\left(\sqrt{2^\ell}\right)$
  - Can we do better?

Small Space Birthday Attack

- **Attack 2:** Select random x_0, define $x_i = H(x_{i-1})$
  - Initialize: $x=x_0$ and $x'=x_0$
  - Repeat for i=1,2,…
    - x:=H(x) now $x = x_i$
    - x':=H(H(x')) now $x' = x_{2i}$
    - If x=x' then break
  - Reset $x=x_0$ and set $x'=x$
  - Repeat for j=1 to i
    - If H(x) = H(x') then output x,x'
    - Else x:= H(x), x' = H(x) Now $x=x_j$ AND $x' = x_{i+j}$

Small Space Birthday Attack



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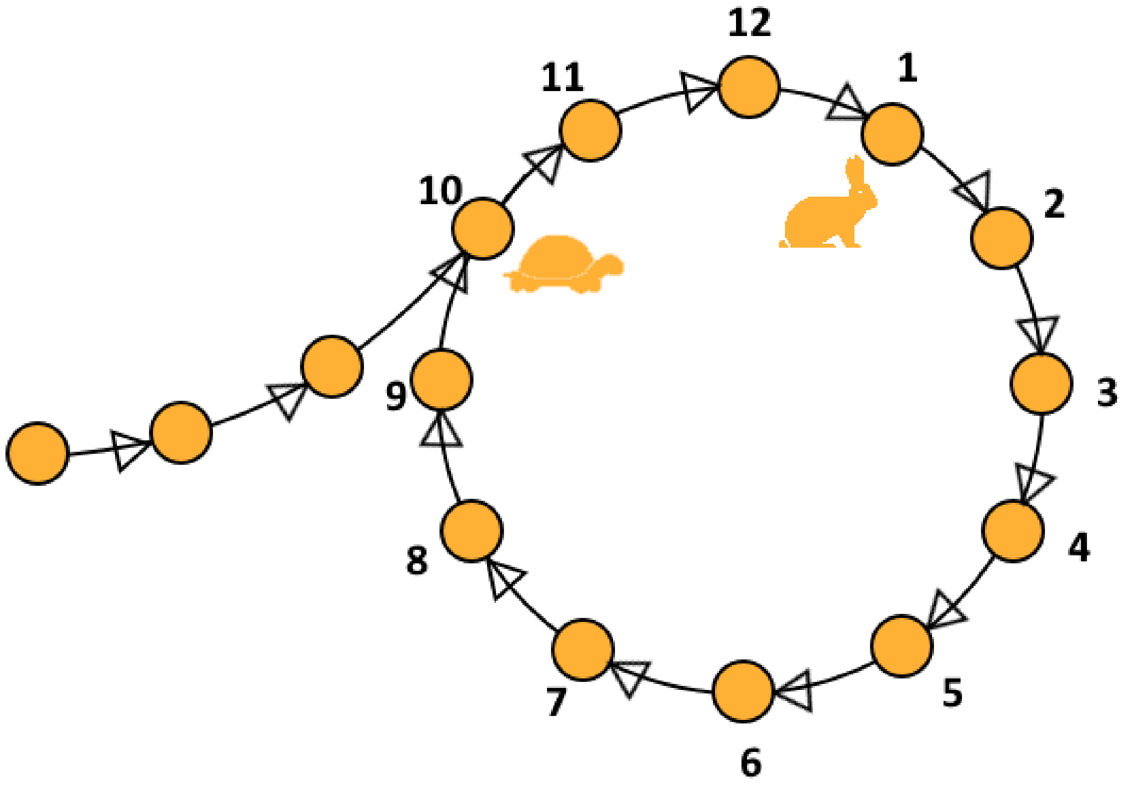
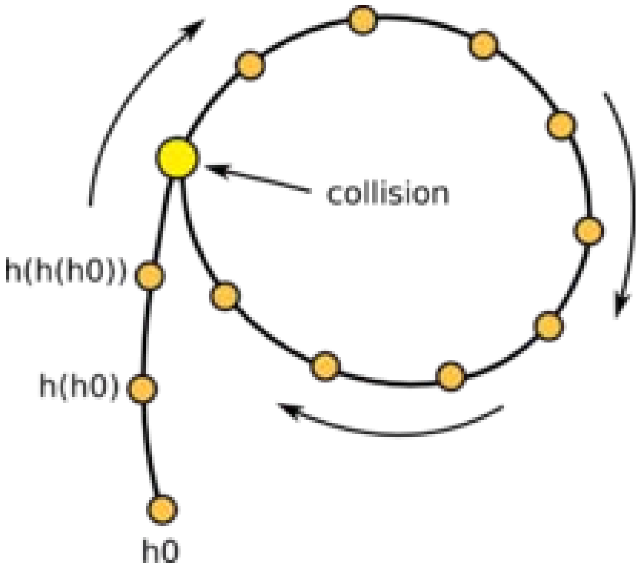
  - Repeat for i=1,2,…
    - $x:=H(x)$ now $x = x_i$
    - $x':=H(H(x'))$ now $x' = x_{2i}$
    - If $x=x'$ then break
  - Reset $x=x_0$ and set $x'=x$
  - Repeat for j=1 to i
    - If $H(x) = H(x')$ then output x,x'
    - Else $x:= H(x)$, $x' = H(x)$ Now $x=x_j$ AND $x' = x_{i+j}$

Finds collision after $O\left(2^{\ell/2}\right)$ steps in expectation

Floyd's Cycle Finding Algorithm



- A cycle denotes a hash collision
- Occurs after $O\left(2^{\ell/2}\right)$ steps by birthday paradox
- First attack phase detects cycle
- Second phase identifies collision



- Analogy: Cycle detection in linked list
- Can traverse "linked list" by computing H

Small Space Birthday Attack

- Can be adapted to find "meaningful collisions" if we have a large message space $\mathrm{O}\left(2^{\ell}\right)$

- **Example**: $S = S_1 \cup S_2$ with $|S_1| = |S_2| = 2^{\ell-1}$
  - S_1 = Set of positive recommendation letters
  - S_2 = Set of negative recommendation letters

- **Goal**: find $z_1 \in S_1$, $z_2 \in S_2$, such that $H(z_1) = H(z_2)$

- Can adapt previous attack by assigning unique binary string $\mathrm{b}(\mathrm{x}) \in \{0,1\}^{\ell}$ of length to each $x \in S$

$$\mathrm{x}_{\mathrm{i}} = H(\mathrm{b}(\mathrm{x}_{\mathrm{i-1}}))$$

Targeted Collision (e.g., Password Cracking)

- Attacker is given y=H(pwd)
- Goal find x' s.t. H(x') = y

- There is an attack which requires
  - Precomputation Time: $O(|PASSWORDS|)$
  - Space: $|PASSWORDS|^{2/3}$
  - On input y finds pwd in Time: $|PASSWORDS|^{2/3}$
- Cracking costs amortize over many users…
- Other time-memory tradeoffs are possible…
- **Defense 1:** y=H(pwd|salt)   [password salting]
- **Defense 2:** Make sure that H is moderately expensive to compute (MHFs)

Targeted Collision (e.g., Password Cracking)

- Attacker is given y=H(x)
- Goal find x' s.t. H(x') = y
- Precomputation (sketch)
  - Store $s = 2^{\ell/3}$ pairs (SP_i,EP_i) where $EP_i = Ht(SP_i)$ and t = $2^{\ell/3}$
- Let $y=y_0$
- For i=1,2…., $2^{\ell/3}$
  - $y_i = H(y_{i-1})$
  - For each j s.t $EP_j=y_i$
    - Check if y is in the hash chain (SP_i,EPi)
    - Yes → Found desired x'

Space: $2^{\ell/3}$
Precomputation Time: $2^{2\ell/3}$

Total #j's $= \frac{st^2}{2^\ell} < O(1)$

Total Runtime $= O(t) = O(2^{\ell/3})$

Success Rate $\approx \frac{1}{4t}$

Targeted Collision (e.g., Password Cracking)

- Attacker is given y=H(x)
- Goal find x' s.t. H(x') = y
- Precomputation (sketch)
  - Store $4st = 4 \times 2^{2\ell/3}$ pairs $\left(SP_i^j, EP_i^j\right)$ where $EP_i^j = Ht(c_j \oplus SP_i)$ and t = $2^{\ell/3}$
- Let $y=y_0$
- For i=1,2…., $2^{\ell/3}$
  - $y_i^j = H(c_j \oplus y_{i-1})$
  - Foreach j s.t $EP_i^j = y_i^j$
  - Check if y is in the hash chain (SP_i,EPi)
    - Yes → Found desired x'

Space: $2^{2\ell/3}$

Precomputation Time: $2^{\ell}=2^{2\ell/3}2^{\ell/3}$

Repeat for each j < t

Total Runtime $= O(t \times t) = O(2^{2\ell/3})$

Success Rate > 0.63

Targeted Collisions (Other Applications)

- Define $H(K) = F_k(x)$

- Suppose attacker obtains a pair $x, F_k(x)$ (chosen plaintext attack)
- There is a key recovery attack with
  - Precomputation Time: $|\mathcal{K}|$
  - Space: $|\mathcal{K}|^{2/3}$
  - Cracking Time: $|\mathcal{K}|^{2/3}$
- Precomputation costs amortize if we are attacking multiple different keys
  - As long as we have $x, F_{k'}(x)$ we don't need to repeat precomputation phase

Week 5: Topic 3:
Random Oracle Model +
Hashing Applications

(Recap) Collision-Resistant Hash Function

Intuition: Hard for computationally bounded attacker to find x,y s.t. H(x) = H(y)

How to formalize this intuition?

- **Attempt 1:** For all PPT A,

$$\Pr\left[A_{x,y}(1^n) = (x,y)\ s.t\ H(x) = H(y)\right] \leq negl(n)$$

- **The Problem:** Let x,y be given s.t. H(x)=H(y)

$$A_{x,y}(1^n) = (x,y)$$

- We are assuming that |x| > |H(x)|. Why?
  - H(x)=x is perfectly collision resistant! (but with no compression)

(Recap) Keyed Hash Function Syntax

- Two Algorithms
  - $\mathrm{Gen}(1^n; R)$ (Key-generation algorithm)
    - Input: Random Bits R
    - Output: ~~Secret~~ key s
  - $H^s(m)$ (Hashing Algorithm)
    - Input: key s and message $\mathrm{m} \in \{0,1\}^*$ (unbounded length)
    - Output: hash value $H^s(m) \in \{0,1\}^{\ell(n)}$

- Fixed length hash function
  - $m \in \{0,1\}^{\ell'(n)}$ with $\ell'(n) > \ell(n)$

When Collision Resistance Isn't Enough

- **Example**: Message Commitment
  - Alice sends Bob: $\mathrm{H}^s(r \parallel m)$ (e.g., predicted winner of NCAA Tournament)
  - Alice can later reveal message (e.g., after the tournament is over)
    - Just send r and m (note: r has fixed length)
    - Why can Alice not change her message?
  - In the meantime Bob shouldn't learn *anything* about m

- **Problem**: Let (Gen,H') be collision resistant then so is (Gen,H)

$$H^s(x_1, \dots, xd) = H'^s(x_1, \dots, xd) \parallel x_d$$

When Collision Resistance Isn't Enough

- **Problem**: Let (Gen,H') be collision resistant then so is (Gen,H)

$$H^s(x_1, \ldots, xd) = H'^s(x_1, \ldots, xd) \parallel x_d$$

- (Gen,H) definitely does not hide all information about input $(x_1, \ldots, xd)$

- **Conclusion**: Collision resistance is not sufficient for message commitment

The Tension

- **Example**: Message Commitment
  - Alice sends Bob: $\mathrm{H}^s(r \parallel m)$ (e.g., predicted winners of NCAA Final Four)
  - Alice can later reveal message (e.g., after the Final Four is decided)
  - In the meantime Bob shouldn't learn anything about m

This is still a reasonable approach in practice!

- No attacks when instantiated with any reasonable candidate (e.g., SHA3)
- Cryptographic hash functions seem to provide "something" beyond collision resistance, but how do we model this capability?

Random Oracle Model

- Model hash function H as a truly random function
- Algorithms can only interact with H as an oracle
  - **Query**: x
  - **Response**: H(x)
- If we submit the same query you see the same response
- If x has not been queried, then the value of H(x) is uniform

- **Real World:** H instantiated as cryptographic hash function (e.g., SHA3) of fixed length (no Merkle-Damgård)

Back to Message Commitment

- **Example**: Message Commitment
  - Alice sends Bob: $\mathrm{H}(r \parallel m)$ (e.g., predicted winners of NCAA Final Four)
  - Alice can later reveal message (e.g., after the Final Four is decided)
    - Just send r and m (note: r has fixed length)
    - Why can Alice not change her message?
  - In the meantime Bob shouldn't learn *anything* about m

- **Random Oracle Model**: Above message commitment scheme is secure (Alice cannot change m + Bob learns nothing about m)
- **Information Theoretic Guarantee** against any attacker with q queries to H

Random Oracle Model: Pros

- It is easier to prove security in Random Oracle Model
- Suppose we are simulating attacker A in a reduction
  - **Extractability**: When A queries H at x we **see this query** and learn x (and can easily find H(x))
  - **Programmability**: We can set the value of H(x) to a value of our choice
    - As long as the value is correctly distribute i.e., close to uniform
- Both **Extractability** and **Programmability** are useful tools for a security reduction!

Random Oracle Model: Pros

- It is easier to prove security in Random Oracle Model

- Provably secure constructions in random oracle model are often much more efficient (compared to provably secure construction is “standard model”

- Sometimes we only know how to design provably secure protocol in random oracle model

Random Oracle Model: Cons

- Lack of formal justification
- Why should security guarantees translate when we instantiate random oracle with a real cryptographic hash function?

- We can construct (contrived) examples of protocols which are
  - Secure in random oracle model…
  - But broken in the real world

Random Oracle Model: Justification

"A proof of security in the random-oracle model is significantly better than no proof at all."

- **Evidence of sound design** (any weakness involves the hash function used to instantiate the random oracle)
- **Empirical Evidence for Security**

  "there have been no successful real-world attacks on schemes proven secure in the random oracle model"

Hash Function Application: Fingerprinting

- The hash h(x) of a file x is a unique identifier for the file
  - Collision Resistance → No need to worry about another file y with H(y)=H(y)

- Application 1: Virus Fingerprinting

- Application 2: P2P File Sharing

- Application 3: Data deduplication

Tamper Resistant Storage

$H(m_1)$

m_1

Dropbox

m_1'

Tamper Resistant Storage

File Index	Hash
1	$H(m_1)$
2	$H(m_2)$
3	$H(m_3)$

Disadvantage: Too many hashes to store

Dropbox

m_1, m_2, m_3

Send file 1

m_1'

Tamper Resistant Storage

Disadvantage: Need all files to compute hash m_1,m_2,m_3

$H(m_1,m_2,m_3)$

m_1,m_2,m_3

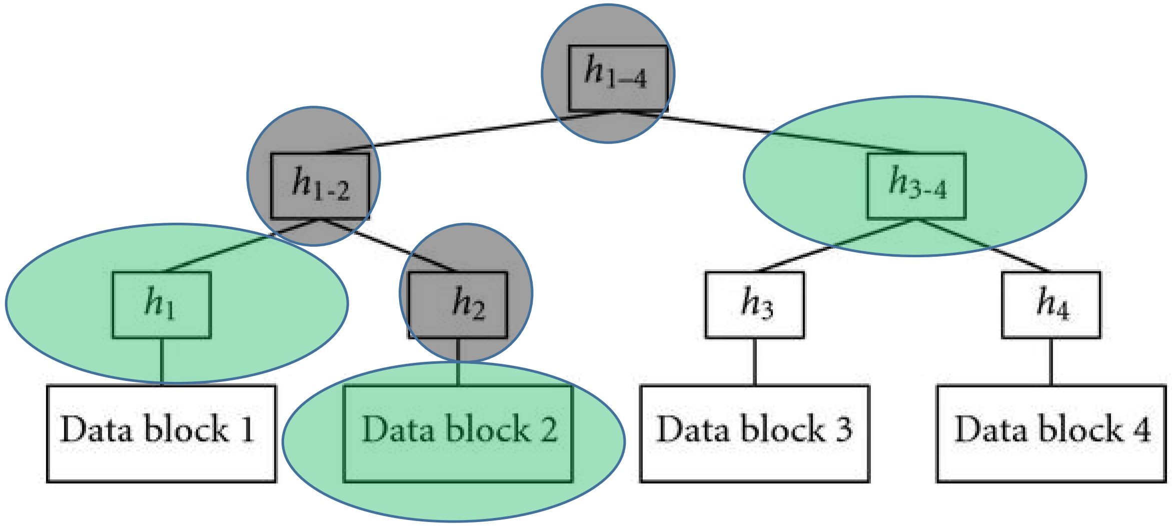
Send file 1

m_1'

Dropbox

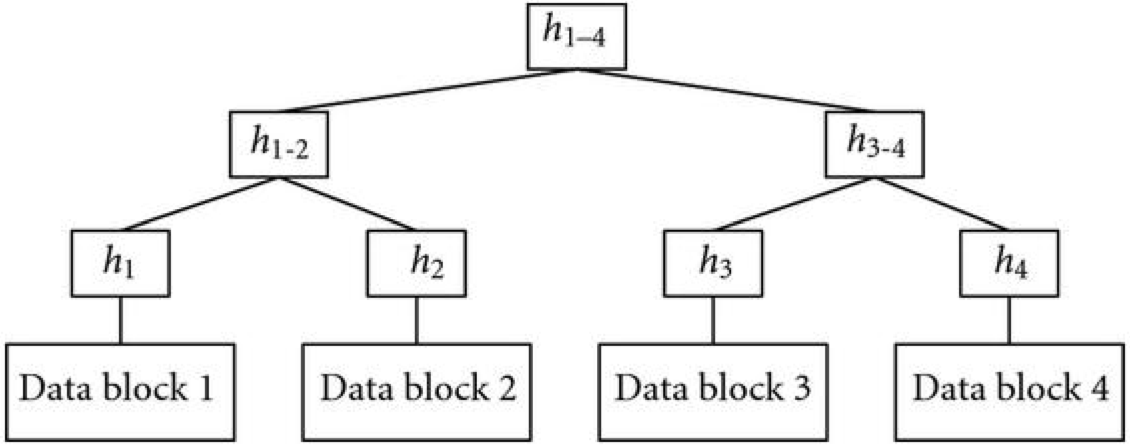
Merkle Trees

- **Proof of Correctness for data block 2**



- **Verify that root matches**
- **Proof consists of just log(n) hashes**
  - **Verifier only needs to permanently store only one hash value**

Merkle Trees



Theorem: Let (Gen, h^s) be a collision resistant hash function and let $H^s(m)$ return the root hash in a Merkle Tree. Then H^s is collision resistant.

Tamper Resistant Storage

Root: $H_{1\text{-}4}$

m_1, m_2, m_3, m_4

Send file 2

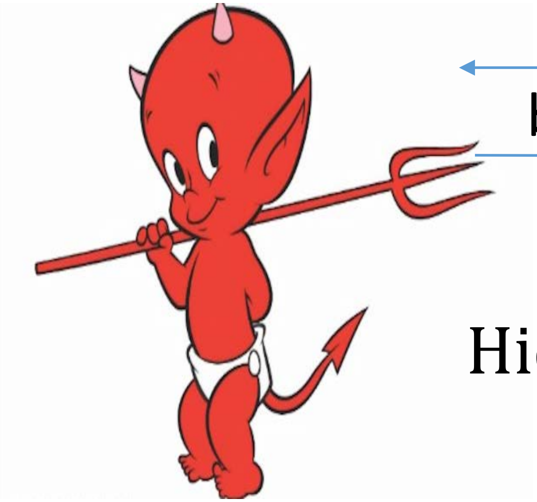
$m_2', h_1, h_{3\text{-}4}$

Dropbox

Commitment Schemes

- Alice wants to commit a message m to Bob
  - And possibly reveal it later at a time of her choosing
- Properties
  - Hiding: commitment reveals nothing about m to Bob
  - Binding: it is infeasible for Alice to alter message

Commitment Hiding $\left(\mathrm{Hiding}_{A,Com}(n)\right)$



m_0, m_1 →

← **commit(r,m_b)**

b' →

$$\mathrm{Hiding}_{A,Com}(n) = \begin{cases} 1 & \text{if } b = b' \\ 0 & otherwise \end{cases}$$

r = Gen(.)

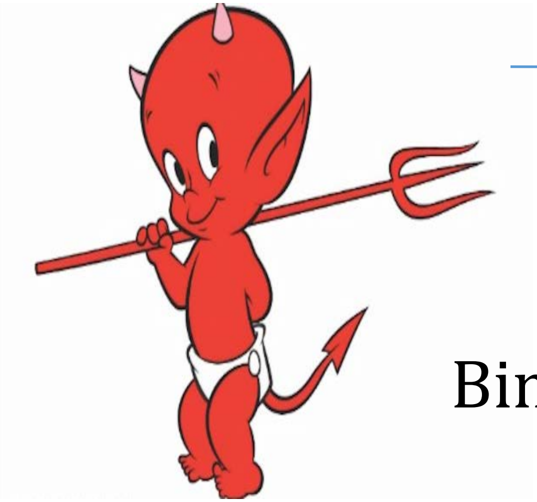
Bit b

$$\forall PPT\ A\ \exists \mu\ (\text{negligible})\ \text{s.t}$$

$$\Pr\left[\mathrm{Hiding}_{A,Com}(n) = 1\right] \le \frac{1}{2} + \mu(n)$$

Commitment Binding ($\text{Binding}_{A,Com}(n)$)

r_0, r_1, m_0, m_1



$$\text{Binding}_{A,Com}(n) = \begin{cases} 1 & \text{if } \mathbf{commit(r_0,m_0) = commit(r_1,m_1)} \\ 0 & otherwise \end{cases}$$

$$\forall PPT\ A\ \exists \mu \text{ (negligible) s.t}$$
$$\Pr[\text{Binding}_{A,Com}(n) = 1] \leq \mu(n)$$

Secure Commitment Scheme

- **Definition:** A secure commitment scheme is **hiding** and **binding**
- **Hiding**

$$\forall PPT\ A\ \exists\mu\ (\text{negligible})\ \text{s.t}$$
$$\Pr\left[\text{Hiding}_{A,Com}(n) = 1\right] \le \frac{1}{2} + \mu(n)$$

- **Binding**

$$\forall PPT\ A\ \exists\mu\ (\text{negligible})\ \text{s.t}$$
$$\Pr\left[\text{Binding}_{A,Com}(n) = 1\right] \le \mu(n)$$

Commitment Scheme in Random Oracle Model

- **Commit**(r,m):=H(m|r)

- **Reveal**(c):= (m,r)

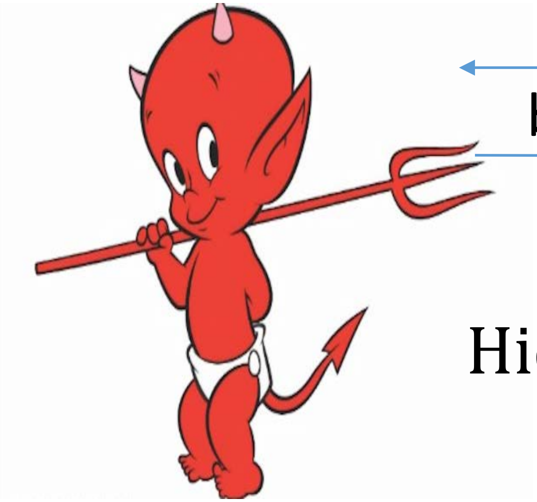
Theorem: In the random oracle model this is a secure commitment scheme.

Commitment Hiding $\left(\text{Hiding}_{A,Com}(n)\right)$

m_0,m_1

$H(r,m_b)$

b’



$$\text{Hiding}_{A,Com}(n) = \begin{cases} 1 & if\, b = b' \\ 0 & otherwise \end{cases}$$

r = Gen(.)

Bit b

$$\forall \cancel{PPT}\ A\ making\ q(n)\ queries\ \text{s.t}$$

$$\Pr\left[\text{Hiding}_{A,Com}(n) = 1\right] \le \frac{1}{2} + \frac{q(n)}{2^{|r|}}$$

Other Applications

- Password Hashing

- Key Derivation

Next Week

- Stream Ciphers
- Block Ciphers
- Feistel Networks
- DES, 3DES
- Read Katz and Lindell 6.1-6.2

Revisit: Building Authenticated Encryption

Attempt 3: (Authenticate-then-encrypt) Let $\mathrm{Enc}'_{K_E}(m)$ be a CPA-Secure encryption scheme and let $\mathrm{Mac}'_{K_M}(m)$ be a secure MAC. Let $K = (K_E, K_M)$ then

$$Enc_K(m) = \left\langle \mathrm{Enc}'_{K_E}(m \parallel t) \right\rangle \text{ where } \quad \mathrm{t} = \mathrm{Mac}'_{K_M}(m)$$

Doesn't **necessarily** work

- Approach is still used in TLS
- Some practitioners still advocate for this methodology

Building Authenticated Encryption

Attempt 3: (Authenticate-then-encrypt) Let $\mathrm{Enc}'_{K_E}(m)$ be a CPA-Secure encryption scheme and let $\mathrm{Mac}'_{K_M}(m)$ be a secure MAC. Let $K = (K_E, K_M)$ then

$$\mathrm{Enc}_{\mathrm{K}}(m) = \left\langle \mathrm{Enc}'_{K_E}(m \parallel t) \right\rangle \text{ where } \ \mathrm{t} = \mathrm{Mac}'_{K_M}(m)$$

A Bad Example:

$$\mathrm{Enc}_{\mathrm{K}}(m) = \left\langle r, \mathrm{F}_{K_E}(r) \oplus (\mathrm{ECC(m)} \parallel t) \right\rangle$$
$$\mathrm{ECC(m)} = w \text{ (codeword for error correction)}$$
$$\mathrm{Decode(w')} = m \text{ if w}' \text{and w are `mostly the same}'$$

Source: The Order of Encryption and Authentication for protecting communication by Hugo Krawczyk
https://eprint.iacr.org/2001/045

Building Authenticated Encryption

$$\mathrm{Enc_K}(m) = \langle r, \mathrm{F}_{K_E}(r) \oplus (\mathrm{ECC(m)} \parallel t) \rangle \text{ where } \begin{array}{l} \mathrm{t} = \mathrm{Mac}'_{K_M}(m) \\ \mathrm{w} = \mathrm{ECC(m)} \end{array}$$

$Dec_K(\langle r, s \rangle)$

- $w \parallel t \leftarrow \mathrm{F}_{K_E}(r) \oplus s$
- $m \leftarrow Decode(w)$
- $output = \begin{cases} m & \text{if } \mathrm{t} = \mathrm{Mac}'_{\mathrm{K_M}}(m) \\ \perp & \text{otherwise} \end{cases}$

Source: The Order of Encryption and Authentication for protecting communication by Hugo Krawczyk
https://eprint.iacr.org/2001/045

Building Authenticated Encryption

$$\mathrm{Enc_K}(m) = \langle r, \mathrm{F}_{K_E}(r) \oplus (\mathrm{ECC(m)} \parallel t) \rangle \text{ where } \begin{array}{l} \mathrm{t = Mac'_{K_M}}(m) \\ \mathrm{w = ECC(m)} \end{array}$$

$Dec_K(\langle r, s \rangle)$

- $w \parallel t \leftarrow \mathrm{F}_{K_E}(r) \oplus s$
- $m \leftarrow Decode(w)$
- $output = \begin{cases} m & \text{if } \mathrm{t = Ma} \\ \perp & \mathrm{o} \end{cases}$

Key Point: Error Correcting Code allows attacker to flip a few bits of s without altering message m.

$$\langle r, s \oplus 10^{n-1} \rangle = \langle r, \mathrm{F}_{K_E}(r) \oplus (w' \parallel t) \rangle$$

Source: The Order of Encryption and Authenticatio
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Building Authenticated Encryption

$$\mathrm{Enc_K}(m) = \langle r, \mathrm{F}_{K_E}(r) \oplus (\mathrm{ECC(m)} \parallel t) \rangle \text{ where } \quad \mathrm{t} = \mathrm{Mac}'_{K_M}(m)$$

$$\mathrm{w} = \mathrm{ECC(m)}$$

Worst Case: Chosen ciphertext attack allows attacker to completely recover plaintext.

Key Point: Error Correcting Code allows attacker to flip a few bits of s without altering message m.

$$\langle r, s \oplus 10^{n-1} \rangle = \langle r, \mathrm{F}_{K_E}(r) \oplus (w' \parallel t) \rangle$$

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