Homework 1

Due Date: September 03, 2019 at 11:59PM on Gradescope.
Instructor: Jeremiah Blocki

Homework Guideline Reminders

- Assignments must be typed. Submit one pdf file to Gradescope by 11:59PM, or else late penalties will apply. The pdf file can include hand-drawn images of figures.
- Each question needs to start with the resources and collaborator (RC) statement. You will not be penalized for using resources or having collaborators if your answers are expressed in your own words. If you consulted no resources outside of course material or had no collaborators, you must state so. A question without a complete RC statement will not be graded.

Question 1 (25 points)

Use mathematical induction (either weak or strong) to show the following statements:

(a) $n! > \pi^n$ for all integers $n \geq 7$.

(b) $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$ for all integers $n \geq 1$.

(c) Any integer $n \geq 2$ can be expressed as the product of 1 or more prime numbers.

Question 2 (25 points)

(a) Rank the following functions representing running times from smallest to largest (in terms of growth rate with respect to n). Group functions together if they are in the same equivalence class (i.e. $f(n)$ and $g(n)$ are in the same equivalence class if $f(n) = \Theta(g(n))$). Logarithms are base 2 unless stated otherwise. You only need to show the final ordering with equivalence classes, no need to explain your answers (except those in part b).

$$(n+1)!\ ,\ 8^{\log n}\ ,\ 5\log n\ ,\ n\log(n^3)\ ,\ 3^{n+2}\ ,\ 3^{n/2}\ ,\ \sqrt{n} + (\log n)^5\ ,\ \log(\log n)^5\ ,\ 0.9^n\ ,$$
$$\frac{\log n}{\log\log n}\ ,\ (\sqrt{2})^{\log n}\ ,\ 2^n\ ,\ n^2(\log n)^4 + 2n^3\ ,\ 3^{n-3}\ ,\ \log\log(n+1)\ ,\ \log(4n^3)$$

(b) Show your complete work when comparing the following function pairs. Provide the C and k values in the formal definition when asserting a big O, Ω, or Θ relationship (e.g. $f(n)$ is $O(g(n))$ if there exists $C > 0$ and $k \geq 0$ such that $f(n) \leq Cg(n)$ for all $n \geq k$).

(i) $3^{n/2}$ vs. 2^n

(ii) $8^{\log n}$ vs. $n^2(\log n)^4 + 2n^3$

(iii) $(\sqrt{2})^{\log n}$ vs. $\sqrt{n} + (\log n)^5$

Question 3 (25 Points)

How many times is function F called in each code segment? Clearly explain your answer and express the bounds in terms of n in big-O notation.

(a) Code Segment:

Algorithm 1 Code Segment 1

```
for (i = n; i > 1; i = i/3) do
    for (j = 0; j < n; j = j + 1) do
        F(i,j)
```

(b) Code Segment:

Algorithm 2 Code Segment 2

```
for (i = 0; i < n^2; i = i + 1) do
    for (j = 0; j < n; j = j + 3) do
        for (k = 0; k < j; k = k + 1) do
            F(i, j, k)
```

Question 4 (25 Points)

Evaluate the following sum exactly in terms of n and m.

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left(2(j-1)+\frac{i^2}{2^j}\right)$$