Static Single Assignment (SSA) Form

A sparse program representation for data-flow.

Computing Static Single Assignment (SSA) Form

Overview:

- What is SSA?
- Advantages of SSA over use-def chains
- “Flavors” of SSA
- Dominance frontiers revisited
- Inserting ϕ-nodes
- Renaming the temporaries
- Translating out of SSA form

R. Cytron, J. Ferrante, B. K. Rosen, M. N. Wegman, and F. K. Zadeck, *Efficiently Computing Static Single Assignment Form and the Control Dependence Graph*, *ACM TOPLAS* 13(4):451–490, Oct 1991

What is SSA?

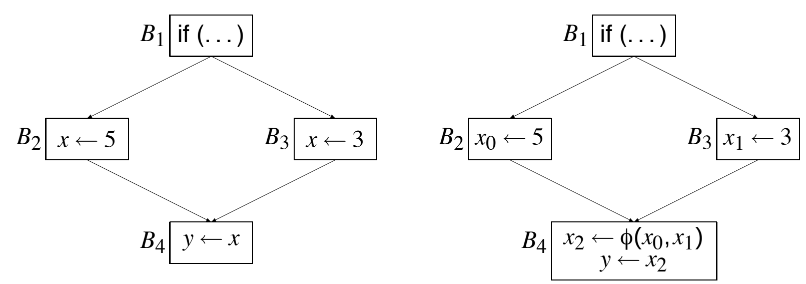
- Each assignment to a temporary is given a unique name
- All of the uses reached by that assignment are renamed
- Easy for straight-line code

$$
\begin{array}{rcl|rcl}
v & \leftarrow & 4 & v_0 & \leftarrow & 4 \\
 & \leftarrow & v+5 & & \leftarrow & v_0+5 \\
v & \leftarrow & 6 & v_1 & \leftarrow & 6 \\
 & \leftarrow & v+7 & & \leftarrow & v_1+7
\end{array}
$$

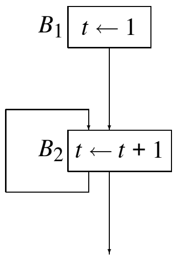
- What about control flow?

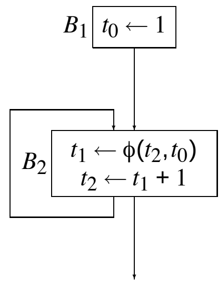
  $\Rightarrow$ ϕ-nodes

What is SSA?



What is SSA?





Advantages of SSA over use-def chains

- More compact representation
- Easier to update?
- Each use has only one definition
- Definitions explicitly merge values
  May still reach multiple ϕ-nodes

"Flavors" of SSA

Where do we place ϕ-nodes?

Condition:
If two non-null paths $x \rightarrow^+ z$ and $y \rightarrow^+ z$ converge at node z, and nodes x and y contain assignments to t (in the original program), then a ϕ-node for t must be inserted at z (in the new program)

minimal
As few as possible subject to condition

semi-pruned by Preston Briggs
As few as possible subject to condition, and t must be live across some basic block

pruned
As few as possible subject to condition, and no dead ϕ-nodes

Dominance Frontiers Revisited

The *dominance frontier* of v is the set of nodes $\text{DF}(v)$ such that:

- v dominates a predecessor of $w \in \text{DF}(v)$, but
  x does not strictly dominate $w \in \text{DF}(v)$

$$\text{DF}(v) = \{w \mid (\exists u \in PRED(w))[v \text{ DOM } u] \wedge v \ \overline{\text{DOM!}} \ w\}$$

- d dominates v, d DOM v, in a CFG *iff all* paths from $Entry$ to v include d
- d *strictly* dominates v:

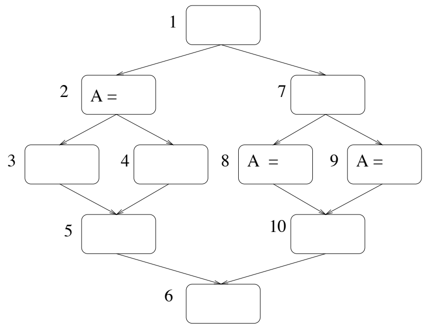
$$d \text{ DOM! } v \iff d \text{ DOM } v \text{ and } d \neq v$$

- The *immediate* dominator of v, $\text{IDOM}(v)$, is the closest strict dominator of v:

$$d \text{ IDOM } v \iff d \text{ DOM! } v \wedge (\forall w \mid w \text{ DOM! } v)[w \text{ DOM } d]$$

$\text{IDOM}(v)$ is v's parent in the *dominator tree*

Dominance Frontier: Example



DF(8) =

DF(9)=

DF(2)=

DF({8,9}) =

DF(10) =

DF({2,8,9,10}) =

Iterated Dominance Frontier

Extend the dominance frontier mapping from nodes to sets of nodes:

$$\mathrm{DF}(S) = \bigcup_{n \in S} \mathrm{DF}(n)$$

The *iterated* dominance frontier $\mathrm{DF}^+(S)$ is the limit of the sequence:

$$\mathrm{DF}_1(S) = \mathrm{DF}(S)$$
$$\mathrm{DF}_{i+1}(S) = \mathrm{DF}(S \cup \mathrm{DF}_i(S))$$

Theorem:

The set of nodes that need ϕ-nodes for any temporary t is the iterated dominance frontier $\mathrm{DF}^+(S)$, where S is the set of nodes that define t

Iterated Dominance Frontier Algorithm: $\mathrm{DF}^+(S)$

Input: Set of blocks S

Output: $\mathrm{DF}^+(S)$

```
workList ← {}
DF+(S) ← {}
foreach n ∈ S do
  DF+(S) ← DF+(S) ∪ {n}
  workList ← workList ∪ {n}
end
while workList ≠ {} do
    take n from workList
    foreach c ∈ DF(n) do
      if c ∉ DF+(S) then
        DF+(S) ← DF+(S) ∪ {c}
        workList ← workList ∪ {c}
      end
    end
end
```


Inserting ϕ-nodes (minimal SSA)

```
foreach t ∈ Temporaries do
  S ← {n | t ∈ Def(n)} ∪ Entry
  Compute DF+(S)
  foreach n ∈ DF+(S) do
    Insert a φ-node for t at n
  end
end
```

Inserting ϕ-nodes for globals (semi-pruned SSA)

Compute *local* liveness: globals are those live across block boundaries (*ie*, used before definition in *any* basic block)

foreach $t \in$ *Temporaries* **do**
  if $t \in$ *Globals* **then**
    $S \leftarrow \{n \mid t \in Def(n)\} \cup Entry$
    Compute $\mathrm{DF}^+(S)$
    foreach $n \in \mathrm{DF}^+(S)$ **do**
      Insert a ϕ-node for t at n
    end
  end
end

Inserting fewest ϕ-nodes (pruned SSA)

Compute *global* liveness: nodes where each temporary is live-in

foreach $t \in$ *Temporaries* **do**
  if $t \in$ *Globals* **then**
    $S \leftarrow \{n \mid t \in Defs(n)\} \cup Entry$
    Compute $\mathrm{DF}^+(S)$
    foreach $n \in \mathrm{DF}^+(S)$ **do**
      if t live-in at n **then**
        Insert a ϕ-node for t at n
      end
    end
  end
end

Renaming the temporaries

After ϕ-node insertion, uses of t are either:

original: dominated by the definition that computes t.

> If not, then $\exists$ path to use avoiding definition, which means separate paths from definitions converge between definition and use, thus inserting another definition.
> *ie*, each use dominated by an evaluation of t or a ϕ-node for t

ϕ: has a corresponding predecessor p, dominated by the definition of t (as before)

Thus, walk dominator tree, replacing each definition and its dominated uses with a new temporary.

Use a stack to hold current name (subscript) for each set of dominated nodes.

Propagate names from each block to corresponding ϕ-node operands of its successors.

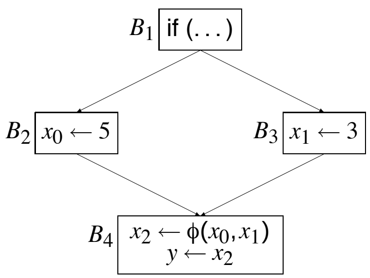
Renaming the temporaries

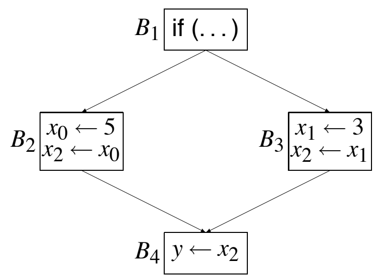
foreach $t \in$ *Temporaries* **do** *count*$[t] \leftarrow 0$; *stack*$[t] \leftarrow$ *empty*; *stack*$[t]$.push(0)
Rename$(Entry)$

proc *Rename*(n) $\equiv$
  foreach statement $I \in n$ **do**
    if $s \not\equiv \phi$ **then foreach** $t \in Uses(I)$ **do**
        $i \leftarrow$ *stack*$[t]$.top
        replace use of t with t_i in I
    foreach $t \in Defs(I)$ **do**
      $i \leftarrow$ ++*count*$[t]$; *stack*$[t]$.push(i)
      replace def of t with t_i in I
  foreach $s \in SUCC(n)$ **do**
    given n is the jth predecessor of s
    foreach $\phi \in s$ **do**
      given t is the jth operand of ϕ
      $i \leftarrow$ *stack*$[t]$.top
      replace jth operand of ϕ with t_i
  foreach $c \in Children(n)$ **do** *Rename*(c)
  foreach statement $I \in n, t \in Defs(I)$ **do** *stack*$[t].pop()$

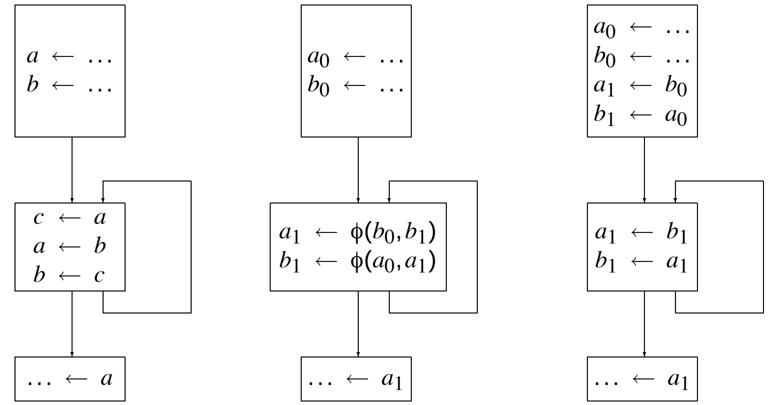
Translating Out of SSA Form

Replace ϕ-nodes with copy statements in predecessors

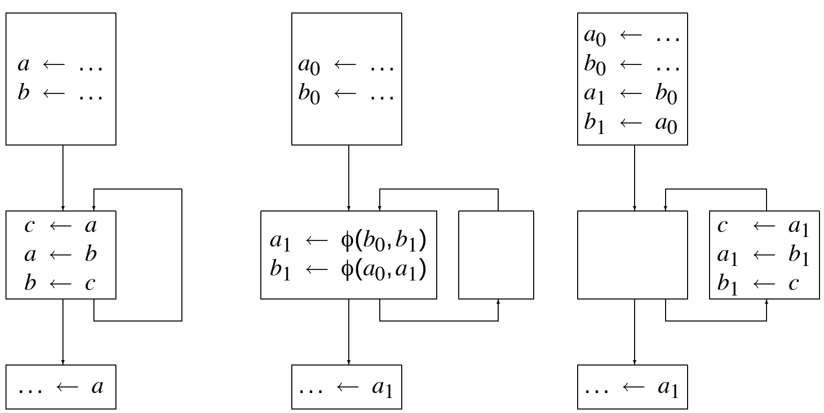




Normal Form, Optimized SSA, Incorrect Translation



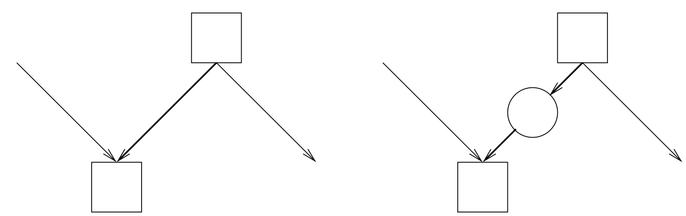
Normal Form, Edge-Split Opt SSA, Correct Translation



Solution: critical edge splitting

Critical Edge:

source has multiple out-edges *and* target has multiple in-edges



Good for other transformations too (*cf* landing pads)

Next Time

Static Single Assignment

- Induction variables (standard vs. SSA)
- Loop Invariant Code Motion with SSA

Wegman & Zadeck, *Constant Propagation with Conditional Branches*, TOPLAS 13(2):181–210, Apr 1991