1.a) 

1.b) 

1.c)
2.a) $(c|1)(01)^*|0^*10^*$
2.b) $(1^*01^*01^*)^*|0^*10^*10^*$
2.c) $a^*b^* \ldots z^*$
2.d) $(a|b|c|d)^+az(a|b|c|d)^+b(a|b|c|d)^+bz(a|b|c|d)^+cz(a|b|c|d)^+dz(a|b|c|d)^+d$  
2.e) We can use the pumping lemma to prove that language $L$ of algebraic expressions with parentheses is not regular. For any given $n$, we can come up with a string $s = xyz$ in $L$ such that $\text{length}(s) > n$ and $xy^*z \not\in L$; one such $s$ is an algebraic expression with more than $n$ levels of redundant parentheses around it.

3.a) \[
\begin{array}{c|cc}
\epsilon & a & b \\
s_0 & s_1, s_9 \\
s_1 & s_2, s_5 \\
s_2 & s_3 \\
s_3 & s_4 & s_5 \\
s_4 & s_8 \\
s_5 & s_6 \\
s_6 & s_8 \\
s_7 & s_8 \\
s_8 & s_1, s_9 \\
s_9 & \\
\end{array}
\]

- $(s_0, \epsilon) = \{s_0, s_1, s_9, s_2, s_5\}$
- $(\{s_0, s_1, s_2, s_5, s_9\}, a) = \{s_3, s_6\}$
- $(\{s_3, s_6\}, b) = \{s_1, s_2, s_4, s_5, s_8, s_9\}$
- $(\{s_3, s_6\}, c) = \{s_1, s_2, s_5, s_7, s_8, s_9\}$
- $(\{s_1, s_2, s_4, s_5, s_8, s_9\}, a) = \{s_3, s_6\}$
- $(\{s_1, s_2, s_5, s_7, s_8, s_9\}, a) = \{s_3, s_6\}$

After state collapsing we get:
\[
\begin{array}{c|ccc}
\epsilon & a & b & c \\
s_0* & s_1 \\
s_1 & s_2 & s_2 \\
s_2* & s_1 \\
\end{array}
\]
3.b) \[
\begin{array}{c|ccc}
\epsilon & 0 & 1 \\
\hline
s_0 & s_1, s_7 & & \\
s_1 & s_2, s_4 & & \\
s_2 & s_3 & & \\
s_3 & s_6 & s_5 & \\
s_4 & s_6 & & \\
s_5 & s_7 & & \\
s_6 & s_8 & & \\
s_7 & s_9 & & \\
s_8 & s_{10} & & \\
s_9 & s_{11} & & \\
s_{10} & s_{12}, s_{14} & & \\
s_{11} & s_{13} & & \\
s_{12} & s_{14}, s_{12} & & \\
s_{13} & s_{14} & & \\
s_{14} & & & \\
\end{array}
\]

- \( \{s_0, s_1, s_2, s_4, s_7\}, 0 \) = \( \{s_1, s_2, s_3, s_4, s_6, s_7\} \)
- \( \{s_0, s_1, s_2, s_4, s_7\}, 1 \) = \( \{s_1, s_2, s_4, s_5, s_6, s_7, s_8\} \)
- \( \{s_1, s_2, s_3, s_4, s_6, s_7\}, 0 \) = \( \{s_1, s_2, s_3, s_4, s_6, s_7, s_8\} \)
- \( \{s_1, s_2, s_3, s_4, s_6, s_7\}, 1 \) = \( \{s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8\} \)
- \( \{s_1, s_2, s_4, s_5, s_6, s_7, s_8\}, 0 \) = \( \{s_1, s_2, s_3, s_4, s_6, s_7\} \)
- \( \{s_1, s_2, s_4, s_5, s_6, s_7, s_8\}, 1 \) = \( \{s_1, s_2, s_4, s_5, s_6, s_7, s_8, s_9\} \)
- \( \{s_1, s_2, s_4, s_5, s_6, s_7, s_8, s_9\}, 0 \) = \( \{s_1, s_2, s_3, s_4, s_6, s_7, s_{10}\} \)
- \( \{s_1, s_2, s_3, s_4, s_6, s_7, s_8, s_9\}, 1 \) = \( \{s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9\} \)
- \( \{s_1, s_2, s_3, s_4, s_6, s_7, s_8, s_9, 0\}, 0 \) = \( \{s_1, s_2, s_3, s_4, s_6, s_7, s_{10}\} \)
- \( \{s_1, s_2, s_3, s_4, s_6, s_7, s_8, s_9, 1\}, 0 \) = \( \{s_1, s_2, s_3, s_4, s_6, s_7, s_{11}, s_{12}, s_{14}\} \)
- \( \{s_1, s_2, s_3, s_4, s_6, s_7, s_8, s_9, 1\}, 1 \) = \( \{s_1, s_2, s_3, s_4, s_6, s_7, s_{11}, s_{12}, s_{13}, s_{14}\} \)
- \( \{s_1, s_2, s_3, s_4, s_6, s_7, s_8, s_9, 0\}, 0 \) = \( \{s_1, s_2, s_3, s_4, s_6, s_7\} \)
- \( \{s_1, s_2, s_3, s_4, s_6, s_7, s_8, s_9, 1\}, 0 \) = \( \{s_1, s_2, s_3, s_4, s_6, s_7, s_{10}\} \)
- \( \{s_1, s_2, s_3, s_4, s_6, s_7, s_8, s_9, 1\}, 1 \) = \( \{s_1, s_2, s_3, s_4, s_6, s_7, s_{12}, s_{13}, s_{14}\} \)
After state collapsing we get:

<table>
<thead>
<tr>
<th></th>
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</tr>
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<tbody>
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<td>$s_1$</td>
</tr>
<tr>
<td>$s_1$</td>
<td>$s_0$</td>
<td>$s_2$</td>
</tr>
<tr>
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<td>$s_3$</td>
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</tr>
<tr>
<td>$s_3$</td>
<td>$s_1$</td>
<td>$s_4$</td>
</tr>
<tr>
<td>$s_4$</td>
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</tr>
<tr>
<td>$s_6^*$</td>
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</tbody>
</table>

3.c)

<table>
<thead>
<tr>
<th></th>
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<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_0$</td>
<td>$s_1$, $s_{14}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s_1$</td>
<td>$s_2$, $s_5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s_2$</td>
<td></td>
<td>$s_3$</td>
<td></td>
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<tr>
<td>$s_3$</td>
<td></td>
<td>$s_4$</td>
<td></td>
</tr>
<tr>
<td>$s_4$</td>
<td>$s_{13}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s_5$</td>
<td>$s_6$, $s_9$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s_6$</td>
<td></td>
<td>$s_7$</td>
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<td>$s_8$</td>
<td>$s_{12}$</td>
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<tr>
<td>$s_9$</td>
<td></td>
<td>$s_{10}$</td>
<td></td>
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<tr>
<td>$s_{10}$</td>
<td></td>
<td>$s_{11}$</td>
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<tr>
<td>$s_{11}$</td>
<td>$s_{12}$</td>
<td></td>
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</tr>
<tr>
<td>$s_{12}$</td>
<td>$s_{13}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s_{13}$</td>
<td>$s_1$, $s_{14}$</td>
<td>$s_{15}$</td>
<td></td>
</tr>
<tr>
<td>$s_{14}$</td>
<td>$s_{15}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s_{15}$</td>
<td>$s_{16}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s_{16}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- $(\{s_0, s_1, s_2, s_5, s_6, s_9, s_{14}\}, 0) = \{s_3, s_{10}\}$
- $(\{s_0, s_1, s_2, s_5, s_6, s_9, s_{14}\}, 1) = \{s_7, s_{15}\}$
- $(\{s_3, s_{10}\}, 0) = \{s_1, s_2, s_5, s_6, s_9, s_{11}, s_{12}, s_{13}, s_{14}\}$
- $(\{s_3, s_{10}\}, 1) = \{s_1, s_2, s_4, s_5, s_6, s_9, s_{13}, s_{14}\}$
- $(\{s_7, s_{15}\}, 0) = \{s_1, s_2, s_5, s_6, s_8, s_9, s_{12}, s_{13}, s_{14}\}$
- $(\{s_7, s_{15}\}, 1) = \{s_{16}\}$
- $(\{s_1, s_2, s_5, s_6, s_9, s_{11}, s_{12}, s_{13}, s_{14}\}, 0) = \{s_3, s_{10}\}$
- $(\{s_1, s_2, s_5, s_6, s_9, s_{11}, s_{12}, s_{13}, s_{14}\}, 1) = \{s_7, s_{15}\}$
- $(\{s_1, s_2, s_4, s_5, s_6, s_9, s_{13}, s_{14}\}, 0) = \{s_3, s_{10}\}$
- $(\{s_1, s_2, s_4, s_5, s_6, s_9, s_{13}, s_{14}\}, 1) = \{s_7, s_{15}\}$
- $(\{s_1, s_2, s_5, s_6, s_8, s_9, s_{12}, s_{13}, s_{14}\}, 0) = \{s_3, s_{10}\}$
- $(\{s_1, s_2, s_5, s_6, s_8, s_9, s_{12}, s_{13}, s_{14}\}, 1) = \{s_7, s_{15}\}$
After collapsing states we get:

\[
\begin{array}{c|cc}
& 0 & 1 \\
\hline
s_0 & s_1 & s_2 \\
\hline
s_1 & s_0 & s_0 \\
\hline
s_2 & s_0 & s_3 \\
\hline
s_3^* & & \\
\end{array}
\]

4) We know that regular languages are closed under complement and union operations. For this reason \( L_1, L_2, L_1 \cup L_2, \) and \( L_1 \cup L_2 \) are regular languages. Since \( L_1 \cap L_2 = \overline{L_1 \cup L_2} \), then \( L_1 \cap L_2 \) is a regular language.

5) We first eliminate left recursion in \( R \) and left factor \( Q \), to obtain the following production rules:

\[
\begin{align*}
L & \to Ra \\
R & \to abaR' \\
Q & \to bQ' \\
L & \to Qba \\
R & \to cabaR' \\
Q & \to bcR' \\
R' & \to bcR' \\
Q' & \to bc \\
R' & \to c \\
R' & \to \epsilon \\
\end{align*}
\]

The First and Follow table for above grammar is:

<table>
<thead>
<tr>
<th>Nullable</th>
<th>Null</th>
<th>First</th>
<th>Follow</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>no</td>
<td>{a, b, c}</td>
<td>-</td>
</tr>
<tr>
<td>R</td>
<td>no</td>
<td>{a, c}</td>
<td>{a}</td>
</tr>
<tr>
<td>R'</td>
<td>yes</td>
<td>{b, \epsilon}</td>
<td>{a}</td>
</tr>
<tr>
<td>Q</td>
<td>no</td>
<td>{b}</td>
<td>{b}</td>
</tr>
<tr>
<td>Q'</td>
<td>no</td>
<td>{b, c}</td>
<td>{b}</td>
</tr>
</tbody>
</table>

The \( LL(1) \) parsing table is:

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>Ra</td>
<td>Qba</td>
<td>Ra</td>
</tr>
<tr>
<td>R</td>
<td>abaR'</td>
<td>-</td>
<td>cabaR'</td>
</tr>
<tr>
<td>R'</td>
<td>\epsilon</td>
<td>R' \to bcR'</td>
<td>-</td>
</tr>
<tr>
<td>Q</td>
<td>-</td>
<td>bQ</td>
<td>-</td>
</tr>
<tr>
<td>Q'</td>
<td>-</td>
<td>bQ'</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>Ra</td>
<td>Qba</td>
<td>Ra</td>
</tr>
<tr>
<td>R</td>
<td>abaR'</td>
<td>-</td>
<td>cabaR'</td>
</tr>
<tr>
<td>R'</td>
<td>\epsilon</td>
<td>R' \to bcR'</td>
<td>-</td>
</tr>
<tr>
<td>Q</td>
<td>-</td>
<td>bQ</td>
<td>-</td>
</tr>
<tr>
<td>Q'</td>
<td>-</td>
<td>bQ'</td>
<td>-</td>
</tr>
</tbody>
</table>
The grammar in question is not $LR(0)$ since there’s a shift-reduce conflict in the two production rules for $A$.

The grammar is $LR(1)$ since there are no conflicts in the above parsing table that uses 1 symbol lookahead.