Lecture 00: Mathematical Toolkit in Computer Science (CS 59000 – MTK)

### Course Content

- Introduce Foundational Theoretical Topics in Computer Science
  - Mathematical Foundations
  - Wide Applications in Computer Science

### About Me

- I am a Theoretical Cryptographer
- Visit webpage to find more about my research

#### Mathematical Basics

- Inequalities: Jensen's Inequality and consequences
- Summations/Integrals
- Probability Basics, Pigeonhole Principle

#### Balls and Bins Problems

- Birthday Paradox, Max Load, Coupon Collectors Problem
- The Power of 2 Choices
- Bloom Filters
- Randomized Routing on Networks

#### Concentration Inequalities

- Markov Inequality, Chebyshev Inequality
- Chernoff-Hoeffding Bound
- Azuma's Inequality
- Talagrand Inequality
- Anti-concentration Bounds: Littlewood-Offord Theorem
- Tightness of Chernoff Bound: Method of Types
- Stirling Approximation

#### Probabilistic Techniques

- Lovász Local Lemma and a few Applications
- Moser-Tardos Algorithm

#### Markov Chains and Random Walks

- Stationary Distributions
- Mixing Time
- Stopping time

### Spectral Graph Theory

- Expanders
- Random walks and concentration
- Averaging Samplers

#### **Error-correcting Codes**

- Gilbert-Varshamov Bound
- Linear codes
- Shannon's Channel Coding Theorem

### Discrete Fourier Analysis on the Boolean Hypercube

- BLR Linearity Testing
- Randomness Extraction
- Hypercontractivity
- KKL Theorem
- Goldreich-Levin Theorem

### Course Outline: Grand Aim

- An Introduction to Foundational Topics in Mathematics
  - Assist your research in diverse topics in Computer Science

## Prerequisites

- Basic Algorithms (CS 58000)
- Mathematical Maturity
- Read the course website and the course policy

#### Course Instruction

- Board-work
- Lecture notes (in next couple of days)
- Pointers to a lot of reading materials
- General Pointers to books and other related courses
  - No official course book
  - A recommended book: "Probability and Computing: Randomized Algorithms and Probabilistic Analysis" by Michael Mitzenmacher and Eli Upfal
  - The course syllabus is flexible and student interest will influence it

# Grading

- 45% Homework (roughly five)
- 20% Midterm (in class)
- 30% Final Exam
- 5% Class Participation

#### How to Use this Course?

- For Grades
  - Submit Homework, perform in Exams, and Participate in Class
- For Research
  - Solve extra-credit problems, read additional materials, discuss with instructor by scheduling appointments, and target to find a research topic of choice

#### Office Hours

- Office hour with Instructor: By Appointment Only
- Office hour with TA: One hour that is agreeable to the TA and all students

## Concluding Remarks

- What is expect of you: Knowledge of "Algorithms"-equivalent course, some Mathematical Maturity and Class Participation
- We will collaboratively learn from each other
- Read the course webpage