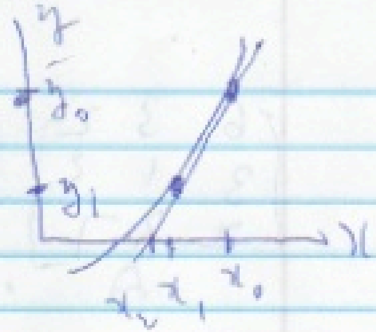
CS314

week 2 Tues

2-2

<Secant Method>

$$x_2 = x_1 - \frac{f(x_1)(x_1-x_0)}{f(x_1)-f(x_0)}$$



<example> $f(x) = \sin x - .5 = 0$

k	x_k	$f(x_k)$	x_{k+1}
0	$x_0=0$	$\sin 0 - .5 = -.5$	
1	$x_1=1$	$\sin 1 - .5 = .3415$	$x_2 = 1 - \frac{.3415(1-0)}{.3415-(-.5)} = .594$
2	$x_2=.594$	$\sin(.594)-.5 = .0597$	$x_3 = .594 - \frac{.0597(.594-1)}{(.0597-.3415)} = .50798$
3	$x_3=.50798$	$\sin(.50798)-.5=-.0135$	

$x_4 = .50798 - .0135 \cdot \frac{(.50798-.597)}{.0135-.50798}$

$= .5235$

<Precise solution>

$\sin x = .5 \quad x = 30^\circ \quad = 30 \frac{\pi}{180} = \frac{\pi}{6} = .5235$

$$f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

If $\Delta x = x_1 - x_0$ The secant method can be rewritten,

<The solution of linear systems>

$Ax = B \Rightarrow$
$$\begin{aligned} 6x+3y+2z &= 29 \\ 3x+y+2z &= 17 \\ 2x+3y+7z &= 21 \end{aligned}$$

The new point x_2 will be the intersection with the x axis of the line btw P_0 & P_1

(1) $$m = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

(2) $$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

from (1) and (2)

$$\frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{-f(x_1)}{x_2 - x_1}$$

$$x_2 - x_1 = -f(x_1)\,\frac{(x_1 - x_0)}{f(x_1) - f(x_0)}$$

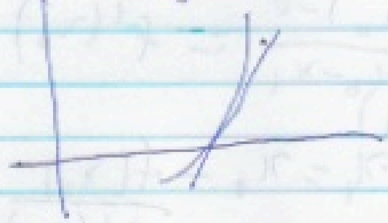
$$\boxed{x_2 = x_1 - f(x_1)\,\frac{(x_1 - x_0)}{f(x_1) - f(x_0)}}$$

Secant method

In fact we could use three terms of the taylor expansion to construct an improved Newton Rapson method that also uses the second derivative. this method produces a function

$x_1 = g(x_0)$

where $g(x_1)$ is quadratic and not linear



Ex) Solve $\sin x = \frac{1}{2}$ $\quad$ x = radians

$f(x) = \sin x - \frac{1}{2} = 0$ $\quad$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$f'(x) = \cos x$

i	x	$f(x)$	$f'(x)$	$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$
0	$x_0 = 0$	$\sin 0 - .5 = -.5$	$\cos 0 = 1$	$x_{k+1} = 0 - \frac{-.5}{1} = .5$
1	$x_0 = .5$	$\sin .5 - .5 = -0.02$	$\cos .5 = .9795$	$.5 - \frac{-0.02}{.9795} = .522$
2	$x_2 = .522$	$\sin .522 - .5 = .0039$	$\cos(.522) = .8668$	$.522 - \frac{.0034}{.8668}$

$x_3 = .5235$ exact solution $\quad$ $= .5235$

$\sin x = \frac{1}{2}$ $\quad x = 30$ or $x = 30° \frac{\pi}{180} = \frac{\pi}{6} = .5235$

This is more precise than false position method and bisection at the $i=2$

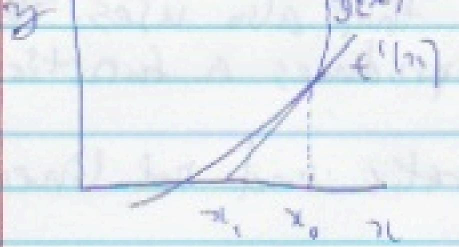
Newton Rapson converges faster than false pos & bisection method.

CS514

Week 2 - Mon note #1

<Newton - Rapson>

If $f(x)$, $f'(x)$, $f''(x)$ are continuous then we can use this information to find the solution of $f(x)$



① $m = \frac{f(x_0) - 0}{x_0 - x_1}$ ② $m = f'(x_0)$

from ① and ②

$$\frac{f(x_0) - 0}{x_0 - x_1} = f'(x_0)$$

$$f(x_0) = f'(x_0)(x_0 - x_1)$$

$$\frac{f(x_0)}{f'(x_0)} = x_0 - x_1, \quad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$ Newton Rapson

<Newton Rapson and taylor expansion>

We can approximate a continuous function using a Taylor polynomial expansion around x_0

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0) \cdot (x - x_0)^2}{2!}$$

Newton Rapson approximates $f(x)$ by using taylor expansion up to the first two terms. We will build the Newton Rapson method again but now using the Taylor expansion.

At x_1 $f(x_1) = 0$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

Substitute $x = x_1$ and use only two terms of Taylor expansion

$$\underset{0}{f(x_1)} = f(x_0) + f'(x_0)(x_1 - x_0) - f(x_0)$$

$$= f'(x_0)(x_1 - x_0) \quad -\frac{f(x_0)}{f'(x_0)} = x_1 - x_0$$

$$x_0 - \frac{f(x_0)}{f'(x_0)} = x_1 \implies \boxed{\text{Newton Rapson} \quad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

$$\begin{bmatrix} 6 & 3 & 2 \\ 3 & 1 & 3 \\ 2 & 3 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 29 \\ 17 \\ 21 \end{bmatrix}$$

$$A \cdot x = B$$

Definitions

$x = (x_1, x_2, x_3 \cdots x_N)$

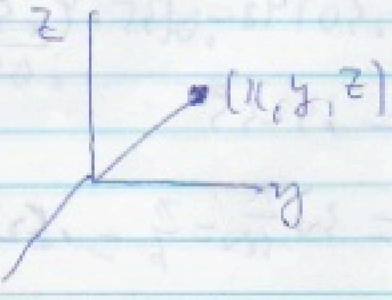
$x_1, x_2 \cdots x_n$ are components of x

N dimensional space

The set of all N Dimensional vectors

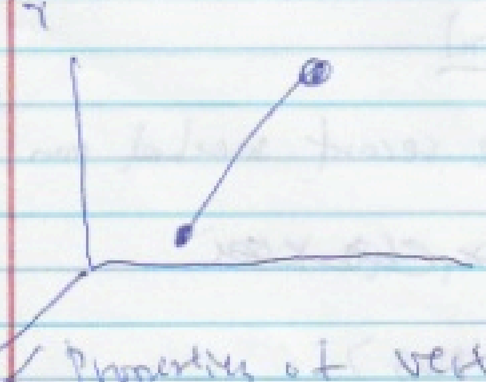
When a vector is used to determine a position it's called a position vector.

<position vector>



When a vector is used to denote movement btw two points in space it's called displacement - vector.

<displacement vector>



<Properties of vectors>

• equality $x = y$ if and only if $x_j = y_j$ for $j = 1, 2, \cdots N$

Sum $x + y = (x_1 + y_1, x_2 + y_2, \cdots x_n + y_n)$

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Week 2 mon June 21 note #2

<Order of convergence>

Assume $x=p$ is a zero of the function $f(x)=0$ and $E=p-p_n$ is the approx error. If two positive constants $A\neq 0$ and $R>0$ exist, and

$$\lim_{n\to\infty} \left|\frac{p-p_{n+1}}{p-p_n}\right|^R = \lim \frac{E_{n+1}}{|E_n|^R} = A$$

Then the sequence is said to converge to p with order of convergence R that is

$$E_{n+1} = A \cdot |E_n|^R$$

R will tell us how fast a method converges.

If $R=1$ then the convergence is linear

If $R=2$ the convergence is quadratic

Usually the more roots (solutions) an equation has the slower the convergence may be.

If p is a simple root the Newton Rapson method has $R=2$ (quadratic convergence)

<Secant Method>

The Newton Rapson method requires the evaluation of two functions at each iteration.

- It also requires the derivative $f'(x)$
- The secant method will requires only one evaluation of $f(x)$
- If $f(x)$ has only one root. The secant method has an order of convergence $R\approx 1.618$ that is close to Newton-Rapson.

The secant method uses two initial points x_0, x_1..