

**BINARY** (N number F fraction S sign) Fixed Point:  $((-1)^S)(NNNN.FFFF)$ , stored as SNNNNFFFF  
Floating Point:  $((-1)^S)(1 + .FFFF) * 2^{(NNNN - 1023)}$ , stored as SNNNNFFFF

**ERROR** (p value, p' approx,  $p = p' + e_1$ ,  $q = q' + e_2$ ) Absolute:  $|p - p'|$ , Relative:  $|p - p'| / |p|$ ,  $p \neq 0$   
 $p + q = p' + q' + (e_1 + e_2)$ ,  $pq = p' + q' + (q'e_1 + p'e_2 + e_1e_2)$

**[F(x) = 0]** Monotone: no sign change, Oscillating = changes signs each iteration

Bisection Method (start with A and B);  $C = (A + B)/2$ ; If  $(A)f(C) < 0$ ,  $C = A$ , Else  $C = B$

Regula Falsi Method (start with A and B);  $C = B - [(f(B)(B - A))/(f(B) - f(A))]$ , same checks/loop

Newton-Raphson Method (from 1<sup>st</sup> order Taylor Series, start with  $p_0$ ):  $p_k = p_{k-1} - (f(p_{k-1})/f'(p_{k-1}))$   
 $f'(x) = (f(x + E) - f(x))/E$  for some very small E; converges order 2 if simple root, 1 if multiple

Secant Method (start with  $p_0$  and  $p_1$ ),  $p_{k+1} = p_k - [((f(p_k)(p_k - p_{k-1}))/f(p_k) - f(p_{k-1}))]$

Horizontal convergence when  $|f(x)| < \text{eps}$ , vertical when  $|x_n - p| < \text{del}$ ,  $f(p) = 0$

Well-conditioned = clear/straight near zero, ill = multi-crossing, close to x-axis

**[Ax = B]** Upper triangular: only 0 below diagonal, Back substituting: solve last equation, substitute up  
Can multiply by scalar, add multiples, move order of rows in a matrix; pivot = largest left in column

Gaussian Elimination: Make upper triangular, back substitute (use if only 1 system)

LU factorization (place next to identity matrix, perform same on both, substitute) (use for >1 system)

**ITERATIVE [Ax=B]** (use on big sparse matrices); Gauss-Seidel Method: Get  $x_k, y_k$  from  $x_{k-1}$  and  $y_{k-1}$   
Jacobi Method: Get  $x_k, y_k$  from  $x_k$  and  $y_k$ , ( $x_{k-1}$  and  $y_{k-1}$  when not possible)

Converges for strictly diagonal ( $|a[k][k]| > \sum_{[j=1; j \neq k; j \rightarrow n]} |a[k][j]|$ ), k goes from 1 to n)

**POLYNOMIAL APPROX.** Horner's Method: derivative of  $(\sum_{[j=n; j \rightarrow 1]} (a_j x^{j+1})/(j+1))$  to approx.  $p(x)$

Lagrange: have n points,  $p[n](x) = \sum_{[k=0; n]} y_k * (\prod_{[j=0; j \neq k; j \rightarrow n]} (x - x_j)) / (\prod_{[j=0; j \neq k; j \rightarrow n]} (x_k - x_j))$

Newton Method (using "divided differences",  $f[g(x)]$ );  $P_n(x) = P_{n-1}(x) + a_n(\prod_{[i=0; n]} (x - x_i))$   
 $f[x_k] = f(x_k)$ ,  $f[x_{k-1}, x_k] = (f[x_k] - f[x_{k-1}]) / (x_k - x_{k-1})$ , get  $a_i$  from  $f[x_{i-1}, x_i]$  or  $f[x_1]$

Padé Approximation:  $R_{n,m}(x) = P_n(x)/Q_m(x)$ ,  $P_n(x) = p_0 + p_1x + \dots + p_nx^n$ ,  $Q_m(x) = 1 + q_1x + \dots + q_mx^m$   
 $f(x)Q_m(x) - P_n(x) = 0$ ;  $a_0 - p_0 = 0$ ,  $a_1 + a_0q_1 - p_1 = 0$ ,  $a_2 + a_1q_1 + a_0q_2 - p_2 = 0$ , etc., get  $a_i$  from Taylor exp. of  $f(x)$