Homework 1 Solution

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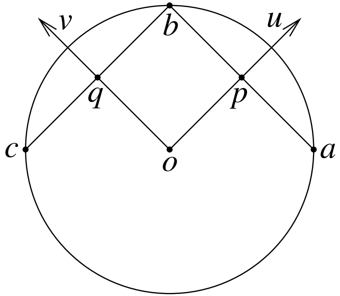
Problem 1: Circle Through Three Points



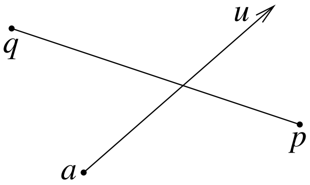
- The perpendicular bisector of ab is $p + ku$ with $p = (a + b)/2$ and $u = (b_y - a_y, a_x - b_x)$.
- Likewise, $q + lv$ for bc.
- The center o is the intersection point of the bisectors.
- Solve $(p + ku - q) \times v = 0$ for $k = (q - p) \times v / u \times v$.
- If the points are collinear, $u \times v = 0$ and there is no solution.

Problem 1: ACP Code

```
class CircleCenter : public Point {
  Point *a, *b, *c;

  DeclareCalculate (PV2) {
    PV2<N> aa = a->get<N>(),
           bb = b->get<N>(),
           cc = c->get<N>(),
           p = (aa + bb)/2, q = (bb + cc)/2,
           u(bb.y - aa.y, aa.x - bb.x),
           v(cc.y - bb.y, bb.x - cc.x);
    N k = (q - p).cross(v)/u.cross(v);
    return p + k*u;
  }
public:
  CircleCenter (Point *a, Point *b, Point *c)
    : a(a), b(b), c(c) {}
};
```

Problem 2: Ray/Line Segment Intersection



When does the ray $a + ku$ intersect the line segment pq?

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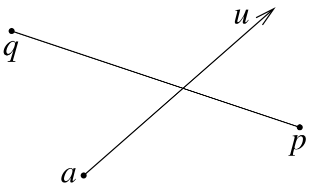
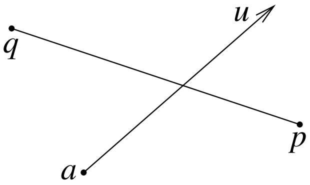


When does the ray $a + ku$ intersect the line segment pq?

1. p and q are on opposite sides of $a + ku$.

Problem 2: Ray/Line Segment Intersection



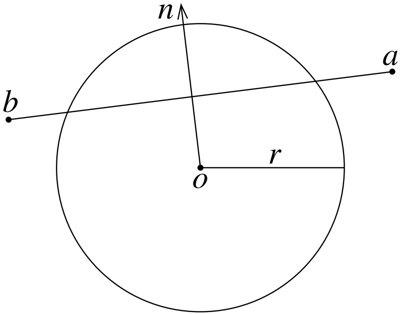
When does the ray $a + ku$ intersect the line segment pq?

1. p and q are on opposite sides of $a + ku$.
2. a is on one side of pq and u points into the other side.

How are these tests performed?

1. $\mathrm{sign}(u \times (p - a)) \neq \mathrm{sign}(u \times (q - a))$
2. $\mathrm{sign}((a - p) \times (q - p)) \neq \mathrm{sign}(u \times (q - p))$

Problem 3: Circle/Line Segment Intersection



- The distance from o to the ab line is less than r: $-r < \frac{n}{||n||} \cdot (o - a) < r$ with $n = (b_x - a_x, a_y - b_y)$.
- Squaring yields $(n \cdot (o - a))^2 < (n \cdot n) r^2$.
- Points a and b are outside the circle: $(a - o) \cdot (a - o) > r^2$ and $(b - o) \cdot (b - o) > r^2$.
- Points a and b are on opposite sides of the line through o and tangent to n: $\mathrm{sign}(n \times (a - o)) = -\mathrm{sign}(n \times (b - o))$.

Problem 4: 3D Triangle Intersection

Two triangles intersect when an edge of one intersects the interior of the other.

An edge *ef* intersects a triangle *abc* when the intersection point of *ef* with the *abc* plane is inside *abc*.

The intersection point, $p = e + ku$ with $u = f - e$, satisfies $n \cdot (e + ku - a) = 0$ with $n = (a - b) \times (a - c)$. It lies on *ef* when $0 < k < 1$.

Let p' be the projection of p onto the coordinate plane of the largest magnitude component of n, e.g. $p' = (p_x, p_y)$ when $|n_z|$ is largest. Apply the 2D point-in-triangle test $\mathrm{LT}(a', b', p') > 0$, $\mathrm{LT}(b', c', p') > 0$, and $\mathrm{LT}(c', a', p') > 0$.

Degenerate Cases

1. *ef* intersects an *abc* edge: one of the LT is zero.
2. *ef* contains an *abc* vertex: two of the LT are zero.
3. e or f is in *abc*: $k = 0$ or $k = 1$.
4. e or f is on an *abc* edge: tests 1 and 3.
5. e or f is an *abc* vertex: tests 2 and 3.
6. e and f are in the *abc* plane: $n \cdot u = 0$, project and compute 2D edge/edge intersections.