Programmable QL-bits from Signed Regular Graphs

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Abstract

We construct and analyze programmable quantum-like bits (QL-bits) as two-level logical encodings realized by eigenmodes of signed graph adjacency matrices. Two k-regular subgraphs coupled by an l-regular bipartite connector produce a composite signed adjacency R. We prove that symmetric regular coupling yields exact logical Hadamard modes $|\pm\rangle_L$ with eigenvalues $k \pm l$. We then give two constructive state-synthesis mechanisms for any real encoded state $(\omega_1 V_A;\ \omega_2 V_B)$: (i) symmetric degree detuning with $\Delta = (k_A - k_B)/(2l)$, and (ii) directed/asymmetric coupling with $\Delta_C = l_A/l_B$. In both cases, parameter/inverse switching avoids singular regimes and covers boundary amplitude ratios. Because degrees are integer in the unweighted signed model, target amplitudes are approximated by rational degree ratios, while uniform degree scaling provides an independent knob for spectral separation/robustness.

Model

We build a single QL-bit from two regular graphs G_A, G_B (adjacency A, B) coupled by a signed bipartite connector. The composite signed adjacency is $R = \begin{pmatrix} A & C_A \\ C_B & B \end{pmatrix}$, with $C_B = C_A^T$ in the symmetric case. For k_A- and k_B-regular subgraphs, the Perron-Frobenius eigenvectors are uniform unit vectors V_A, V_B, giving an encoded basis

$$|0\rangle_L = \begin{pmatrix} V_A \\ 0 \end{pmatrix}, |1\rangle_L = \begin{pmatrix} 0 \\ V_B \end{pmatrix}.$$

Goal: choose integer degree parameters so that $|\psi\rangle_L = (\omega_1 V_A;\ \omega_2 V_B)$ is an eigenvector of R with $\omega_1^2 + \omega_2^2 = 1$.

Lemma: Regular Coupling

Regular coupling gives Hadamard-like modes.
If A, B are k-regular (same size) and $C \in \{-1, 0\}$ is l-regular (l directed couplings per vertex across the cut), then

$$|+\rangle_L = \tfrac{1}{\sqrt{2}} \begin{pmatrix} V_A \\ V_B \end{pmatrix} \quad \text{and} \quad |-\rangle_L = \tfrac{1}{\sqrt{2}} \begin{pmatrix} V_A \\ -V_B \end{pmatrix}$$

are eigenvectors of $R = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}$ with eigenvalues $k - l$ and $k + l$ (ordering depends on sign convention).

Theorem: Tuning arbitrary states

State synthesis reduces to tuning degree ratios. Two mechanisms realize any real-amplitude encoded state $|\psi\rangle_L = (\omega_1 V_A; \omega_2 V_B)$: (i) symmetric degree detuning and (ii) directed/asymmetric coupling. Because degrees are discrete and the base formulas have "blow-up" loci, we use a switching rule in each mechanism by also allowing the inverse parameterization.

Theorem 3.1: Mechanism A – Symmetric detuning + switching
With symmetric coupling $C_A = C_B^T = C$ and detuned subgraph degrees $k_A \neq k_B$, define

$$\Delta \coloneqq \frac{k_A - k_B}{2l}.$$

For target $|\psi\rangle_L = \omega_1 |0\rangle_L + \omega_2 |1\rangle_L$ ($\omega_1^2 + \omega_2^2 = 1$, real),

$$\Delta = \frac{\omega_2^2 - \omega_1^2}{2\omega_1\omega_2}$$

makes $|\psi\rangle_L$ an eigenvector.

Switching (symmetric): where the above ratio becomes large in magnitude, use the inverse parameter Δ^{-1}.

Lemma 4.1: Mechanism B – Directed coupling + switching
Allow directed regular couplings with degrees l_A, l_B:

$$R = \begin{pmatrix} A & C_A \\ C_B & B \end{pmatrix}, \qquad C_A V_B = -l_A V_A, \qquad C_B V_A = -l_B V_B,$$

and set $k_A = k_B = k$. Define

$$\Delta_C \coloneqq l_A / l_B, \text{then } \Delta_C = \omega_1^2 / \omega_2^2$$

makes $|\psi\rangle_L$ an eigenvector.

Switching (directed): Similarly to the symmetric case, use Δ_C^{-1} when Δ_C becomes unbounded.

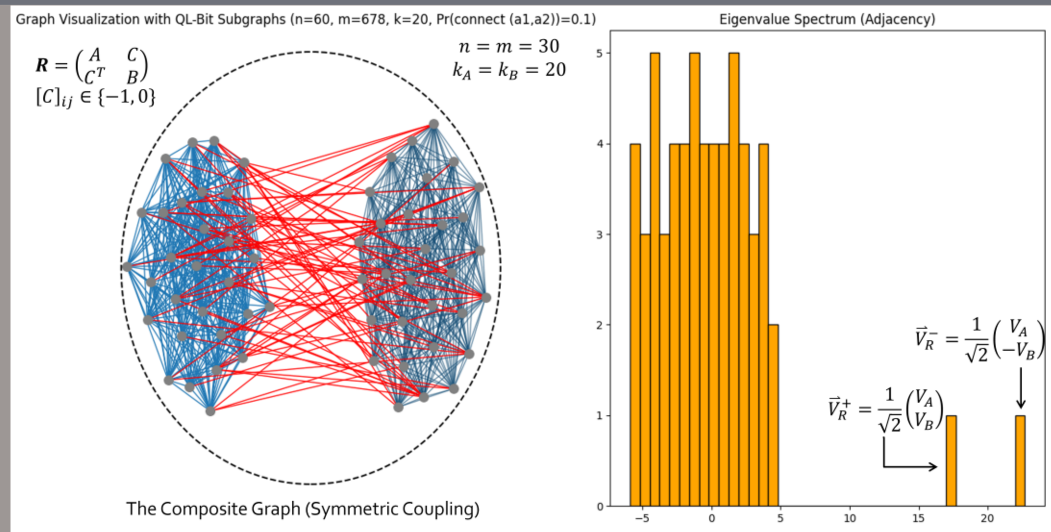

Fig. 1. Example composite QL-bit graph formed by two k-regular subgraphs coupled by an ℓ-regular bipartite connector. The adjacency spectrum shows two emergent eigenvalues near $k \pm \ell$, corresponding to the encoded Hadamard-like eigenmodes. Gap persists under degree scaling / perturbations.

Discrete Feasibility: Integer Degrees ⇒ Rational Approx.

In the signed/unweighted model $[C]_{ij} \in \{-1, 0\}$, regularities are integers, so the tunable parameters live in the rationals. This is the source of a clean "accuracy scaling" story: increasing graph size (or denominators) increases how finely one can approximate a desired amplitude ratio.

Symmetric coupling constraints (simple graphs, $2n$ vertices; n per side):

$$k_A, k_B, l \in \mathbb{Z}, \qquad k_A, k_B, l \neq 0, \qquad |k_A - k_B| < n, \qquad |l| < n.$$

Hence,

$$\Delta = \frac{k_A - k_B}{2l} \in \mathbb{Q}, \qquad |\Delta| < n \text{ (maximal setting).}$$

Directed coupling constraints:

$$l_A, l_B, k \in \mathbb{Z}, \qquad l_A, l_B, k \neq 0, \qquad |l_A - l_B| < n.$$

Hence,

$$\Delta_C = l_A / l_B \in \mathbb{Q}, \qquad |\Delta_C| < n \text{ (maximal setting).}$$

Unified Design Equation and Random-Walk Lens

A single constraint unifies the symmetric-detuning and directed-coupling constructions. Allow all four regularities to vary (k_A, k_B, l_A, l_B) in $R = \begin{pmatrix} A & C_A \\ C_B & B \end{pmatrix}$, and require an encoded eigenvector $|\psi\rangle_L = (\omega_1 V_A;\ \omega_2 V_B)$. Then the regularities must satisfy (Appendix C)

$$\omega_2^2 l_A - \omega_1^2 l_B + \omega_1\omega_2(k_B - k_A) = 0$$

Specializing recovers both mechanisms:

- $l_A = l_B$ gives the symmetric detuning relation (Theorem 3.1).
- $k_A = k_B$ gives the directed ratio $\Delta_C = l_A / l_B = \omega_1^2 / \omega_2^2$ (Lemma 4.1)

Random-walk interpretation (Sec. 5): on the undirected composite graph, the stationary distribution is degree-proportional. Under symmetric detuning,

$$\pi_i = \frac{k_A + l}{n(k_A + k_B + 2l)} (i \in A), \qquad \pi_i = \frac{k_B + l}{n(k_A + k_B + 2l)} (i \in B),$$

while the encoded QL amplitudes depend only on the ratio parameter (Δ or Δ_C), leaving absolute scaling available to tune secondary criteria (e.g., separation/robustness) without changing $|\psi\rangle_L$.

Practical Considerations and Extensions

With signed unweighted edges $[C]_{ij} \in \{-1, 0\}$, the regularities are integers, hence Δ and Δ_C are rational; larger graphs/denominators give finer amplitude resolution, and switching to Δ^{-1} or Δ_C^{-1} avoids singular regimes. Keeping ratios fixed while scaling degrees tunes spectral separation. Next: complex discrete weights (e.g., $\mu_4 = \{\pm 1, \pm i\}$) for phases and explicit gate/multi-bit constructions.

References

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