NETWORKS and MATRIX COMPUTATIONS

CS 59000-NMC
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T-Th 10:30-11:45
CIVL 2123

Course Teaser
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the social network

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We like books …
NETWORKS and MATRIX COMPUTATIONS

Why looking at networks of data as a matrix is a powerful and successful paradigm.
the social network
Citation network

<Insert your favorite network here>
TOPICS
How to state network problems as matrix problems. Matrices over a semi-ring and how this yields instant parallel algorithms!

Relationships with MapReduce computations.
Adjacency matrix
$A : n \times n, A = A^T$
$A_{i,j} = 1$ if $(i, j) \in E$
$-d_{\text{max}} \leq \lambda(A) \leq d_{\text{max}}$

Laplacian matrix
$D = \text{diag}(Ae)$
$L = D - A$
$0 \leq \lambda(L) \leq 2d_{\text{max}}$

Normalized Laplacian matrix
$\tilde{L} = D^{-1/2}LD^{-1/2} = I - D^{-1/2}AD^{-1/2}$
$0 \leq \lambda(\tilde{L}) \leq 2$

Random walk matrix
$P = D^{-1}A$

Modularity matrix
$d = Ae$
$M = A - 1/(2|E|)dd^T$

Signless Laplacian matrix

Incidence matrix
(It is incidentally discussed)

Seidel matrix

Heat Kernel

Everything is undirected. Mostly connected components only too.
TROPICAL semi-rings

http://www.ultimatebeachviews.com/wallpaper/Tropical-010/
MARKOV CHAIN theory

Random walks on networks
Perron Frobenius theory
State space classification

How to “solve” a large linear system with a random walk.

How to “solve” Markov chain problems on compressed graphs
Deep dive into one particular application. Is it a Markov chain or a linear system?

How to compute it, FAST.

How to manipulate PageRank.
Just PageRank?
SimRank
BlockRank
TrustRank
ObjectRank
HostRank
Random walk with restart
GeneRank
DiffusionRank
IsoRank
ItemRank
ProteinRank
SocialPageRank
FoodRank
FutureRank
TwitterRank
PageRank formulations and theory

Graph or Web graph → Substochastic matrix → Strongly preferential PageRank → PseudoRank

Codes → Theory

Weakly preferential PageRank → Eigensystems

Sink preferential PageRank → Linear systems

Other transformations

\( \mathbf{v} \) teleportation vector
\( \tilde{\mathbf{P}} \) substochastic matrix (for algorithms)
\( \mathbf{d} \) dangling node vector \( (\mathbf{d} = \mathbf{e} - \mathbf{P}^T \mathbf{e}) \)

\( \tilde{\mathbf{P}} + \mathbf{vd}^T \rightarrow \mathbf{P} \) Strongly preferential PageRank
\( \tilde{\mathbf{P}} + \mathbf{ud}^T \rightarrow \mathbf{P} \) Weakly preferential PageRank \((\mathbf{u} \neq \mathbf{v})\)

\( \mathbf{P} \) PageRank stochastic matrix (for theory)

\( (\mathbf{I} - \alpha \mathbf{P}) \mathbf{x} = (1 - \alpha) \mathbf{v} \) PageRank linear system
SPECTRAL GRAPH theory

The Fiedler vector, the Laplacian matrix and graph cuts

Semi-definite approximation problems and properties

Local partitioning
How to split a graph without even seeing it all!
Higher order graph analysis with tensors.

Network alignment

Affine eigenvectors and centrality

SVD graph analysis.

Matrix based graph models.
NETWORK ALIGNMENT

maximize $\alpha w^T x + \frac{\beta}{2} x^T S x$

subject to $Ax \leq e, x_i \in \{0, 1\}$

History

- Maximum Common Subgraph
- Pattern Recognition
- Ontology Matching
- Similarity Flooding
- Graph Similarity
- Bioinformatics

Sparse problems

Sparse $\mathcal{L}$ largely ignored (exception Klau)

Our paper tackles that case explicitly
<Insert your favorite application/topic/area here>
THE WORK
A project.
A proposal
A paper
A presentation
A lecture.

Take a paper and present it to the class using the matrix paradigm. OR

Present a paper that uses the matrix paradigm.
OTHER WORK

Survey
Homework (2-4?)
Quizzes
Writing
Quizzes?

Taken/Missed
These are for me, not you.
QUESTIONS?

www.cs.purdue.edu/homes/dgleich/nmcomp