NETWORKS *and* MATRIX
COMPUTATIONS

CS 59000-NMC
David F. Gleich
T-Th 10:30-11:45
CIVL 2123

Course Teaser
17 August 2011

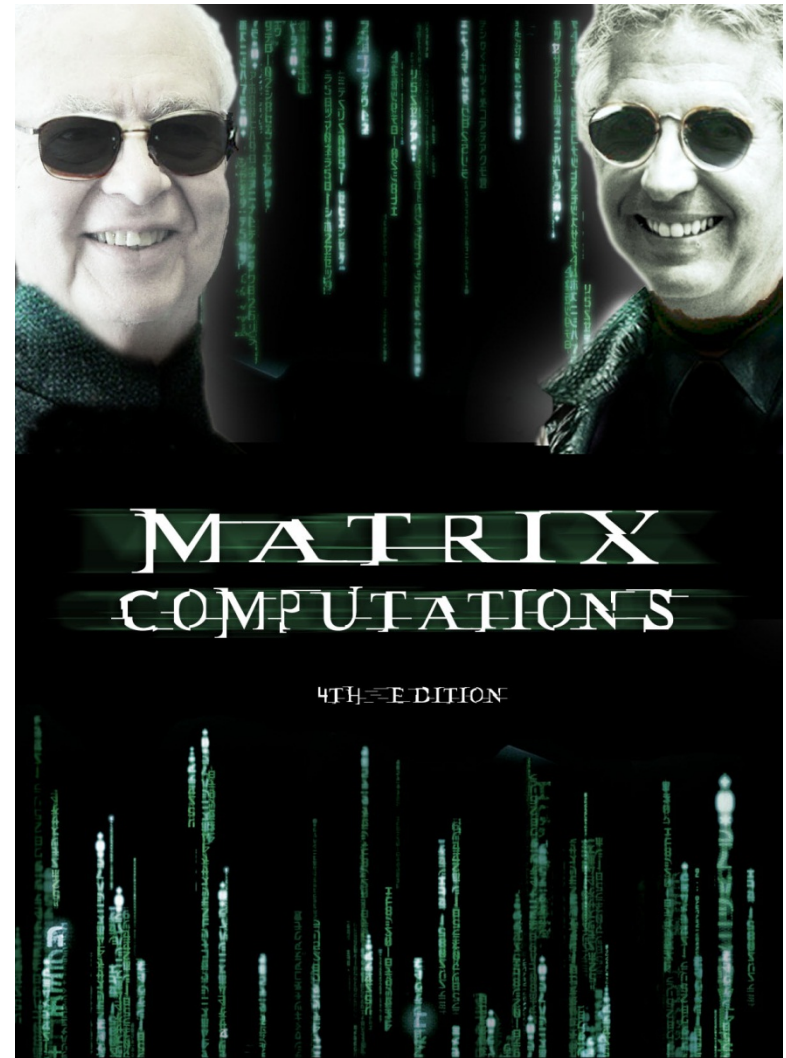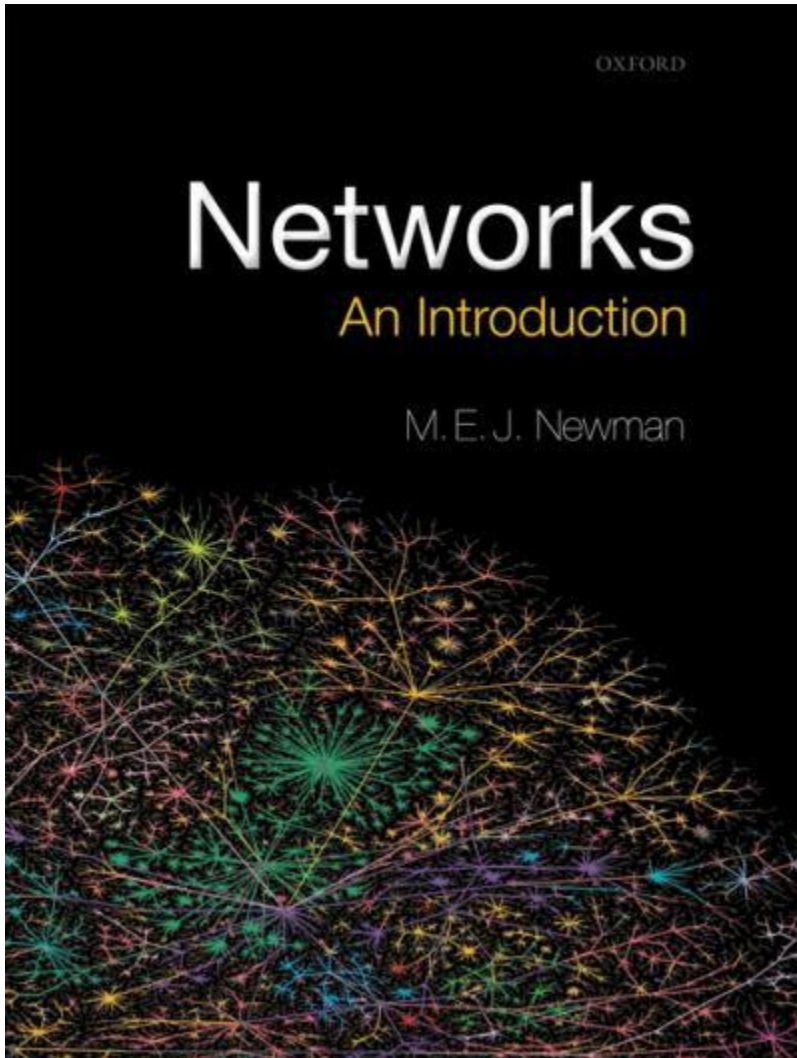the social network

KEANU REEVES   LAURENCE FISHBURNE

MATRIX

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We like books …

Gene H. Golub Charles van Loan

NETWORKS *and* MATRIX COMPUTATIONS

*Why looking at **networks** of data as a **matrix** is a **powerful** and **successful** paradigm.*

Web graph

the social network

Citation network

time ← Oct, 2007 Mar, 2007 Aug, 2006 Jan, 2006

<Insert your favorite network here>
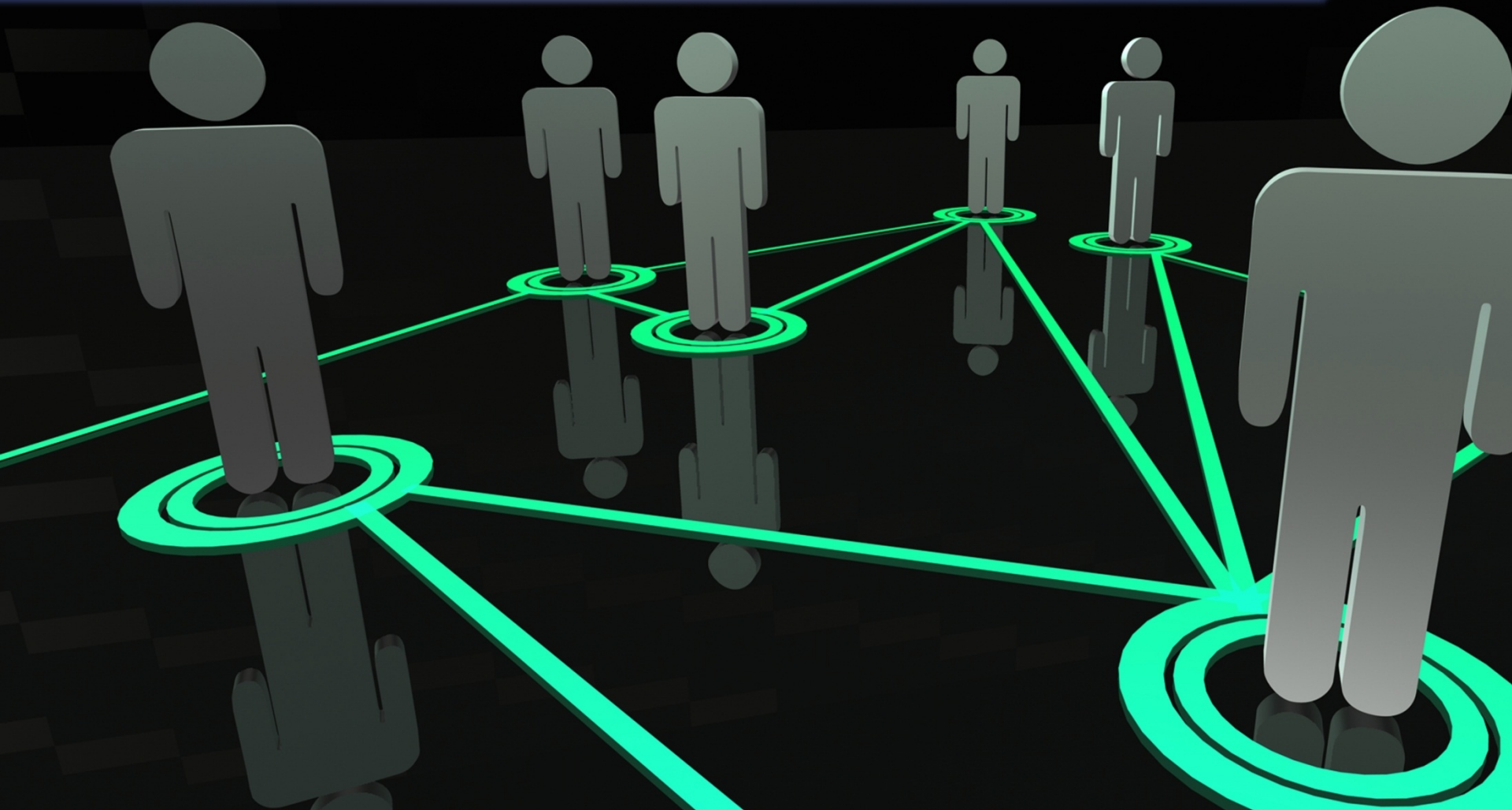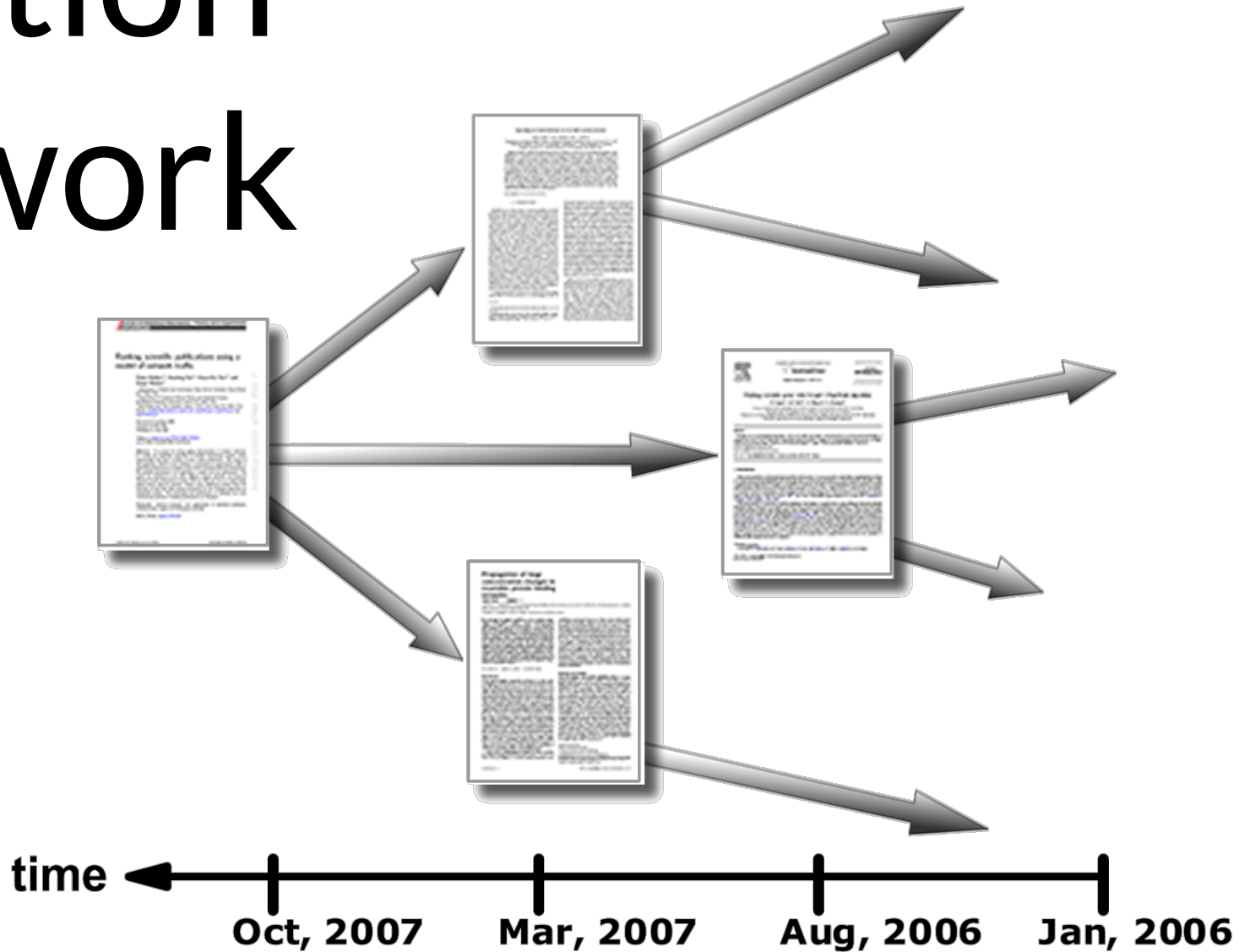
TOPICS

BASICS

How to state network problems as matrix problems.
   Matrices over a semi-ring and how this yields instant parallel algorithms!

Relationships with MapReduce computations.

Adjacency matrix

$$\boldsymbol{A} : n \times n, \boldsymbol{A} = \boldsymbol{A}^T$$

$$A_{i,j} = 1 \text{ if } (i,j) \in E$$

$$-d_{\max} \leq \lambda(\boldsymbol{A}) \leq d_{\max}$$

Laplacian matrix

$$\boldsymbol{D} = \text{diag}(\boldsymbol{A}\boldsymbol{e})$$

$$\boldsymbol{L} = \boldsymbol{D} - \boldsymbol{A}$$

$$0 \leq \lambda(\boldsymbol{L}) \leq 2d_{\max}$$

Normalized Laplacian matrix

$$\tilde{\boldsymbol{L}} = \boldsymbol{D}^{-1/2}\boldsymbol{L}\boldsymbol{D}^{-1/2} = \boldsymbol{I} - \boldsymbol{D}^{-1/2}\boldsymbol{A}\boldsymbol{D}^{-1/2}$$

$$0 \leq \lambda(\tilde{\boldsymbol{L}}) \leq 2$$

Random walk matrix
$$\boldsymbol{P} = \boldsymbol{D}^{-1}\boldsymbol{A}$$

Modularity matrix
$$\boldsymbol{d} = \boldsymbol{A}\boldsymbol{e}$$
$$\boldsymbol{M} = \boldsymbol{A} - 1/(2|E|)\boldsymbol{d}\boldsymbol{d}^T$$

Signless Laplacian matrix

Incidence matrix
 (It is incidentally discussed)

Seidel matrix

Heat Kernel

Everything is undirected.   Mostly connected components only too.

TROPICAL semi-rings

MARKOV CHAIN theory

Random walks on networks
Perron Frobenius theory
State space classification

How to "solve" a large linear
    system with a random walk.

How to "solve" Markov chain
    problems on compressed
    graphs

PAGERANK

Deep dive into one particular application. Is it a Markov chain or a linear system?
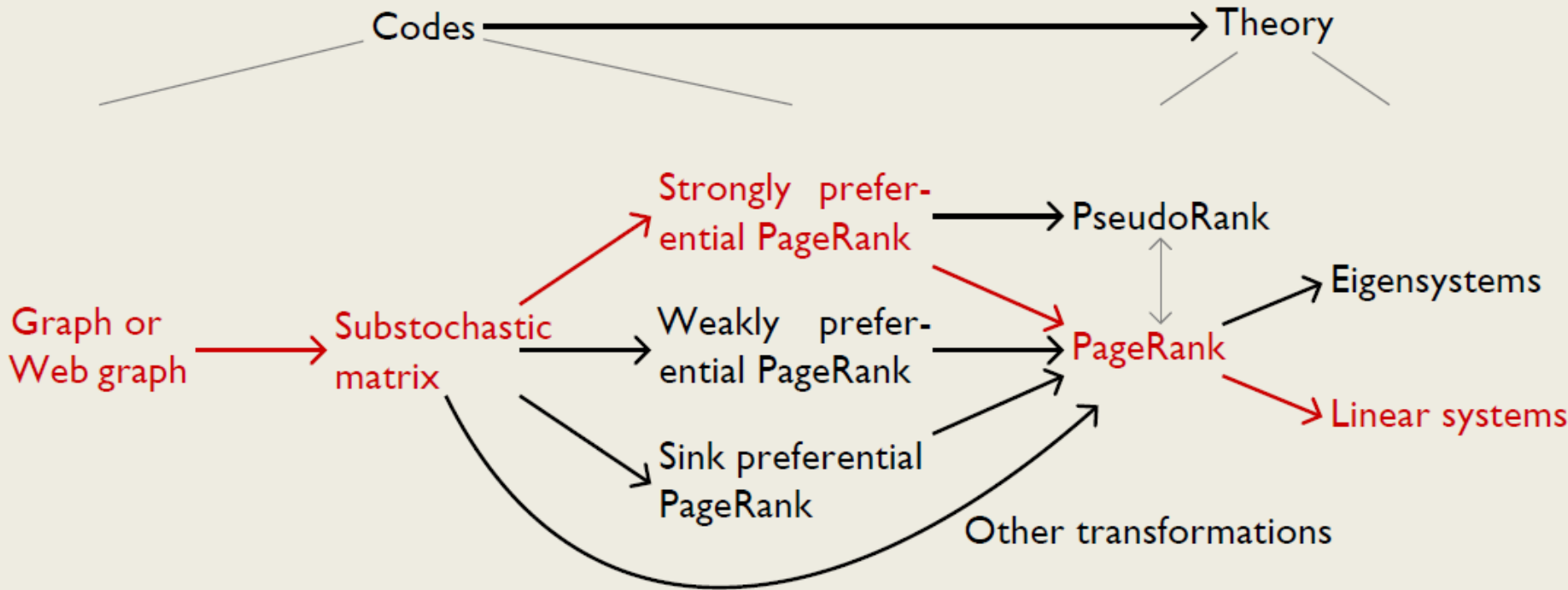
How to compute it, FAST.

How to manipulate PageRank.

Just PageRank?

SimRank
BlockRank
TrustRank
ObjectRank
HostRank
Random walk with restart
GeneRank
DiffusionRank
IsoRank
ItemRank
ProteinRank
SocialPageRank
FoodRank
FutureRank
TwitterRank

PageRank formulations and theory

Codes ─────────────────────────────→ Theory

Strongly prefer-
ential PageRank ──────→ PseudoRank

Graph or
Web graph ──→ Substochastic
matrix ──→ Weakly prefer-
ential PageRank ──→ PageRank ──→ Eigensystems

Sink preferential
PageRank

Other transformations ──→ Linear systems

$\mathbf{v}$ teleportation vector

$\bar{\mathbf{P}}$ substochastic matrix (for algorithms)

$\mathbf{d}$ dangling node vector ($\mathbf{d} = \mathbf{e} - \mathbf{P}^T\mathbf{e}$)

$\bar{\mathbf{P}} + \mathbf{v}\mathbf{d}^T \to \mathbf{P}$ Strongly preferential PageRank

$\bar{\mathbf{P}} + \mathbf{u}\mathbf{d}^T \to \mathbf{P}$ Weakly preferential PageRank ($\mathbf{u} \neq \mathbf{v}$)

$\mathbf{P}$ PageRank stochastic matrix (for theory)

$(\mathbf{I} - \alpha\mathbf{P})\mathbf{x} = (1 - \alpha)\mathbf{v}$ PageRank linear system

SPECTRAL GRAPH theory

The Fiedler vector, the
Laplacian matrix and
graph cuts

Semi-definite approximation
problems and properties

Local partitioning
*How to split a graph without
even seeing it all!*

QUITE A BIT MORE

Higher order graph analysis with tensors.

Network alignment

Affine eigenvectors and centrality

SVD graph analysis.

Matrix based graph models.

NETWORK ALIGNMENT

maximize $\quad \alpha \mathbf{w}^T \mathbf{x} + \frac{\beta}{2} \mathbf{x}^T \mathbf{S} \mathbf{x}$

subject to $\quad \mathbf{A} \mathbf{x} \leq \mathbf{e}, x_i \in \{0, 1\}$

History

- MAXIMUM COMMON SUBGRAPH
- PATTERN RECOGNITION
- ONTOLOGY MATCHING
- SIMILARITY FLOODING
- GRAPH SIMILARITY
- BIOINFORMATICS

Sparse problems

Sparse $\mathcal{L}$ largely ignored (exception Klau)
Our paper tackles that case explicitly

<Insert your favorite application/topic/area here>

THE WORK

BIG ITEM one

A project.
 A proposal
 A paper
 A presentation

BIG ITEM two

A lecture.
Take a paper and present it to the class using the matrix paradigm. OR
Present a paper that uses the matrix paradigm.

OTHER WORK

Survey
Homework (2-4?)
Quizzes
Writing

Quizzes?

Taken/Missed
These are for me, not you.

QUESTIONS?

www.cs.purdue.edu/homes/dgleich/nmcomp