

CS 59000-NMC, 30 August 2011

Your name:

Please answer the following questions. You may not use any outside references or technology. Justify and explain all answers. This quiz is for my own evaluation, so that I can provide better instruction in the course.

Question

Consider the vector 1-norm. Show that

$$\|\mathbf{x}\|_\infty \leq \|\mathbf{x}\|_1 \leq n\|\mathbf{x}\|_\infty.$$

When is the inequality an equality?

Solution

$$\|\mathbf{x}\|_\infty = \max_i |x_i| \leq \sum_i |x_i| = \|\mathbf{x}\|_1 \leq \sum_i |x_{\max}| \leq n\|\mathbf{x}\|_\infty$$

The first two are equal when $\max_i |x_i| = \sum_i |x_i|$. This means that \mathbf{x} can have only a single non-zero component, otherwise the sum will always be greater.

The second two are equal when $\sum_i |x_i| = n|x_{\max}|$. Put another way, this means that the average magnitude must be equal to the maximum magnitude. This will only happen when $|x_{\max}| = |x_i|$ for all i . So the vector must have elements with equal magnitude, but possibly different signs. Over \mathbb{C} , we can change any element by a complex rotation $e^{i\theta}$, which does not alter the magnitude.