UNCONSTRAINED MINIMIZATION IN 1D

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Everything will be 1d or univariate until further notice. Let $f(x) : \mathbb{R} \to \mathbb{R}$, for instance.

The material here is from Chapter 2 in Nocedal and Wright.

1 A BASKET OF DEFINITIONS

DEFINITION 1 (global minimizer) *A point x^* is a global minimizer if $f(x^*) \leq f(x)$ for all x in the domain of f.*

DEFINITION 2 (strict global minimizer) *A point x^* is a strict global minimizer if $f(x^*) < f(x)$ for all x in the domain of f except for x^**

DEFINITION 3 (local minimizer) *A point x^* is a local minimizer if $f(x^*) \leq f(x)$ for all x in an open neighborhood of x^*.*

DEFINITION 4 (strict local minimizer) *A point x^* is a strict local minimizer if $f(x^*) < f(x)$ for all x in an open neighborhood of x^* except for x^*.*

DEFINITION 5 (isolated minimizer) *A point x^* is an isolated (local) minimum if its a local minimizer and there is a neighborhood of x^* where x is the only minimizer.*

1.1 EQUIVALENCE?

Let $f(x)$ be continuously differentiable. Can a strict minimizer be non-isolated?

Isolated implies Strict

Strict implies Isolated?

2 RECOGNIZING MINIMIZERS

The key idea *Taylor's theorem ...*

2.1 TAYLOR'S THEOREM (UNIVARIATE)

Let $f(x)$ be twice continuously differentiable.

$f(x_0 + h) =$

$f(x_0 + h) =$

where α

2.2 NECESSARY CONDITIONS FOR A LOCAL MINIMIZER

First order

Second order

Conclusion So any local minimizer has:

2.3 SUFFICIENT CONDITIONS FOR A LOCAL MINIMIZER

Necessary vs. Sufficient

Necessary can be used to

Sufficient can be used to

First order

To think about ... What do these conditions say about $f(x) = x^4$?