Computational methods in optimization

David F. Gleich

Purdue University
CS 520
Course objectives

To understand why optimization is hard
To understand when we can optimize
To understand how to optimize

To be able to optimize a function
To understand optimization software
What is optimization?

- (Cheeky) I hope you know, you decided to take this class!
- Mathematical programming
- Operations research
What is optimization?

My attempt at a definition –

Optimization is the umbrella term for the

theory / software / methodology / models / process

of finding the

extreme / improved / better points

of a mathematical function representing a useful quantity
while satisfying constraints on the feasible inputs to that function
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Syllabus
Course outline
Background

Software
Least Squares
Prerequisites

Calculus
Linear algebra (QR, SVD, Eigenvalues)
1d quadratic functions

Numerical linear algebra (CS515):
- Cholesky / iterative methods / etc.
Unconstrained optimization

- Non-linear equations
- Newton methods
- Line search
- Trust region
- Quasi-newton

minimize $f(x)$
Constrained optimization

• Linear programming
• Quadratic programming
• Large-scale

minimize \( f(x) \)
subject to \( l \leq \begin{bmatrix} x \\ Ax \\ c(x) \end{bmatrix} \leq u \)
Modern topics

- Convex
- Integer
- Stochastic
Questions about topics?
Your first quiz!
2. You are at the center of a 20m equilateral triangle with a raptor at each corner. The top raptor has a wounded leg and is limited to a top speed of 10 m/s.

(Not to scale)

The raptors will run toward you. At what angle should you run to maximize the time you stay alive?
solution in Julia
Who cares?
The new model

choose direction to run $v_p[j]$ for $j = \{1, \ldots, N\}$

to minimize “likelihood” of being eaten

subject to raptor motion

$$r_i[j + 1] = r_i[j] + hv_i \frac{p[j] - r_i[j]}{\|p[j] - r_i[j]\|}$$

human motion


Thanks to Nick Henderson for many slides.
How it’s done

model → modeling environment (AMPL) → solver (SNOPT)

web service (NEOS) → direct (Matlab, C, Fortran)

Thanks to Nick Henderson for many slides.

Monday, January 13, 14
time = 0.01 sec
time = 2.65 sec
Aerospace Applications of NPSOL and SNOPT

OTIS #1
What are you applications?
Course logistics!
Numerical computing software
Why I’ve moved everything to Julia

• Free, free, free! (Run it anywhere.)
• Close enough to Matlab
  • A(5,6) vs. A[5,6]
  • [V,D] = eig(A) vs. V,d = eigen(A)
  • “my string” vs. ‘my string’
• For-loops are “more efficient” (used a lot here!)
• Many helpful tools for numerical analysis
  • BigFloat has enhanced precision
• Great optimization tools in Julia! (the de-facto standard now?)
  • World leading automatic differentiation!
What I’m worried about

• Julia is still under active development
• There are many rough edges
• Graphics / plotting are still very (2017) rough (2020) not always reliable (2023)
• Time-to-first-plot/function/etc is slow... 😞
  • Should be much better in the next 1-2 years
How to use Julia

On your own computer

• VSCode – what I use for normal development
  • (but our demos don’t work because I used a bunch of interactive stuff ... sigh)
• Pluto (what I would redo scripts with now...)
• Jupyter notebook (Julia-Python-R)
  • What you will see in class!

• Julia command line (not recommended)
  • or interact
• Atom/Juno