Recall the standard form for a linear program:

\[
\begin{align*}
\text{minimize} & \quad c^T x \\
\text{subject to} & \quad Ax = b \\
& \quad x \geq 0.
\end{align*}
\]

We'll revisit the dual later. I won't have enough time to talk about it in this lecture. But for reference, my favorite way of defining the dual is via the "simple LP"

\[
\begin{align*}
\text{minimize} & \quad c^T x \\
\text{subject to} & \quad Ax \leq b.
\end{align*}
\]

It's Lagrangian is:

\[
L(x, \lambda) = c^T x - \lambda^T (b - Ax),
\]

which can be “tranposed” to yield:

\[
L(x, \lambda) = x^T c + x^T A^T \lambda - b^T \lambda = (-b)^T \lambda - x^T (A^T \lambda - c)
\]

which is the Lagrangian of

\[
\begin{align*}
\text{minimize} & \quad -b^T \lambda \\
\text{subject to} & \quad c \leq A^T \lambda, \lambda \geq 0
\end{align*}
\]

where \(x\) are now the Lagrange multipliers.

The Lagrange multipliers \(\lambda\) are often called dual variables for this reason.

**2 KKT CONDITIONS ARE NECESSARY AND SUFFICIENT**

Let \(\lambda\) be the Lagrange multipliers for the equality constraints and \(s\) be the multipliers for the inequality constraints. Then the KKT conditions for the primal LP are:

\[
\begin{align*}
A^T \lambda + s &= c \\
Ax &= b \\
x &\geq 0 \\
s &\geq 0 \\
x^T s &= 0.
\end{align*}
\]

For the rest of the course, you might want to commit these conditions to memory! They’ll be very important.

In general, the KKT conditions are only the necessary conditions for a local minimum. However, for an LP, we'll show that they are also sufficient. In other words, any point \(x\) that satisfies these conditions is a solution, that is, a local minimizer and a global minimizer.

First, note that for any solution \((x^*, \lambda^*, s^*)\) that satisfies the KKT conditions, we have that

\[
c^T x^* = b^T \lambda^*.
\]

**Quiz** Show this using the KKT conditions!
Now, consider any other feasible point $f$ where $Af = b$, $f \geq 0$. We can show that $c^T f \geq c^T x^*$ directly:

$$c^T f = (A^T \lambda^* + s^*)^T f = b^T \lambda^* + f^T s^* \geq b^T \lambda^* = c^T x^*$$

because $f \geq 0$ and $s^* \geq 0$. 
