Computational methods in analysis optimization

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CS520

Thanks to Nick Henderson for many slides.
Course objectives

To understand why optimization is hard
To understand when we can optimize
To understand how to optimize

To be able to optimize a function
To understand optimization software
Course outline
Background

Software

Least Squares

Matrix calculus
Unconstrained Optimization

Non-linear equations
Newton methods
Line search
Trust region
Quasi-newton

\[
\text{minimize } f(x)
\]
Constrained Optimization

Linear programming

Quadratic programming

Large-scale

minimize \( f(x) \)

subject to \( l \leq \begin{bmatrix} x \\ Ax \\ c(x) \end{bmatrix} \leq u \)
Modern Topics

Convex

Integer

Stochastic
Questions about topics?
Your first quiz
Miss Lenhart couldn't be here today, so she asked me to substitute.

Mr. Munroe: I've put out your tests. Please get started.

Mr. Munroe, Miss Lenhart never taught us this.

That's because Miss Lenhart doesn't understand how important certain kinds of math are.

But this just looks--

This material is more vital than anything you've ever learned.

But--

No buts.

Source:  http://xkcd.com/135/
2. You are at the center of a 20m equilateral triangle with a raptor at each corner. The top raptor has a wounded leg and is limited to a top speed of 10 m/s.

(Not to scale)

The raptors will run toward you. At what angle should you run to maximize the time you stay alive?

Raptors move at 25 m/s
You move at 6 m/s
But who cares?
The new model

choose direction to run

to minimize “likelihood” of being eaten

subject to raptor motion

human motion

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The new model

choose direction to run $v_p[j]$ for $j = \{1, \ldots, N\}$

to minimize “likelihood” of being eaten

subject to raptor motion

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$\sum_{j=1}^{N} \sum_{i=1}^{3} \frac{1}{||p[j] - r_i[j]||^2} dt$

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Monday, January 13, 14
The new model

choose direction to run \( v_p[j] \) for \( j = \{1, \ldots, N\} \)

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The new model

choose direction to run $v_p[j]$ for $j = \{1, \ldots, N\}$

to minimize "likelihood" of being eaten

$$\sum_{j=1}^{N} \sum_{i=1}^{3} \frac{1}{\|p[j] - r_i[j]\|^2} dt$$

subject to raptor motion

$$r_i[j+1] = r_i[j] + hv_i \frac{p[j] - r_i[j]}{\|p[j] - r_i[j]\|}$$

human motion


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How it's done

model → modeling environment (AMPL) → solver (SNOPT)

web service (NEOS) → direct (Matlab, C, Fortran)

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Solve!
time = 0.01 sec
time = 2.65 sec
Aerospace Applications of NPSOL and SNOPT

OTIS #1
What are your applications?