Lecture 7

HW 2 posted soon.

Why is
$$\max_{x \neq 0} \frac{f(Ax)}{f(x)} + \frac{f(Bx)}{f(x)}$$
$$\le \max_{x,y} \frac{f(Ax)}{f(x)} + \frac{f(Ay)}{f(y)}$$

Even if $A \succ 0$, will
$x_{k+1} = (I - \alpha A)x_k + \alpha b$ N
will converge to $x^* = A^{-1}b$?

$|1 - \alpha\lambda| < 1$ No, eigenvalue

If that then yes, I think it I've understood correctly.

How do we pick α in Richardson?

Why are
$\|AB\|_{mat} \le \|A\|_{mat} \cdot \|B\|_{mat}$
and $\|Ax\|_{vec} \le \|A\|_{mat} \|x\|_{vec}$
Both submultiplicative?

- Do we need to read + write Julia on the exam?
- Will this class help us find an AI related job?
- Why do we change the algorithm when solving diff. systems?

- Which is more parallel friendly? CSC or CSR?
- How are these distributed across a machine?

How to choose step in gradient descent automatically?

How do we check convexity?

Typo: $\det(A) \neq 0$ not $\det(A) = 1$

Solve non-convex via gradient descent?

- Time complexity for $Ax = b$?

- If $Ax = b$ has infinite soln, can we use iterative methods to find one?

Submult. property is "weird"

$f_n(x) = \|x\|$ when x is an $\in \mathbb{R}^n$

$f_m(x) = \|x\|$ when x is in $\mathbb{R}^m$.

$f_{m \times n}(AB) \le f_{m \times k}(A) \cdot f_{k \times n}(B)$

$x_i 2 = 0$
for $i = 1 : n$
$x_i 2 += X[i] * X[i]$
~~end~~

What does this compute?

Which is more parallel friendly CSR? CSC?

How to pick α in Richardson?

if $A \succ 0$,

then $\alpha^* = \frac{2}{\lambda_1 + \lambda_n}$ (I think).

Extreme Eigenvalues.

$\lambda_1 \ge 0$

$\lambda_n \le \|A\|$ for any norm.

Use $\alpha = \frac{2}{\min(\|A\|_\infty, \|A\|_1)}$ as a first guess.

$f(x) = \frac{1}{2} x^T A x - x^T b$

$\min f(x) \iff$ find $Ax = b$

when A is sym. pos. def.

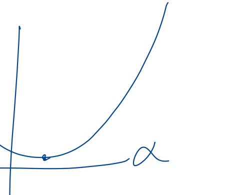
At each step of Richardson

$x_{k+1} = x_k + \alpha r_k$

1 variable function in α.

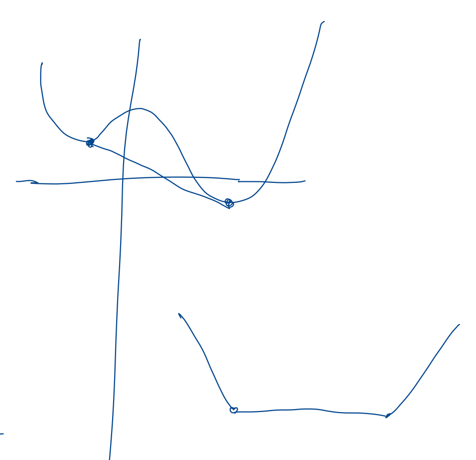
$f(\alpha) = \frac{1}{2}(x_k + \alpha r_k)^T A (x_k + \alpha r_k) - (x_k + \alpha r_k)^T b.$

$\to \min_\alpha f(\alpha)$



$f'(\alpha) = 0$

$f(\alpha) = a\alpha^2 + b\alpha + c$



$A \succ 0, A = A^T$

$f(x) = \frac{1}{2} x^T A x - x^T b$

has a unique soln!

$H(x) = A$.

If $A \succeq 0, A = A^T$

Then we have and $A \not\succ 0$, then

There exists z s.t.

$Az = 0$. $(z \neq 0.)$

$Ax = b$

$A(x + z) = Ax + Az$
$= b + 0$.

$\min \frac{1}{2} x^T A x - x^T b$

s.t. $\|x\|$ is as small as possible.

MINRES will solve this! (one little trick).

$x^+ = A^+ b$

pseudo-inverse has this property!