Lecture 6

HW 1 due Monday!

TA office hrs TODAY!

Mine tomorrow!

1. Operator Norms → we will!
2. What norm to use? ||
3. $p \in [0,1]$ Norms? What fails? Where used?

Why not $\|\cdot\|$ for Condition vs. $\|\cdot\|$?
$\|\cdot\| = \|\cdot\|$ is definition!

What method is Using in Julia?
↳ Use factorize. I think.

Do we Even Compute Error as?

$\|\text{error}\| \le \|A^{-1}\| \cdot \|b - Ax\|$?
↳ No! We use it as a bound!

When is a matrix good?
Spectral norm?
Cond. #?

Can we get O run time for Neumann Series?
Yes, but unless A has struct, it isn't very interesting.

Why is Sym. pos. def. useful?
due to non-normal for $A \ne A^T$.

Sparse matrix formal defn? → See Lecture 5.

What if $\rho(A) = 1$ but $|\lambda_i(A)| < 1$ for all but 1.

Matrix Norm that is not Sub-mult?

What Else is $\rho(A)$ used for?

Transition Matrix: When to use it as transpose?

When to use T vs. T^T for a transition matrix?

- It depends on what you want to compute + how you are compute it.

$A = A^T$, $A \succ 0$

$A \Leftrightarrow$ Sym. pos. def.

$x^T A x > 0$

So Sym. pos. def. "generalizes" positive in the sense of a quadratic form.

If $A \ne A^T$ and we look at

$x^T A x \Leftrightarrow$

$\frac{1}{2} x^T (A + A^T) x$

Why?

$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

Operator Induced Norms

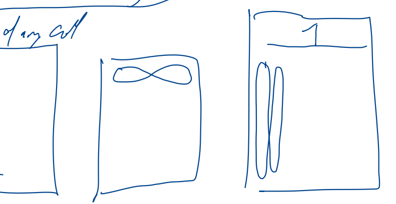
$\|A\| = \max \|Ax\|$ s.t. $\|x\| = 1$

for Some vector norm $\|\cdot\|$.

$\|A\|_2$ for $\|x\|_2$
$= \sigma_{max}$ (largest sing. value)

$\|A\|_1 = \max \|\cdot\|_1$ of any col

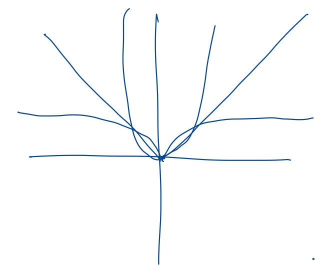
$\|A\|_\infty = \max \|\cdot\|_1$ of any row.



$p \in [0,1)$ Norms

$p = 0$ Norm $\Leftrightarrow$

# of Entries of a vector.



Non sub-mult. Matrix norm.

$\|A\|_\Delta = \max_{ij} |A_{ij}|$

$= \|\text{vec}(A)\|_\infty$

$= \|A\|_{F,\infty}$.

$\|A\|_\infty = \uparrow$ these in ML.

$\|A\|_\infty$ = Op induced ∞ norm.

= max $\|\cdot\|_1$ over rows.

THIS CLASS

Opnorm in Julia

Not sub-mult!

When is A "good" if we want algs like

$\sum_{k=0}^{\infty} A^k$ or $\Rightarrow$ $\boxed{\text{we need } \rho(A) < 1}$

$\lim_{k \to \infty} A^k$ $\Rightarrow$ More interesting λ_1 vs. λ_2 gap. i.e. $\frac{\lambda_1}{\lambda_2}$

for these, we want

$\kappa(A) = \|A\| \cdot \|A^{-1}\|$

We'll do this in a bit.

Thm: $\lim_{K \to \infty} \sum_{k=0}^{K} A^k$ Converges iff $\rho(A) < 1$.

This thm does not imply $\left(\sum A^k\right) \underline{v}$ to Compute $(I - A)^{-1} v$.

works for some $\underline{v}$.

$A \Rightarrow$ Eigenvectors

$[\underline{v}_1 \quad V_R]$ ← $n \times n-1$ matrix

$|\lambda| = 1$ $\qquad$ $|\lambda_i| < 1$.

If $\underline{x} \in \text{range } V_R \Leftrightarrow$

$x = V_R y$ for some y.

Then $\sum_{k=0}^{\infty} A^k v$ Will Converge.