Lecture 4

Announcements

- My Office Hours This week TODAY! 1-2pm
- HW1 posted
- HW1 note on Ed Stem
- New Section in Book
- Julia Demos.

(*) A formal definition of a Matrix.

(*) HW Q's not allowed ☹

Orthogonal M preserve other norms?

HW must be typeset.

Do cols of Square orthog Matrix form a basis for range A.

Partial pts.

Is neg. def. a thing?

Why column pointer over indexed Storage?

In Candyland How do we use a matrix to Compute the Expected # of moves.

Does CSC save Space asymptotically or just a Constant?

Does $A^TA = I$ (A square) $\Leftrightarrow$

$\|Ax\|_1 = \|x\|_1$?

No!

But Sometimes the one other the path

if P is a column stoch. matrix, $\boxed{P_{ij} \geq 0, \sum_i P_{ij} = 1 \ \forall j}$

then if $x \geq 0$ too

$\|Px\|_1 = \|x\|_1$

Do cols of Square orthog. matrix

A: $n \times n$, Square, orthog.

$\text{range}(A) = \boxed{\{ y \mid Ax = y, x \in \mathbb{R}^n \}}$

Cols of A span $\mathbb{R}^n$ $\Leftrightarrow$

Cols of A are an orthog basis for range (A).

Neg. def.?

Yes but if A is neg. def. then

$-A$ is pos. def.

So we focus on the pos. def. case WLOG..

Min $\frac{1}{2}x^TAx - x^Tb$

If A is neg. def...

$x^TAx < 0$ for any x.

if $x^TAx < 0$ for some x.

Then $(\alpha x^T A \alpha x) = \alpha^2 \underbrace{x^TAx}_{\sigma}$

$= \alpha^2 \sigma < 0$

CSC / CSR

random access to colums

random access to rows.

$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = e_1 e^T$

I	J	V
1	2	5.0
1	3	3.0
2	3	-1.0
5	1	1.0
4	2	2.0

Sort by colum index $\rightarrow$

I	J	V
5	1	1.0
4	2	2.0
1	2	5.0
1	3	3.0
2	3	-1.0

$\rightarrow$

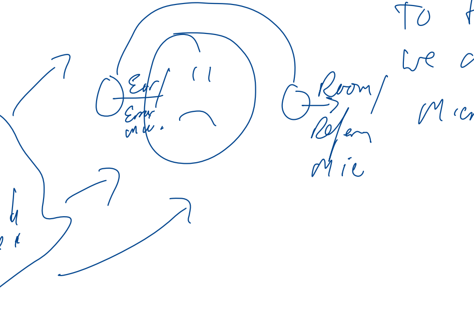
I	V
5	1.0
4	2.0
1	5.0
1	3.0
2	-1.0

colptr

1	1
2	2
4	3
6	

Toeplitz + Noise Cancellation

Setup



To fix this, we add 2 Microphones

Signals

$x(t)$ - Source Signal $\rightarrow \underline{x}$

$r(t)$ - room mic $\rightarrow \underline{r}$

$e(t)$ - Earphone mic. $\rightarrow \underline{e}$

$f(t)$ - Noise Canc. filter $\rightarrow \underline{f}$

$d(t)$ - "disturbance" or noise in $e(t)$. $\rightarrow \underline{d}$

All Signals are discrete!

index = Sample at time $i \cdot h$.

Suppose $\underline{f} = 0$

$\underline{e} = \underline{x} + \underline{d}$

~~$e(t) = x(t) + d(t)$.~~

~~$d(t) =$~~ $\boxed{f(r(t))}$

$\underline{d}$ = Causal function of $\underline{r}$.

$$\boxed{d_i = \sum_{k=0}^{L-1} p_k r_{i-k}}$$

We assume this form. w/ unknown $\underline{p}$

Idea: fit $\underline{p}$.

Then we can compute $\underline{d}$ Ahead of time!

Use f = Correction s.t.

$x + d + f = x$.

Goal: find p.

$d_L = r_1 p_{L-1} + r_2 p_{L-2} - r_L p_0$

$d_{L+1} = r_2 p_{L-1} + r_3 p_{L-2} \cdots r_{L+1} p_0$

$\underline{d} = R \, \underline{p}$

$$\begin{bmatrix} r_L & & r_2 \, r_1 \\ r_{L+1} & r_L & r_2 \\ & & \ddots \\ & & \end{bmatrix} \begin{bmatrix} p_0 \\ \vdots \\ p_{L-1} \end{bmatrix}$$