Lecture 29

Q: What is the difference between SVD and EVD on

$A = A^T, A \succeq 0.$

Find:

· Why $\begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}$ in the first place? ✓✓

· What, in practice, are λ-vals of $\begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}$ ✓

if $\begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \sigma \begin{bmatrix} x \\ y \end{bmatrix}$

Do we know $x = y$?

No! $x \neq y$ (may not even have same dims)

· Golu
vs.

· How h

- Kahan Bidiag...

Lawson, Hanson, Chan.

Use Golub Kahan? ✓

Why this matrix?

$$\begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}$$

$$A^TA, \quad AA^T$$

$$\begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}\begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix} =$$

$$\begin{bmatrix} AA^T & 0 \\ 0 & A^TA \end{bmatrix}$$

Wielandt matrix.

Gene Golub was famous for figuring o
how to get QR on

$$\begin{bmatrix} 0 & B \\ B^T & 0 \end{bmatrix} =$$

w/o forming C.

$$U^TAV \to B$$ a bid
Matr

t

hy:

n ?

Use Double Implicit Shifted QR...

$$\begin{bmatrix}0 & A\\ A^T & 0\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \sigma\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}Ay\\ A^Tx\end{bmatrix}$$

$$\begin{bmatrix}0 & A\\ A^T & 0\end{bmatrix}\begin{bmatrix}x\\ -y\end{bmatrix} = \begin{bmatrix}-Ay\\ A^Tx\end{bmatrix} = \begin{bmatrix}-\sigma x\\ \sigma y\end{bmatrix}$$

$$= -\sigma\begin{bmatrix}x\\ -y\end{bmatrix}$$

$$Ay = \sigma x$$

$$A = U\Sigma V^T \Rightarrow AV = U$$

$$A\underline{v_i} = \sigma_i$$

What if

$A = A^T$, $A \succeq 0$.

$A = Q \Lambda Q^T$ is EVD

$A = U \Sigma V^T$ is SVD.

But if $A = A^T$ ~ $U \neq V$ but if $A = A^T$, $A \succeq 0$, then

Σ

u_i

Uniqueness of SVD.

If $\sigma_i \neq \sigma_j$ for all $i \neq j$, then $\underline{u}_i, \underline{v}_i$ are unique! (up to scaling).

If $\sigma_i = \sigma_j$ for some pair, we are unique in subspace.....

So if $A = A^T$, $A \succeq 0$, and $\sigma_i \neq \sigma_j$ $i \neq j$, the EVD = SVD always.

$$\text{Golub-Kahan} \Longleftrightarrow \boxed{\text{Lanczos for } \begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}}$$

Golub-Kahan gives

$$A X_k = Y_{k+1} B_{k+1,k} = \begin{bmatrix} \alpha_1 & & \\ \beta_2 & \ddots & \\ & & \alpha_k \\ & & \beta_{k+1} \end{bmatrix}$$

$$X_k^T X_k = I$$

$$Y_k^T Y_k = I.$$

σ-vals of $A \simeq \sigma$-vals of B_k.

$$B_{k+1} = \tilde{U}_k \tilde{\Sigma}_k \tilde{V}_k^T, \quad \therefore A X_k = Y_{k+1} U_k \Sigma_k V^T$$

$$A \underbrace{X_k \tilde{V}_k}_{V_k} = \underbrace{Y_k \tilde{U}_k}_{U_k} \Sigma_k$$

$$A \approx U_k \Sigma_k V_k^T$$

$$U_k^T A V_k = \Sigma_k.$$