Lecture 26

HW due dates

Q: How to do sym. diagonal scaling on a sparse matrix A in CSR form?

$A \to \tilde{A}$

$\tilde{A} = D^{-1/2} A D^{-1/2}$

D: diag(A)

- Why do we want to keep symmetry?
- $\boxed{\text{Can we think of rank } k \text{ correction as preconditioner?}}$
- Can we permute to be more diag dominant? Open Research. (Eg. RCM) then apply SDD preconditioner? Use AMD + reorder.
- Does A need to be SPD for sparsified Cholesky to work?

- Are precond always linear or are there non-linear maps?
- Are there combos that don't work? What is effective?
- What about variable preconditioners?
- Theory vs. practice of precond in 2025 vs 2019?
- Why does rank k correction the soln A has k distinct λ-vals?

Compute $\tilde{A} = D^{-1/2} A D^{-1/2}$

$$\tilde{A}_{ij} = \frac{A_{ij}}{\sqrt{d_i d_j}}$$

Alg: get diag entries.

Why symmetry?

→ The algs for sym. matrices are better!

Thm 10.2.5 in GvL v3

IHB finish in $r+1$ steps for rank r B.

$A x = b$

$\downarrow$

$$(A + UV^T) y = c$$

s.t. x is easy to get from y.

⊕ Randomized Nyström precond.

Incomplete Chol / LU

- Okay theory for $A = A^T$, $A \neq A^T$
- More practice than theory

(Exist for $A = A^T$, $A \neq A^T$.)

Randomized Sparse Cholesky / LU

- good theory for SDD.
- more theory than practice.

Only theory for SDD.

Non-linear preconditioner

Need bijective/invertible maps between vectors

$\mathbb{R}^n \leftrightarrow \mathbb{R}^n$

Probably some literature!

Not mainstream!

Variable precond.

Given $P_1, \ldots, P_k, \ldots$

Use P_i at step i of your method.

1. Rules out Krylov/Subspace methods.
2. No issues for Stationary method.
3. Not sure why not more widely used.

(HW) Suppose GMRES finishes in k steps, then $\exists\, p(x)$ s.t.

~~$A p(x) = b$~~

~~when p has degree~~

when p has degree k.

$\boxed{p(A)} b = 0$

$\downarrow$

Krylov space ends early!

If A has r distinct λ-vals, then the char. poly of A has r distinct roots $\Rightarrow p(A) b = 0$ w/in $r+1$ steps

If $A = I + B$ where B has rank r, then the same thing happens.

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