Lecture 19

HW4 due next week!

- Prof gave next week.

If we turn $f(x+\delta x)$ into $f(x)+\delta x$ does that give us BW stability?

BW Stability proving it mathematically vs. Numerically.

Problems where we can't get low BW Error?

You got me! I don't know!

- Why $2\log(\varepsilon)/\log(\alpha)$ iterations for PR?

- Can errors cancel and improve accuracy?

- How to study growth factor in GE in Real Examples?

- Algs used in practice that aren't BW stable? What's the benefit?

Can we sacrifice accuracy in PR?

GE vs. QR for BW stability?

What causes large variance in FP solution?

Does BFloat16 give better numerical stability? Why does ML use them?

The $(x+y)(1+\delta)$ comes from Exponential Representation? Are there other ways to store/approx. IR that have diff. trade offs? → yes...

How does Growth factor inform bound on E? $\tilde{L}\tilde{U} = A+E$.

Is BW stability comparable in any sense? PR/Richardson etc.

Page Rank

$(I-\alpha P)\underline{x} = (1-\alpha)\underline{v}$

P is column stochastic

$P_{ij} \ge 0, \quad \sum_i P_{ij} = 1 \quad \forall j.$

$P^T\underline{e} = \underline{e}$

$\|P\|_1 = 1.$

$\underline{v}$ is stochastic, i.e. $v_i \ge 0$

$\sum v_i = 1. \iff \underline{v}^T\underline{e} = 1.$

$\underline{x}^{(k+1)} = \alpha P\underline{x}^{(k)} + (1-\alpha)\underline{v}$

Richardson
$$\begin{cases} Ax = \underline{b} \\ \underline{x}^{(k+1)} = \underline{x}^{(k)} + \omega \underline{r}^{(k)} \\ r^{(k)} = b - Ax^{(k)} \end{cases}$$

$x^{(k+1)} = x^{(k)} + ((1-\alpha)\underline{v} - (I-\alpha P)x^{(k)})$

$= \alpha P x^{(k)} + (1-\alpha)\underline{v}$

$$\begin{aligned} x^{(k+1)} &= \alpha P x^{(k)} + (1-\alpha)\underline{v} \\ -\ x &= \alpha P x + (1-\alpha)\underline{v} \end{aligned}$$

$(x^{(k+1)} - x) = \alpha P(x^{(k)} - x)$

$\|x^{(k+1)} - x\|_1 \le \alpha \|P\|_1 \cdot \|x^{(k)} - x\|_1$

$= \alpha \|x^{(k)} - x\|_1$

$\|x^{(0)} - x\|_1 \le \|x^{(0)}\| + \|x\|$ if $x^{(0)}$ is stochastic? $\le 2.$

if $x^{(k)} \ge 0,\ e^T x^{(k)} = 1.$

$x^{(k+1)} = \alpha P x^{(k)} + (1-\alpha)\underline{v}$

$\Rightarrow x^{(k+1)} \ge 0$

also

$e^T x^{(k+1)} = \alpha e^T P x^{(k)} + (1-\alpha)\underline{e}^T\underline{v}.$

$(P^T e)^T = e^T P = e^T.$

$= \alpha \boxed{e^T x^{(k)}} + (1-\alpha)$

$= 1.$

$\|x^{(k)} - x\|_1 \le \alpha^k \|x^{(0)} - x\|$

$\le 2\alpha^k$

if you want $\|x^{(k)} - x\|_1 \le \varepsilon,$

you need...

$2\alpha^k \le \varepsilon$

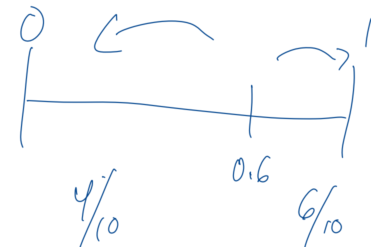
$\log(2\alpha^k) \le \log(\varepsilon)$

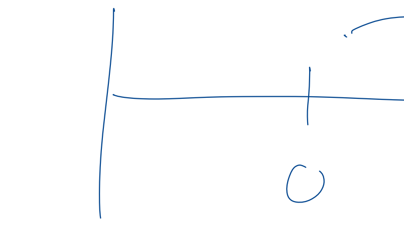
$\log(2) + k\log(\alpha) \le \log(\varepsilon)$

$k \le \dfrac{\log(\varepsilon) - \log(2)}{\log(\alpha)}$ iteration.

$(x+y)(1+\delta)$ gives a rel. ~~Error~~ approx. guaranteed

5.235×10^9





Round can give ε Error at Each Step.

$\Rightarrow n$ Steps $\Rightarrow n\varepsilon$ Error...

$\tilde{L}\tilde{U} = A+E$

when $\|E\| \le 2un^2 G$