

Leve 18

HW 4 posted,
one question
pending

Unusual hard to replace
FP in hardware?

Is Reading/Writing FP #s
Expensive?

→ Test?
→ Binary?

FP #s are expensive
Special in log-space
bins... down to some
limit

What else to reduce error
besides increasing the
precision?

$\|\cdot\|_A$ - what does
this mean?

Least square cond. # vs.
Algorithm of matrix in LS.

Why not FW stability?
Does BW stable $\Rightarrow$ FW stable?

How do we get y in
Relative Error?

Why is it
BW stable? $\checkmark$
addition...
 $f(x(1+\delta) + y(1+\delta))$ vs.
 $f(x(1+\delta)) + f(y(1+\delta))$

In PR, we
can jump to
any node.

How do preconditioning
work?

BW or FW Error?

How to use them in
data analysis.

x : input, f : function
 y : output (math) $y = f(x)$
 $\hat{y}$: output (computer) $\hat{y} = \text{ALG}(x)$

FW Error
 $\frac{\|\hat{y} - y\|}{\|y\|}$

BW Error
 $\frac{\|x - \hat{x}\|}{\|x\|}$ is small

When $\hat{y} = f(\hat{x})$.

Existence of
Computer/ALG.

BW stability & addition
 $x \oplus y = f(x+y)$

$f(x, y) = x + y$
 $\text{ALG}(x, y) = x \oplus y$
 $= f(x+y)$
 $= f(f(x, y))$
 $= (x+y)(1+\delta)$
When $|\delta| \leq u$

$\Rightarrow x(1+\delta) + y(1+\delta)$
 $= f(x(1+\delta), y(1+\delta))$
 $\Rightarrow$ BW stable!

$\frac{2}{x-y}$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{x+y}{2}$

denominator #s

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Classical "Error" in FP.

Catastrophic Cancellation
 $x - y$ if $x, y > 1$
 $\Rightarrow$ Bigger difference

$\frac{2}{x-y}$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{x+y}{2}$

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$\|x\|_A$ is related to
 $x^T A^{-1} x$ for distance
on the ellipse.

$A = V D V^T$
 $x^T A^{-1} x =$
 $x^T V D^{-1} V^T x$

$Ax = b, A = A^T A \succeq 0$
Then we have a
quadratic function

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