Lecture 15

HW3b soon!

Exams.

- Use Givens vs Householder?
  - ↳ Stability?
  - ↳ Efficiency, types (dense, sparse)
- Other Inner-products for Gram-Schmidt?
- QR of a vector a? What happens to elements of A?
- QR + Non-linear least squares? different orthogonality defn.

Product of Givens done by Mat-mat mul vs. rotation / Notation.

Is Steepest descent / Newton on a 1d cross-section? Yes.

What does a Givens Rotation do?

GS vs. Householder vs. Givens + Econ / full QR

Why doesn't GS give orthogonal?

Full QR vs thin + thin works?

QR + Eigenvalues?

Exam Results

$$[Q_1 \; Q_2]\begin{bmatrix} R_1 \\ 0 \end{bmatrix} = A$$

Min $\|b - Ax\| \Rightarrow$

$R_1 x = Q_1^T b$

$\Rightarrow$ Only lin. indep. rows impact soln.

Use Givens to solve the rotation problem? $\to x = G(\theta)^T$.

$G(\theta)X = e_1 e_1^T + e_2 e_2^T = I$

GS is preferred over other algs?

Givens

+ implicit form of orthogonal matrix.
+ allows us to use "sparsity" (of some form)
- slightly slower ☹

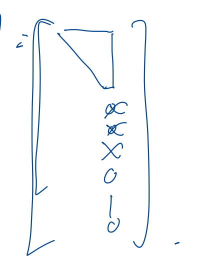
Householder

+ implicit representation of orthogonal matrices

GS

- Worst orthogonality in floating pt. $\|Q^T Q - I\|$ is largest.
+ slightly faster to compute.

$H(x) = I - 2xx^T$ w/ $\|x\|_2 = 1$

$A =$ 

$Q, R = qr(A)$

In Matlab, you get $Q: m \times m$, $R = m \times n$ Explicitly as matrix.

$Q, R = qr(A, \text{`Econ'})$

Then you get $Q: m \times n$, $R = n \times n$.

In Julia:

$Q, R = qr(A)$ gives implicit rep. of Q via WY representation.

$I - 2xx^T = H(x)$

$Q(W, Y) = I - 2WY^T$ is orthog if $\boxed{}$

Wrong, but gives the right idea quickly.

Let $B = A^T A$.

If $A = QR$ then

$B = R^T Q^T Q R$

$= R^T R$

$=$

R = Cholesky factor of $A^T A$!

$A = QR$

$= Q S S R$

S is a diag. sign matrix

$S_{ij} = 0 \quad i \neq j$

$S_{ii} = \pm 1$.

So $S^{-1} = S$.

$$G(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \Rightarrow G_{ij}(\theta) = \begin{array}{c} \\ i \\ j \\ \end{array}\begin{bmatrix} 1 & & & & \\ & \ddots & & & \\ & & c & \cdots & s \\ & & -s & \cdots & c \\ & & & & \ddots \end{bmatrix}$$

$$= \begin{bmatrix} c & s \\ -s & c \end{bmatrix}$$

$\boxed{G = \text{planerot}(c, s, i, j)}$

$c^2 + s^2 = 1 \iff c = \cos(\theta)$, $s = \sin(\theta)$ for some θ.

QR of a vector a w/ Givens Rotation

$$G_{56}(\theta_1)\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \to G_{45}(\theta_2)\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5' \\ 0 \end{bmatrix} \to \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4' \\ 0 \\ 0 \end{bmatrix} \cdots \Rightarrow a =$$

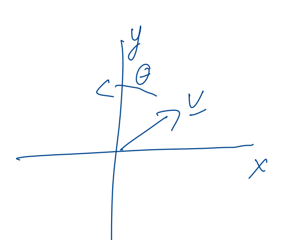
$|a_5'| = \left\| \begin{bmatrix} a_5 \\ a_6 \end{bmatrix} \right\|$.

$|a_4'| = \left\| \begin{bmatrix} a_4 \\ a_5 \\ a_6 \end{bmatrix} \right\|$.

$$G_{56}(\theta_1)G_{45}(\theta_2)G_{34}(\theta_3)G_{23}(\theta_4)G_{12}(\theta_5)\begin{bmatrix} a_1' \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

$|a_1'| = \|a\|$.

What does a Givens Rotation do?



3d rotation matrix:

$$\begin{bmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma \\ & \end{bmatrix}$$

Other inner-products

$x^T A y$ for any sym. pos. def. A.

$\iff$ alt. inner-prod.

$A = FF^T$

$F^{-1}A = QR$, then

$A = FQR$ and $Q^TQ = \ldots$

Wrong, but the idea is close.