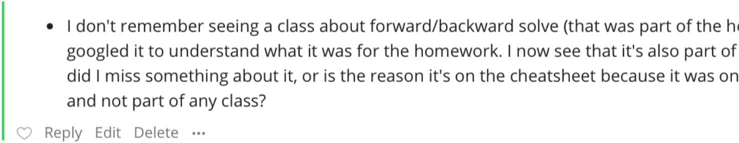
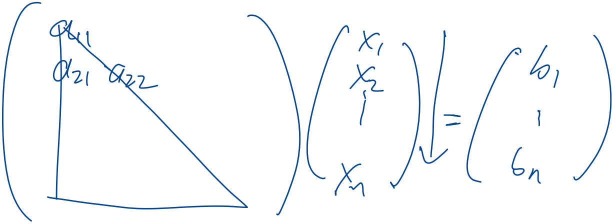
Lecture 13

Exam Thursday!

Wed Night Review
on Zoom. ≈ 9pm.

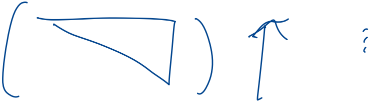


- I don't remember seeing a class about forward/backward solve (that was part of the h
googled it to understand what it was for the homework. I now see that it's also part of
did I miss something about it, or is the reason it's on the cheatsheet because it was on
and not part of any class?



$$a_{11} x_1 = b_1 \Rightarrow x_1 = b_1 / a_{11}$$

$$a_{21} x_1 + a_{22} x_2 = b_2 \Rightarrow x_2 = \frac{b_2 -}{a_{22}}$$



omework) so I
the cheatsheet,
the homework

- I understand the Schur complement represents the remaining system after eliminating part of the variables, but if we still need to perform the same amount of work to compute the first block (like A1^−1), what is the practical advantage of it?

It's an analysis tool!

K Steps of Elimination ⇒ Schur Complement.

$a_{21} x_{11}$

Gradient descent was mentioned in lectures and hinted at in HW2, but there was no specific problem to implement it on. Could you go over it in a general manner just to distinguish it from the other descent algorithms?

Will the content of Lecture 13: "QR and Least Squares, Ch 15" (10/7) be included in the exam?

The column permutation matrix Q was not included in Homework 4-1. What's the criterion for using it in an LU decomposition?

Could you explain th

$$\underline{x}_{k+1} = \underline{x}_k + \alpha_k \underline{r}_k$$

α_k = fixed ⇒ Richardson / gradient descent.

α_k f "optimal" ⇒ gradient descent.

No,

⇒ - Need Q / column perm. for complete pivoting / Rook pivoting.

- Need it for rank revealing factorizations.
- Often used for sparsity.

ne coordinated descent on solving Ax=b? Especially two coordinate version.

$$f(\underline{x}) = \tfrac{1}{2}\underline{x}^T A \underline{x} - \underline{x}^T \underline{b}$$

$$g(\underline{x}) = A\underline{x} - \underline{b}$$

$$\underline{x}_{k+1} = \underline{x}_k + \gamma_k \underline{e}_{k_j}$$

Coord we pick at kth step

Just change 1 coord.

we pick γ_k st

$$\min_{\gamma} \tfrac{1}{2} x_{k+1}^T A x_{k+1} - x_{k+1}^T b$$

$$\underline{x}_{k+1} = x_k + E_{r,s}^{(k)} \underline{c}$$

$$n\begin{bmatrix} \underline{e}_r & \underline{e}_s \end{bmatrix}^{2} \quad {}^{2}\begin{bmatrix} \gamma \\ \sigma \end{bmatrix}$$

$$E_{r,s}^{(k)} \qquad \underline{c}$$

Question: Could you explain the Neumann series in the viral spreading model?

pick $\gamma, \sigma \Leftrightarrow \underline{c}$ st.

$$\min_{\underline{c}} \tfrac{1}{2} x_{k+1}^T A x_{k+1} - x_{k+1}^T \underline{b}$$

$$\min \tfrac{1}{2}(x_k + E_{r,s}\underline{c})^T (A)($$

$$- (x_k + E_{r,s}\underline{c})^T \underline{b}$$

$$=$$

$$\cancel{\tfrac{1}{2} x_k^T A} x_k +$$

$$x_k^T A E_{r,s} \underline{c} +$$

$$\tfrac{1}{2} c^T E_{r,s}^T A E_{r,s} \underline{c} \quad ✓$$

$$- \cancel{x_k^T \underline{b}} - c^T E_{r,s}^T \underline{b}$$

$x_k + E_{r,s} \underline{c}$)

$\Leftrightarrow$ 2x2 linear system for c.

$$(E_{r,s}^T A E_{r,s}) \Rightarrow 2\times 2.$$

$$\underline{c} = (E_{r,s}^T A E_{r,s})^{-1} (E_{r,s}^T \underline{b} - E_{r,s}^T A x_k)$$

Neumann Series in Vira
Spreading.

$\eta_i^{(k)}$ ⇒ prob node i infect
at time k.

$x_i^{(k)}$ = prob. node i infected
before time k

$$x_i^{(k)} = \left(1 - \prod_{\ell=0}^{k} (1 - \eta_i^{(\ell)})\right)$$

$$\approx 1 - \sum \eta_i^{(k)}$$

$$\approx 1 - \sum (\rho A)^k_i$$