Lecture 9

- What order should we use to update Equations? ✓
  - ↳ How many to try?
- What class of problem are Jacobi / GS used on? ↗ if converges fast. → D.D. or Sym. pos. def.
- Do SVD / Eigenvals help write the permutation for Jacobi/GS?

In

Use a
Equ

To
Perm
always
update
(afte

notes (to derive)

map between variables /

tion

analyze: use a

ution Matrix and

use row i to

the variable i.

a permutation)

to solve $Ax = b$

We use a permutation P

s.t. we solve

$$PAx = Pb \longrightarrow \tilde{A}x =$$

(* this reorders the equations *)

Apply

here

$i \to i$

Jacobi on $\tilde{A}$

$$\tilde{A}\underline{x} = \underline{\tilde{b}} \Rightarrow$$

$$(D - N)\underline{x} = \underline{\tilde{b}}$$

diag elem ↑ ↑ off diag

$$D\underline{x}^{(k)} = \underline{\tilde{b}} + N\underline{x}^{(k-1)}$$

$$\underline{x}^{(k)} = D^{-1}N\underline{x}^{(k-1)} + D^{-1}\underline{\tilde{b}}$$

$\tilde{b}$

Jacobi/GS w/ map.

Idea: max-weight Matching of rows/eqns to cols/variables

-W

- Can gradient descent be used to solve min-max saddle pt. problems?

E.g. $\min_x \max_y f(x,y)$

hat are the differen

Coordinate descent / ascent

Steepest descent

gradient descent

Stochastic GD

- Why do we need so many methods?

- Newman, Richardson

GD, CD,

Jacobi, GS

if we can do $\underline{x} = A \backslash \underline{b}$?

⊕

Neumann

$$x^{(k)} = \sum_{i=0}^{k} (I-A)^i b$$

Richardson / Gradient Descent.

$$x^{(k)} = \sum_{i=0}^{k} (I-\omega A)^i b$$

S

$$x^{(k}$$

==Designed== to get Big Picture Right

Need to verify precise details.

teepest Descent

Coord Descent

Jacobi

$$) = \prod_{i=0}^{K} (I - \alpha_i A) b \quad \textcircled{*} \longrightarrow$$

Steepest Descent a variable at a time.

Solve/update all vars at once

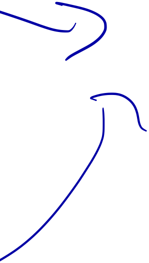
Not quite right

③ off by a factor.

$$x^{(k)} = \sum (D^{-1}N)^k D^{-1} \underline{b}$$

Stochastic GD: (we will randomly

GS



See more methods later.)

Block versions on all these algs!

$$\begin{bmatrix} A_1 & A_2 & A_3 \\ A_4 & \boxed{A_5} & A_6 \\ A_7 & A_8 & A_9 \end{bmatrix}$$

Block Jacobi...

$$x^{(k+1)} = \begin{bmatrix} A_1 & & \\ & A_2 & \\ & & A_3 \end{bmatrix}^{-1} \left(\begin{pmatrix} 0 & -A_2 & -A_7 \\ -A_4 & 0 & -A_5 \\ -A_7 & -A_8 & 0 \end{pmatrix} \right)$$

$$\underline{x}^{(k)} + \underline{b} \Bigg] .$$