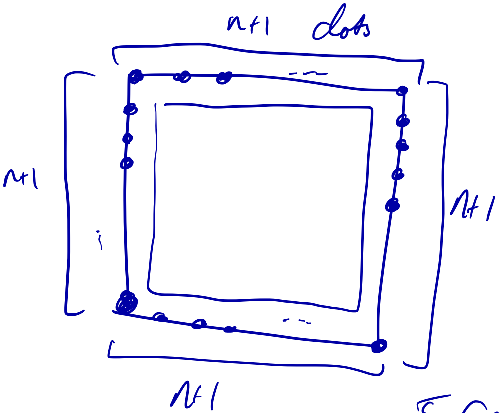
Lecture 8

- $(n+1)^2$ total dots
- $4(n+1)-4$ dots on boundary.

$\uparrow$ double count corners.

- Poisson nnz



nnz $\Rightarrow$ 5 for each interior node

$\Rightarrow$ 1 for each boundary.

- $5(n-1)^2 + 4(n+1) - 4$
- $5(n+1)^2 - 4[4(n+1)-4]$

- How spectral radius is introduced?

$$\max_{g \neq 0} \frac{g^T A g}{g^T g} = \rho(2)$$

- HW Soln? ⇒ No

- Norm matrix in Coordinate transform

- Constants in norm Equiv (How to find them?)

$$C_1 = \max f(x) \quad \text{s.t. } g(x) \le 1$$

- Discuss Poisson matrix

- T Sub-mult. is $\max f(Ax)/f(x)$ the only way? Ch[illegible] norm

? 2

vector

?

- Do other 4 props imply sub-mult? ✓

- How to set step size in steepest descent / gradient descent?

- How to set α in GD ✓

What does P of

- Section 2.3, A
What is a steep

- when is coord. faster than grad

A mean? ✓

✓

= diag dom.
M-Matrix

pest descent?

descent ✓

dre descent?

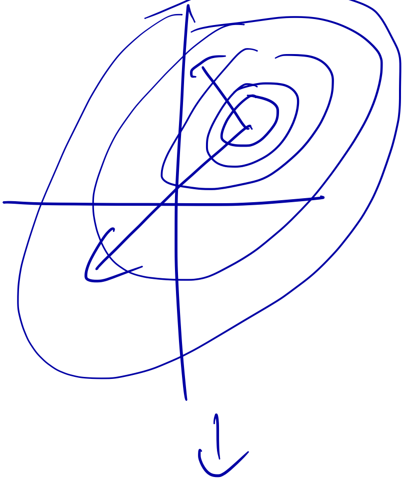
Spectral Radius +
Symmetric Matrices.

$$\rho = \max \frac{|x^T A x|}{|x^T x|}.$$

Intuition:

$$f(x) = \frac{1}{2} \underline{x}^T A \underline{x} - \underline{x}$$

$f: \mathbb{R}^2 \to \mathbb{R}$



$^T \underline{b}$

$A = V \Lambda V$

(spectral de

$$\max \frac{\underline{x}^T A \underline{x}}{\underline{x}^T \underline{x}}$$

$$\Downarrow$$

$$\max \frac{\underline{x}^T V \Lambda V}{\underline{x}^T \underline{x}}$$

^{T}

omp.)

$$\frac{{}^T\underline{x}}{}$$

$$\rightarrow \text{Max} \frac{\underline{x}^T V \Lambda V^T \underline{x}}{\underline{x}^T V V^T \underline{x}}$$

(V is square, orth.)

$$\Downarrow$$

$$y = V^T \underline{x} \quad \text{Max} \frac{y^T \Lambda y}{y^T y}$$

$$\Downarrow$$

$$\text{Max} \frac{\sum \lambda_i y_i^2}{\sum y_i^2} = \lambda_{\max}$$

the spec. radius.

Sub. mult.

$$\|Ax\| \le \|A\| \cdot \overset{\swarrow}{\|x\|}$$

$$\|AB\| \le \|A\| \cdot \|B\|$$

$$\|Ax\|_{\textcircled{1}} \le \|A\|_2 \cdot \|x\|_{\textcircled{1}} \quad ?$$

If $\|x\|_2 \le \|x\|_1$

$$\boxed{\|x\|_\infty \le \|x\|_2 \le \|x\|_1}$$

Giv

$c_1 \|$

How t

look at

idea

$C =$ max

st.

any 2 norms,

$\|x\|_A$, $\|x\|_B$

$x\|_A \le \|x\|_B \le C_2 \|x\|_A \implies \boxed{\dfrac{\|x\|_B}{\|x\|_A} \le C_2}$

find C_1, C_2 ?

then like:

$\|$

max $\|x\|_B$

s.t. $\|x\|_A \le 1$

because norms are sensiti

$\le$

$\|x\|_B$ vs

$f(x)$

$g(x) \le 1$

max $\|x\|_B$

s.t. $\|x\|_A \le 1$

vs.

max $\|x\|_A$

s/t $\|x\|_B \le 1$

$\Rightarrow$

e

Scale

C_2