Lecture 6

Is least square the L_2 norm of the residual? ✓

Why is $\|X\|_\infty = \max|X_i|$ ✓

How do we denote i^{th} row? j^{th} col of A? ✓

My office hours 12:30-1:30pm on Friday! In my office or on Zoom (Sometimes)

i^{th} row

$A[i,:]$

$A(i,:)$

$$A = \begin{bmatrix} a_1^T \\ \vdots \\ a_n^T \end{bmatrix} \Rightarrow a_i^T$$

$A_{i,:}$

j^{th} col

$A = [a_1 \; \dots \; a_n] \Rightarrow a_j$

$A[:,j]$, $A(:,j)$

$A_{:,j}$

Why is $\|X\|_\infty = \max_i |X_i|$?

Limiting arguments:

Let $\|X\|_p = \left(\sum |X|^p\right)^{1/p}$

$\lim_{p\to\infty} \|X\|_p$

Consider: $\|(1, 1.1, 1.2, 0.9)^T\|_p =$

$(1^p + 1.1^p + 1.2^p + 0.9^p)^{1/p}$

$p = 1e6$

$1.1^p = 4.8\times10^{41392}$

$1.2^p = 1.76\times10^{79181}$

≈ 1.2

Let's suppose Everything is the largest.

$\|(1,1,1,1)\|_4 = \sqrt[4]{\ }$

$\|X\|_p \le \|\max|X_i|\, e\|_p = \max|e_i| \bullet \|e\|_p$

$\|e\|_p = \boxed{\sqrt[p]{n}}$

as $p\to\infty \Rightarrow 1$

This is true for all p.

$\Rightarrow \|X\|_\infty \le \max|X_i|$

More Q's

$\|\text{Error}\| = \|y - x\|_2$

$\|\text{Residual}\| = \|b - Ay\|$

Why 1-norm for Residual, 2-norm for Error? ✓

If λ-vals are neg. is Neumann still valid? ✓

Convergence of $I + A + A^2 + \dots + A^n$ for any A? ✓

↳ Other decomp? ✓

- What happens to Neumann if ∞-solns? No solns?
- Norms are Equiv. $\Rightarrow$ all soln Equiv.? ✓
- Spectral decomp. vs Eigenval decomp? ↳ Same! ✓

Least Squares

$\Rightarrow \min \|Ax - b\|_2$

Linear System

$\Rightarrow$ find x s.t. $Ax = b$

$\Rightarrow$ find x s.t. $\|Ax - b\|_2$ is minimized.

If you know

$\|Ax_a - b\|_\infty \le \varepsilon$

and

$\|Ax_b - b\|_1 \le \varepsilon$ →

Are x_a, x_b Equally close to the true x?

Suppose $x = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ $\quad x_a = \begin{bmatrix} 1.1 \\ \vdots \\ 1.1 \end{bmatrix}$

Approach:

Test on Computer!

Try Spectral decomp.

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