Lecture 30

- Q's for the day ✓
- Q's for the final ✓
- Q's for the HW. ✓
- Last notes from me!
- Plan for Review Session. Zoom, Monday afternoon

✓

Will adjust HW 7 on Re-balance all components:

- CG w/ a good initial guess? ✓ ✓
- 1-1 meeting from class. ✓
- Algs over $\mathbb{C}$? over $\mathbb{Z}$? ✓
- A adj $\Leftrightarrow$ Reverse all links
- Lanczos for λ-vals ✓
- Lanczos for non-Sym matrices? ✓
- Project ideas based on the course.

- Subspace / QR methods to only get top-k λ-vals? ✓
- Is content from this week testable on final? ✓

formula sheet for final? ✓

HW: how to estimate $\underline{v}$ in $I + \underline{v}\underline{v}^T$? ✓

CG w/ Know good guess!

$\hat{x} \approx x^*$ (soln)

I explain CG from

$x_0 = 0 \iff$

$\mathcal{K}_k = (b, Ab, \ldots A^k \underline{b})$

Simple! Use the Residual System!

$\hat{r} = A\hat{x} - \underline{b}$

$Ay = \hat{r} = A\hat{x} - b =$ → $x' =$

$y =$

$=$

A

$A^{-1}(A\tilde{x}-b)$

$\hat{x} - x^{*} \Rightarrow$

$x^{*} = \hat{x} - y.$

Algs over $\mathbb{C}$ → all!

Algs over $\mathbb{Z}_p$ → basically nothing ☹.

(or they are very differ)

$A_{ij} = 1$ if dir edge $(i \to j)$

if G is undir, add 2 dir edges $(i \to j, j \to i)$

→ $A^T \Leftrightarrow$ reversing links.

Lanczos for λ-vals

$$A \xrightarrow{k\text{-step Lan}} V_k \quad T_k.$$

λ-vals of T_k are "close" to λ-vals of A.

- always re-orthog V_k to get orthogonality.

Restart w/ better guess to avoid memory issues.

Block Power method does give top-k

$Y = AX$ $\qquad X_0 =$

$X^{(\text{next})} = QR(Y).$

λ-ub!

rand(n, k)

$\uparrow$

Q: How to get $\underline{v}$ in $A = I + \underline{v}\underline{v}^T$

A: do a mat vec!

Compute $y = Aw =$

$$\rho = \underline{v}^T \underline{w}$$

$$w + \rho \underline{v}$$

$$y = \underline{w} + \underbrace{\rho \underline{v}}_{\text{unknown.}}$$

$$\underline{z} = y - w = \rho \underline{v} \leftarrow$$

$$z^T w = \rho v^T w = (v^T w)^2 = \rho^2. \checkmark$$

[illegible] Low rank

Update: Cholesky factorization

$A \succ 0$, $A = A^T$

then $A = LL^T$ or

$A = R^T R$ ← upper-tri.

Idea:

$\hat{A} = A + \underline{u}\underline{u}^T \Rightarrow$

$$\hat{A} = \begin{bmatrix} R \\ \underline{u} \end{bmatrix}^T \begin{bmatrix} R \\ \underline{u} \end{bmatrix}$$

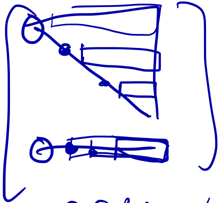
$\overset{?}{=}$

Idea: update (like QR)

$$\begin{bmatrix} R \\ \underline{u} \end{bmatrix} \rightarrow \begin{bmatrix} R' \\ 0 \end{bmatrix}$$

$A = QR$, then

$A^T A = R^T R$



$\Rightarrow O(n^2)$ work for update of Givens