Lecture 28

- range + focus for final vs. midterm. ✓
- Why does Givens rotation avoid iteration but Jacobi rotation doesn't in reduction a sym. matrix to tridiagonal ✓
- How Easy is it to find H s.t. $Hx = e_1$? ✓ in deflation

- Can QR be used w/ Repeat x-v
- Multi-grid - like methods for general spatial data. ✓
  - ↳ geometric MG.
- Add check to Arnoldi/Lanczos for Early termination? ✓
- One shot k-step GMRES vs. per-iteration. ✓
- How to handle numerical orthogonality in deflation ✓

rds?

find H s.t.

$$Hx = e_1$$

=> Easy! we did it for QR!

$$H = I - 2vv^T$$

$$v = e_1 - x \quad ??$$

In deflation, given

A, λ, x where

$Ax = \lambda x$ we

Compute

$$HAH = \begin{bmatrix} \lambda & 0 \cdots \\ 0 & B \end{bmatrix}$$

then "Recurse" on B.

But! you can just do

$H \cdot (A \cdot (H \cdot y))[2:\text{end}]$

to implement it w/o computing B.

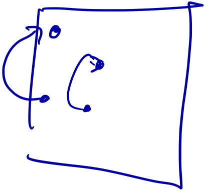
Householder / Givens for $A \to$ Tridiag

Use a fixed # of Steps.

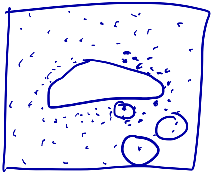
Jacobi

for A → Diag

Use an iterative method.



Geometric Multigrid



if you.

QR $\iff$ Subspace w/

$X_0 = I$

$\uparrow$

a subspace S is invariant if

$x \in S \implies$

$Ax \in S$.

i.e. $S = \{\alpha \underline{v}\}$ is a subspace.

if S is any linear combo of k - λ-vecs, then.

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Check for Early termination in Arnoldi / Lanczos?

↳ for linear systems, yes! (sorta)

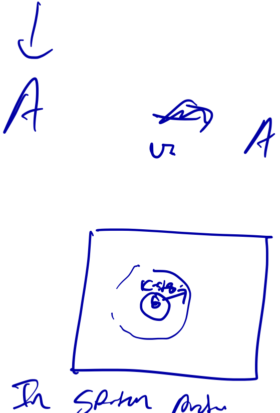
↳ for λ-vals, no! (sorta).

Early termination in theory

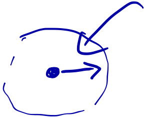
↳ $\beta_k = 0$.

But... $\beta_k = 2 \cdot 10^{-15}$.

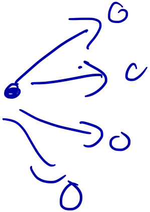
And $\|r_k\| = 0$ (in theory!) ✓



K- Step of GMRES at once.



How many pieces of data do I need to do for K-matvec?

K