Lecture 27

- How do we effectively use the Householder method for finding λ-vals? ✓
- Preconditioners for Earlier Methods? SO? ✓
  CO?
- Multigrid w/o a PDE/geometry? ✓

- If we use ✓

  ②

  $$A^{-1} = \underbrace{I + (I-A) + (I-}_{k\text{-terms}}$$

  do we actually compute this matrix?

- λ-vals of (A,B) vs. AB, BA, etc.

- → if A,B commute, then this is E
  if B is a polynomial in A, then A,
  →

$A^2 + -$

...ay!

B commute.

Idea: Given $A=A^T$, $\lambda, \underline{v}$ where $\lambda, \underline{v}$ is an Eigenpair, how do we find all the others (with, e.g. the power method).

$$\underline{v}^T\underline{v} = 1$$

H: householder reflector to take

$$\underline{v} \xrightarrow{H} \underline{e}_1$$

$$H\underline{v} = \underline{e}_1$$

(H is sym.

$H^T A H$ a

$$HAH^T = \begin{bmatrix} \lambda & 0 \\ 0 & B \end{bmatrix}$$

How to prove?

$$A = V\Lambda V^T$$

$$H\overset{\overset{[\underline{v}\ \underline{Q}]}{\shortparallel}}{V} = \begin{bmatrix} \underline{e}_1 & \underset{\uparrow}{HQ} \end{bmatrix} \begin{bmatrix} \lambda & \\ & \Lambda_2 \end{bmatrix} \begin{bmatrix} e_1^T \\ HQ \end{bmatrix}$$

$$HQ = \begin{bmatrix} 0 \\ U \end{bmatrix} \Big[$$

$$=$$

$$= \cdots$$

$$= \begin{bmatrix} \lambda & 0 \\ 0 & U^T \Lambda_2 U^T \end{bmatrix}.$$

do

No

I

Via

$$A^{-1} \simeq \underbrace{I + (I-A) + (I-A)^2}_{K\text{-term}} + \cdots = B_K$$

~~we~~ actually compute A^{-1} approx?

! We work implicitly!

Can compute

$$B_K \cdot \underline{v}$$

an algorithm w/ K matvecs w/ A

To solve the preconditioned problem

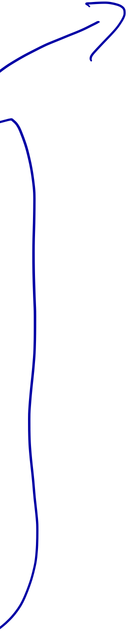
$$\boxed{(B_K A x) = B_K \underline{b}}$$

You just apply matvecs w/ A. to compute

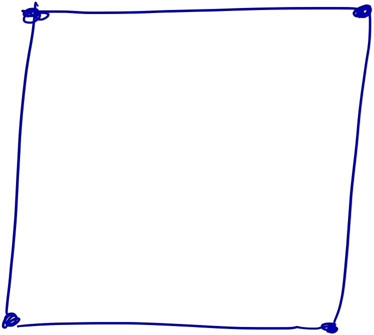
$A\underline{v}$ or

$B_K \underline{v}$.

⊕ don't be scared by non-sym of



④ Todo, on Complete it Email.



Idea: we can solve low-res quickly, and use that to solve high res problem

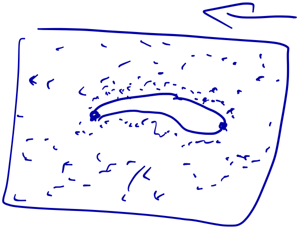
ersion

to

faster.

Algebraic Multigrid tries to relax this assumption.

⊛ If your problem has space, but it's more complicated, it works.



$$\Big((I-A) +$$

$$(I-A)\underline{w} +$$

$$(I-A)$$

$$(I) \cdot \overbrace{Av}^{= w} = w$$

$$I)(Av) =$$

$$w$$

$$+$$

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