Lecture 25

- CG vs. SD for ill-condition systems? ✓
  - * better!
- CG in poly form vs. subspace form? ✓
- Smoothness of flow
  - CG, Newton, SD.
- Jacobi, GS + Preconditioners? ✓
  - ↳ Coming soon!
- Even vs. Odd fns orthogonal?

- γ_i in CG vs. orthog. polys ⟺ γ_i in SD. ✓
  - * Coincidence?
- GMRES, diagonalization + k-step convergence ✓
- GMRES slower than MINRES/CG? ✓
- MINRES better than CG for spd matrices? ✓
- Distance between subspaces? ✓

Distance between Subspaces

⊕ → See Golub-van Loan for this!

$\underbrace{\text{CG is only}}$ vs. Su

↓

BiCG, BiCGStab, ...

for non-sym probs.

Less Robust.

☹

space ?

Arnoldi

High Compute Cost ☹

If A is Sym.

GMRES is slower than MINRES..

Is MINRES ever better than CG for SPD A?

ⓐ In Residual? yes!

ⓑ In Error? Sometimes..

Both $\frac{1}{2}x^TAx - x^Tb$ - CG

$\|b - Ax\|$ - MINRES

are proxys for Error.

Ju in

+ C

Ju

Orthopoly func

G

$$= \frac{g^T A g}{g^T g}.$$

?

If A has K-distinct λ-vals, then GMRES will converge in k-steps to the true Soln.

for any A

↳ Not needed to be diagonalizable.

GMRES will only breakdown if you have a Soln. ✓

Smoothness.

Min $\frac{1}{2}\underline{x}^T A \underline{x} - \underline{x}^T \underline{b}$

SD vs. CG vs. Newton

Smoother

Middle part..

↳ 1 step.

least smooth.

$X_{k+1} = A^{-1}(\underline{b} - A\underline{x}_0)$.

Simp

w/

↳

ide

⊛ A

Imple

$\underline{V}_0$ = close

to X-vec.

lifs the Power Method

Arnoldi / Lanczos.

You can use these

ao to find Eigenvalues!

ARPACK.

Iterate w/ Lanczos

$V_k^{(0)} \longrightarrow V_{k+p}^{(0)}$

ct - - - - ..

$V_k^{(1)} \longrightarrow V_{k+1}$.

Over $x = [-1, 1]$

if $f(x) = -f(-x)$ (odd) $\rightarrow f$

$f(x) = f(-x)$ (Even) $\rightarrow g$

$$\int_{-1}^{1} \underbrace{f(x) g(x)}\, dx = \dots = 0 \text{ yes!}$$

Flip sign on axis.

$$\underbrace{\int_0^1 f(x) g(x)\, dx}_{S} + \underbrace{\int_{-1}^{0} f(x) g(x)\, dx} \overset{=}{\checkmark} - \int_0^1 f(x$$

$) g(x)\, dx = 0$