Lecture 24

- Motivation behind MINRES + CG ✓
- What is total complexity of CG?

↳ at each iteration

[Matvec

+

a few linear $O(n)$ operations

(E.g. $x^T y$ or scaling vectors.

Error/Resid is something like.

$$\|b-Ax_k\| \le \left(\frac{\sqrt{\kappa}-1}{\sqrt{\kappa}+1}\right)^k \|b-Ax_0\| .$$

Idea of

Simple Alg

$$x = \sum (I-A)^k \underline{b}$$

$$= p(A)\, \underline{b} \Leftarrow$$

$$\approx A^{-1} \underline{b}$$

MINRES / CG

Subspace Alg

CG

k^{th} step of CG

$$\min_{\underline{c}} \ \tfrac{1}{2}\underline{x}_k^T A \underline{x}_k - \underline{x}_k^T \underline{b}$$

s.t. $\underline{x}_k = [\underline{b}, A\underline{b}, \ldots, A^k]\,\underline{c}$

$[I\ (I-A)\ (I-A)^2 \ldots (I-A)^k]\,\underline{b}$

$\Updownarrow$

$(I\ A\ A^2 \ldots A^k)\,\underline{c}$

for some $\underline{c}$.

$\longrightarrow$ Make $\underline{c}$ a variable we choose @ every iteration. $\Rightarrow$

MINRES

k^{th} step of MINRES

$$\min_{\underline{c}} \|b - Ax_k\|_2^2$$

s.t. $x_k = [b, Ab, \ldots, A^{k-1}\underline{b}]c$