# Lecture 23

- Can we use Lanczos to find small λ-vals? ✓
- What is preconditioning in the Krylov space? → future lectures! ✓
- Krylov space vs. Eigenvector space ✓
- Are CG / Krylov methods better for solving $Ax = b$ } See last in-class video. ✓

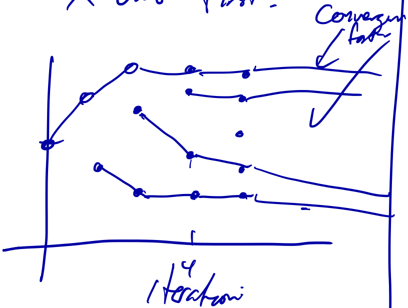
- Does CG converge w/in n iterations to the Exact Soln? ✓
- To compute f(A) Do we Ever try $V f(\Lambda) V^T$? ✓
- Is Arnoldi BW Stable? ✓
- When to compute $A = V \Lambda V^T$ Explicitly? ✓

Lanczos + Small
$\lambda$-vals?

⟹ Yes!

Lanczos / Arnoldi

Converges to Extreme
$\lambda$-vals first.



$\underbrace{K_k(A, b)}$ vs. top $k$ $\lambda$-vals $\uparrow$

as $k$ gets big than the.

Easy to Compute

Much harder to compute.

- Does CG work w/in $n$ iterations to give an exact answer?

↳ In theory, yes!

Why: $\dim(\mathcal{K}_k(A,b)) \le n$.

In practice, No.

⇒ Re-invented as an iterative method about 20 yrs later.

⊛ Consider MINRES LSMR.

Is ARNOLDI BW stable?

I don't know, but I suspect so.

Given $A$, $b$, then we compute $\tilde{V}$, $\hat{H}$ s.t.

$$(A+E)\hat{V}_k = \hat{V}_{k+1}\hat{H}_{k+1}$$

for some $E$.

w/ $\|E\|$ small $O(u)$

Science - David F. Gleich - 2023 CS 515 - Purdue University - Computer Science - David F. Gleich - 2023 CS 515 - Purdue University - Computer Science - David F. Gleich - 2023 CS