Lecture 22

- Why is $K_k(A,b)$ in the monomial basis ill-conditioned?
- Seems like a ~~lot of work?~~ Does it work faster? ✓
- Arnoldi in the context of the Krylov space?
-

→ the matrix

$[b\ Ab$

is ill-conditioned

as $k \to \infty$

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$A^2 \underline{b} \ \cdots \ A^k \underline{b} \,] \rightarrow$ singular as $k \to \infty$.

$\Uparrow$ and always if ($k > n-1$)

dom. λ-vec.

$\downarrow$

$A^k \underline{b} \rightarrow \rho_k \underline{v}$.

lots of

↳ Not SD

$\min \ \frac{1}{2} \underline{x}^T A \underline{x} - \underline{x}^T \underline{b}$

s.t. $x_k \in K^k$ new direction in $\mathcal{K}_k(A,b)$

Work?

really!

rs.

CG

$$\min_{x \in \mathcal{K}_k(A,b)} \tfrac{1}{2}\underline{x}^T A \underline{x} - \underline{x}^T \underline{b}$$

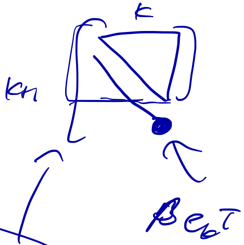
Arnoldi

orthogonal

$$A V_k = V_{k+1} H_{k+1}$$

gives V_k as orthogonal basis for $\mathcal{K}_k(A,b)$.

$$V_k^T A V_k$$



Suppose A is symmetric!

$$V_k^T A V_k = V_k^T V_{k+1} \cdot H_{k+1}$$

$$= \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} H_k \\ \beta e_k \end{bmatrix}$$

sq.

$$= \bar{H}_k$$

$$A = A^T \Rightarrow \bar{H}_k = \bar{H}_k^T \Longrightarrow \boxed{\bar{H}_k \text{ is tridiagon}}$$

$$= (V_k^T A V_k)^T = (\bar{H}_k)^T$$

Why is there a degree n poly s.t. $p(A) = A^{-1}$?

Because a matrix satisfies its own characteristic poly!

$$p(\lambda) = \det(A - \lambda I)$$

$= $ poly of degree n

$$p(\lambda) = \prod_{i=1}^{n} (\lambda$$

$$\boxed{p(A) = 0.}$$

Why does $p(A) =$

there exist $q(A)$

$$p(A) = p_0$$

$$A^{-1} p(A) = p_0 A^{-1} + p_1$$

$$-p_0 A^{-1} \quad 0 = \boxed{p_0 A^{-1}} + p_1$$

Eigenvalues of A :

$-\lambda_i)$

$0 \Rightarrow$

s.t. $q(A) = A^{-1}$

$+ p_1 \cancel{A} + \dots + p_{n-1} A^{n}$

$I + \dots p_n A^{n-1}$

$I + \dots p_n A^{n-1}$

$$A^{-1} = \frac{1}{p_0}(p_1 I + \dots + p_n A^{n-1})$$

$p_0 \neq 0$ because :

Neumann :

$$A^{-1} = \underbrace{I + (I-A) + (I-A)^2 + \dots (I-}_{\text{fixed coefficient for}}$$

Cayley - Hamilton

$$A^{-1} = \frac{1}{p_0}(p_1 I + \dots + p_n A^{n-1})$$

$p_0 \dots p_n$ depend on A!

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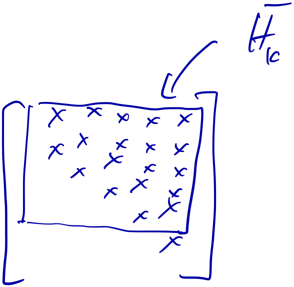
$AJ^{n} + +$

ALL matrices!

$H_k =$



$$H_{k+1} = \begin{bmatrix} \bar{H}_k \\ \beta_{k+1} e_k^T \end{bmatrix}$$