Lecture 20

- generalization of SMW for $f(A)$?

  $f(A) \to f(A + \underline{u}\underline{v}^T)$ ✓
- Backwards Euler? ✓
- Inverse Power Method ✓
- ~~G~~ Factorization Solution?

Excellent Q!

SMW ⇒ if f

then this r

if $f(A) = A^K$

Possible?

Q: $f(A) = A^2$

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given $y = f(A)$

Compute $f(A + \underline{u}\underline{v}$

$f(A) = A^{-1}$

is possible !

is this

$= A^2$

$^T) =$

$(A + \underline{u}\underline{v}^T)^2 =$

$A^2 + \underbrace{A\underline{u}\underline{v}^T}_{} + \underbrace{\underline{u}\underline{v}^T A}_{} + \underbrace{\underline{u}\underline{v}^T\underline{u}\underline{v}^T}_{} =$

$Y + \underline{w}\underline{v}^T + \underline{u}y^T + \gamma\underline{u}\underline{v}^T =$

$Y + (w + \gamma u)\underline{v}^T + \underline{u}y^T$

$\underline{w} = \boxed{A\underline{u}} = \frac{f(x)}{x} \cdot \underline{u}.$

$f(x) = x^2 =$

$f(A) = e^A$?

Backwards Euler

$$\frac{dx}{dt} = A x(t)$$

Setting is a dynamical system.

↳ linear

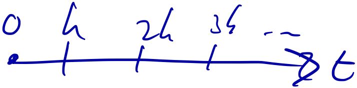
↳ very simple!

Soln: $\underline{x}(t) = \underbrace{e^{At}}\, \underline{x}(0)$ ⊛

$$f(A) = e^{At} \Rightarrow ?$$

→ Euler approx.

↳ discretize time.



given $\underline{x}(0)$ what is $\underline{x}(h)$ for h small?

$$\boxed{\frac{dx}{dt}(0) \approx \frac{x(h) - x(0)}{h}}$$

$$\frac{x(h) - x(0)}{h} = A \underline{x}(0)$$

$$\underbrace{\underline{x}(h) = hAx(0) + x(0)}_{\text{forward Euler!}}$$

$\underline{x}(h)$

$x(2h)$

Ⓐ CS

BW Euler

$$\frac{dx}{dt}(h) \approx \frac{x(h) - x(0)}{h} = A\underline{x}(h)$$

$$\underline{x}(h) - hA\underline{x}(h) = x(0)$$

$$\boxed{(I - hA)\underline{x}(h) = \underline{x}(0)}$$

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$= (I - hA)^{-1} \underline{x}(0).$

$= (I - hA)^{-1} \underline{x}(h).$

$\vdots$

514 !

Inverse Power Method

$$\underline{x}^{(k+1)} = A^{-1} \underline{x}^{(k)}.$$

Power Method converges to the largest magnitude λ-val.

$$\lambda(A^{-1}) = \frac{1}{\lambda(A)}$$

Suppose A is sym, pos. def

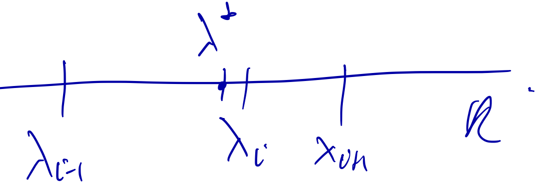
→ Power Meth → $\lambda_{max}(A)$

Inverse Pow Meth → $\dfrac{1}{\lambda_{min}(A)}$.

if λ^* is really close to λ_i, then

$$(A-\lambda^* I)^{-1} \xrightarrow{\text{has eigvals}} \frac{1}{\lambda_i-\lambda^*}$$

will be "big" at λ_i.



Goal:
Run Power method on

$$(A-\lambda^* I)^{-1}.$$

$$\underline{x}^{(k+1)} = (A-\lambda^* I)$$

Compute an LU deco

$$P(A-\lambda^* I) = LU$$

then use, P, L, U to

at each step.

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F = lufact(A - λI) # once
while not done
  Xnext = F \ X
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Q:

$^{-1} x^{(k)}$

mp.)

Equiv.

Solve

Prove

$x \otimes y$ is BW Stable.

$$x \otimes y = xy(1+\epsilon)$$

$$= \underbrace{x(1+\epsilon)} \cdot \underbrace{y}$$

$$= x\sqrt{1+\epsilon} \cdot y\sqrt{1+\epsilon}$$