Lecture 15

Householder
$I - vv^T$ vs.
$I - 2vv^T$
(why 2??) ✓

QR from Variable Elimination? ✓

How to get Eigenvectors w/ the same Eigenvalue?

- When do we use Givens vs. Householder?
- Difference between Cost/Stability of Givens/Householder ✓
- Is Gram-Schmidt ever preferred over QR
  almost never..

- $G_{ij} \underline{v} = \underline{w}$
  $w = \begin{bmatrix} \pm\sqrt{v_i^2 + v_j^2} \\ 0 \end{bmatrix}$
  why the $\underline{w}$?

- Flop count for LU
  $\sum_{i=1}^{n-1} (n-i) + 2(n-i) + 3($
  In LU update

- Recursive Var Elimination
  Schur Complements.

W_c

W_j

$(n-i)^2$

vs. ✓

"Numerically orthogonal"
vs. Orthogonal? ✓

Is QR unique? ✓

Numerical stability of QR. ✓

Very stable! ✓

$I - \underline{v}\underline{v}^T$ (not full rank)

$(I - vv^T)\underline{v}$

$(I - 2\underline{v}\underline{v}^T)\underline{v} =$

$v - 2v = -\underline{v}$

$= \underline{v} - \underline{v} = 0.$

Givens vs. Householder

Slower

Faster (2x)

Target structure / sparsity.

Less structure possible.

"Numerically orthogonal factorization"

(full-rank A)

$A = \underset{\uparrow}{Q}R$ (thin-QR)

R is full-rank

$AR^{-1} = Q$

$\underset{\uparrow}{\boxed{QD}}D^{-1}R$

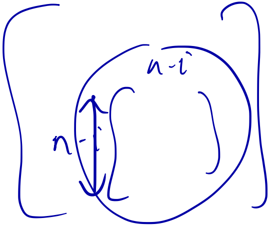
if sign of R is fixed,
then thin QR is unique!

LU update

$$U[i, i{+}1:\text{end}] = A[i, i{+}1:\text{end}]/\alpha \rightarrow$$

(n-i) Compute to find pivot.

$$L[i, i{+}1:\text{end}] = A[i, i{+}1:\text{end}]/\alpha \rightarrow n-i \text{ flops}$$

$$A[i{+}1:\text{end}, i{+}1:\text{end}] = \boxed{A[i{+}1:\text{end}, i{+}1:\text{end}] - \frac{u \cdot v^T}{\alpha}} \rightarrow 2(n-i)^2 \text{ flops}$$