Lecture 13

- Does it ever make sense to solve LS via $A^TAx = A^Tb$? ✓
- If $m < n$, what is QR? ✓
- Is R, when A is square, always invertible? ✓
  A invertible ⟺ R invertible.
- Is spd a common assumption for linear systems? ✓
  spd ⟺ Convexity?
- Range for midterm ✓

- practice questions
  QR?
- QR vs. LU
  $\underline{A}\underline{x} = \underline{b}$?
- QR vs. LU,
  QR better?
  Complexity
- total pivoting
- full QR vs.
- Why (X : y,
  in notes?

ns for ✓

for

faster ?

? ✓ ✓

full QR. ✓

Z_2 ✓

Why is λ-vec still valid in Power of Repeated λ-val? ✓

for PLU factorization is $P = P^{-1}$? ✓

Midterm

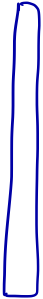
- range of topics:

↳ See online

Comments on $A^TAx = A^Tb$

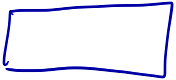
↳ Yes! it does make sense Sometimes!

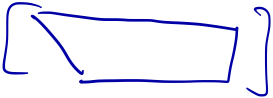
if A is very tall

$A =$ 

$\Rightarrow$ A^TA is tiny!

if $m < n$

$A =$ 

$QR(A) = Q$ 

$\Leftrightarrow$ do $A = [\underset{\text{square}}{B} \quad C]$

then do QR on B.

$B = QR$, then

$$\rightarrow A = Q[R \quad Q^T c]$$

$$A\underline{x} = b$$

$\underline{LU}$

$$PA = LU \Rightarrow$$

$$x = U\text{-solve}(L\text{-solve}(\underset{\uparrow}{P^T}\underline{b}))$$

$$O(n^3 \text{ ops})$$

full vs skinny QR

QR

$A = QR$

$\underline{x} = R\text{-solve}(Q^T \underline{b})$

$O(n^3 \text{ ops})$

(2-5x slower)
w/ no structure.

$A: m \times n \quad m > n$

$$A = \underset{\substack{\uparrow \\ m \times m}}{Q} \; \underset{\substack{\uparrow \\ m \times n}}{R}$$

$$= \underbrace{\begin{bmatrix} \overbrace{Q_1}^{n\text{ cols}} & Q_2 \end{bmatrix}}_{Q} \underbrace{\begin{bmatrix} R_1 \\ 0 \end{bmatrix}}_{R} \begin{matrix} \} \, n\text{-rows} \\ \end{matrix}$$

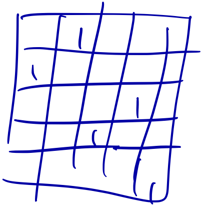
$$A = \underset{\substack{\uparrow \\ m \times n}}{Q} \; \underset{\substack{\uparrow \\ n \times n}}{R}$$

$A = Q_1 R_1$

P is a permutation

Permutations are orthogonal!

$P^{-1} = P^T$.



$\Rightarrow P^T = P$ an

$P^{-1} = P$.

but

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$P\underline{x} = \begin{bmatrix} x_2 \\ x_3 \\ x_1 \end{bmatrix}$$

P

$$P^T = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P^T\underline{x} = \begin{bmatrix} x_3 \\ x_1 \\ x_2 \end{bmatrix}$$

In power method, why are repeated λ-vals (dominant) okay?

Worst case for power

$$\lambda_1 > \lambda_2 > \lambda_3 \ge \dots \ge \lambda_n$$

But $\lambda_n = -\lambda_1$

$$\boxed{\begin{matrix}\lambda_1 = \lambda_2 \\ \underline{v}_1 \quad \underline{v}_2\end{matrix}} > \lambda_3 \ge \lambda_n$$

Power will converge to

if $\lambda_1 = \lambda_2$, then any linear combo of $\underline{v}_1, \underline{v}_2$ is also an λ-vec, so you have a 2d subspace.