Lecture 11

- HW 3a. ✓
- Lec 10 video ☹ ✓
- λ-vals / diagonalization / normality (last class) ✓

- $\lim_{k \to \infty} A^k = 0 \iff$
  Is it true?
- When is LU possi
- How do we get L
  Variable Elimination
- Why chol vs. LU
  spd.?
- LU gives us LOU
  But we need $A^{-1} =$

$\rho(A) < 1$ ✓

ble? ✓

U from ✓

i

for ✓

✓

$U^{-1} D^{-1} L^{-1}$

Which Elements to update w/ Jacobi? ✓

- Summary of Jacobi theory ✓

⇒ Update Error

- Each Eq. to update One Variable? ✓

Exam

- Code? ✓
- Review sheet? ✓
- Practice? ✓

HW

- T vs. T^T?

λ-vals, eti

✓ Symmetric
Hermitian ⇒

Non-normal

Normal: defn

$AA^T =$

$\iff A = U$

Normal ⇒ Diagonalizable   Non-Diag.

= (Diagonalizable ∪ Non-diag) ← Normal.

is

$A^T A$

ΛU^T (orthonorm λ-vecs)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix} \quad A^T = [$$

A is circulant!

$$AA^T = \begin{bmatrix} 14 & 11 & 11 \\ 11 & 14 & 11 \\ 11 & 11 & 14 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 14 & 11 & 11 \\ 11 & 14 & 11 \\ 11 & 11 & 14 \end{bmatrix}$$

$$\downarrow$$

$$\begin{matrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{matrix}\Bigg)$$

Diag.

$$\begin{bmatrix} 3 & 0 & 3 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 3 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3/4 \\ 0 \\ 1/4 \end{bmatrix} = \begin{bmatrix} 9/4 + 3/4 \\ 0 \\ 3/4 + 1/4 \end{bmatrix}$$

Non-normal ☹

But, Diag.

$\lambda = 0 \quad 2 \quad 4$ (or flip)

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 3/4 \\ 0 \\ 1/4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

Non-normal

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Eg: Any upper-triangular Matrix of repeated vals on the diagonal.

$$A = \begin{bmatrix} 0.9 & 0 \\ 0 & 1.1 \end{bmatrix}$$

$$A^k \Rightarrow 0 \text{?} \quad \text{No!}$$

But $A^k \begin{bmatrix} \alpha \\ 0 \end{bmatrix} \rightarrow$

for any α.

⊕ w/ pivoting.

- When is LU possible?

if A is full-rank.

LU from Varah

↳ check notes on Example.

O

Eliminin.

for

We don't need A^{-1} !!

at all!

If we do LDU,

$A = LDU$

then to solve $A\underline{x} = \underline{b}$

we solve $LDU\underline{x} = \underline{b}$

L

when D

D

whe U

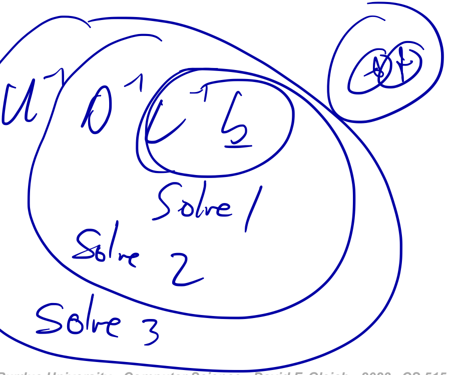
$x =$

$y = \underline{b}$ (Solve 1)

$\boxed{U\underline{x} = y} \Rightarrow$

$\underline{z} = y$ Solve 2

$\underline{x} = \underline{z}$ (Solve 3)



Jacobi + Error

pre-permute $A\underline{x} = \underline{b}$

$A = (D - N)$ then

$$(D - N)\underline{x} = \underline{b} \Leftrightarrow$$

$$D\underline{x} = \underline{b} + N\underline{x}$$

$\Rightarrow$ iteration

$$D\underline{x}^{(k+1)} = \underline{b} + N\underline{x}^{(k)} \Rightarrow$$

$$\boxed{\underline{x}^{(k+1)} = D^{-1}(\underline{b} + N\underline{x}^{(k)})}$$

Error.

let x^* be the true soln.

Then

$$Ax^* = b$$

$$(D - N)x^* = b \Rightarrow$$

$$Dx^* = b + Nx^*$$

$$Dx^{(k+1)} = b + Nx^{(k)}$$

$$D(x^* - x^{(k+1)}) = 0 + N(x^* - x^{(k)})$$

$$x^* - x^{(k+1)} = D^{-1}N(x^* -$$

By same argum

$$x^* - x^{(k)} = D^{-1}N(x$$

$$\vdots$$

$$x^* - x^{(k+1)} = (D^{-1}N$$

$$x^{(k)})$$

$$-$$

$$^{t} - x^{(k-1)})$$

$$)(D^{-1}N(x^{t} - x^{(k-1)}))$$