Lecture 10

- What is the fastest way of doing Gaussian Elimination? $O(n^3)$? ⑩ ✓
- Example of non-sym matrix that doesn't have a spectral decomp?
- Non-sym matrix that has a spectral decomp?
- What is p_k in $x_k = p_k A^k x_0$ ✓

- What is the matrix decomp for continuous op Δ? (not discrete?)
- Prove the convergence of power method
- $A^k \to$ converg for power even though $\rho(A)$
- Random x_0 in GS/
- Doesn't $(D_M - L_M)$ break dependence?

✓

✓

d!

✓

>1?

Jacobi? ✓

-1

✓

o make it parallel?

Gauss-Seidel

↳ Seq. Var. updates.

$$x_i^{(k+1)} = \frac{1}{A_{ii}}\left(b - \sum_{j=1}^{i-1} A_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} A_{ij} x_j\right.$$

$$\vdots$$

$$(D+L)\underline{x}^{(k+1)} = \underline{b} - U\underline{x}^{(k)}$$

$$x^{(k+1)} = (D+L)^{-1}(b - Ux^{(k)}) \quad -$$

$^{(k)})$

$\to (D+L)^{-1}$ is going to be dense (usually).

$\Rightarrow$ NO Efficiency

Power method

Convergence

$$x_{k+1} = \frac{A x^{(k)}}{\|A x^{(k)}\|}$$

$$x_{k+1} = \frac{1 \cdot A^k x^{(0)}}{\|A x^{(k)}\| \cdot \|A x^{(k-1)}\|}$$

$$= \rho_k A^k x^{(0)}$$

$\ldots \|Ax^{(0)}\|$

Let $\|\cdot\|$ be any
Sub-mult. Matrix norm.

then if $\underline{x}$ is an Eigenvec

$$\|Ax\| = \|\lambda x\| = |\lambda|\cdot\|x\| \le \|A\|\cdot\|x\|$$

$$\Rightarrow |\lambda| \le \|A\|.$$

$$\underline{x}^{(k+1)} = \rho_k A^k \underline{x}^{(0)}$$

$$= \rho_k (V \Lambda V^T)^k \cancel{x} x^{(0)}$$

$$\underline{x}^{(k+1)} = \rho_k V \Lambda^k V^T \underline{x}^{(0)}$$

$$\vdots$$

$$V^T \underline{x}^{(k+1)} = \rho_k \Lambda^k V^T \underline{x}^{(0)}$$

$$y^{(k+1)} = \rho_k \Lambda^k y^{(0)}$$

$$y^{(k+1)} = \rho_k \cdot \lambda_1^{(k)} \cdot \frac{\Lambda^k}{\lambda_1^{(k)}} y^{(0)}$$

when $\lambda_1 = \rho(A)$

$$\begin{bmatrix} \pm 1 & & \\ & \frac{\lambda_2^k}{\lambda_1^k} & \\ & & \ddots \end{bmatrix}^k$$

$$\left| \frac{\lambda_i^k}{\lambda_1^k} \right| <$$

because $\lambda_1 \neq$

and this is unique

$$\frac{\lambda_n^{(k)}}{\lambda_1^{k}} \Bigg] \rightarrow \begin{bmatrix} \pm 1 & & \\ & 0 & \\ & & \ddots & \\ & & & 0 \end{bmatrix}$$

$$x_k^{(k)} \rightarrow v$$

$$= v_1$$

dom.

$$y^{(k)} \rightarrow \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$\rho(A)$

$\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

the

λ-vec.

if $\lambda_1 = \lambda_2$, then

$$y^{(k)} \to \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$\lambda_1 = -\lambda_2$$

$$y^{(k)} \to ?$$

GE

(if

GE

faster than $O(n^3)$

no permutation $\overset{u}{\frown}$)

in $O(n^{\omega})$ time.)

∂ is the

$$\frac{\partial u^2}{\partial x^2}$$

$$\frac{\partial u^2}{\partial x^2}$$

2nd deriv ∂

$+ \frac{\partial u^2}{\partial y^2}$ in 2d.

$+ \frac{\partial u^2}{\partial y^2} + \frac{\partial u^2}{\partial z^2}$ in 3d.

$A = A^T$

$(D^{-1}A)^T (D^{-1}A) =$

$A^T D^{-1} D^{-1} A$