Homework 8

Please answer the following questions in complete sentences in a typed, clearly prepared manuscript and submit the solution by the due date on Blackboard (Friday, November 13th, 2015, 5pm, with all the usual extensions...)

Problem 0: Homework checklist

- Please identify anyone, whether or not they are in the class, with whom you discussed your homework. This problem is worth 1 point, but on a multiplicative scale.
- Make sure you have included your source-code and prepared your solution according to the most recent Piazza note on homework submissions.

Problem 1: Gershgorin disks

Theorem 7.2.1 in Golub and van Loan states the Gershgorin circle theorem. This theorem provides an easy check to find regions containing the eigenvalues of a matrix.

1. Read this theorem. Explain it in your own words.
2. Write a function to plot the Gershgorin circles in Matlab.
3. Plot the Gershgorin circles for the matrix:

   \[
   n = 10;
   on = ones(n,1);
   A = spdiags([-2*on 4*on -2*on],-1:1,n,n);
   \]

Problem 2: More eigenvalues and convergence theory

You need to solve one of the following two problems:

1. (GvL 11.2.1) Show that the Jacobi iteration converges for 2-by-2 symmetric, positive, definite systems.
2. (GvL 11.2.2) Show that if \( A = M - N \) is singular, then we can never have \( \rho(M^{-1}N) < 1 \) even if \( M \) is non-singular.