Lecture 21

- This video overview
  - ↳ Secant Method
  - ↳ Newton's Method

  > Univariate Roots of $f: \mathbb{R} \to \mathbb{R}$

- Secant Analysis - Another video
- Multivariate $\to$ $f: \mathbb{R}^n \to \mathbb{R}^m$ or $\mathbb{R}^m \to \mathbb{R}^n$ or $\mathbb{R}^n \to \mathbb{R}^n$
  - ↳ "Bisection"
  - fixed point
  - Newton's Method.

Secant Method

↳ Like False Position but you use the last 2 points

Given x_0, x_1, form a line

$$f(x) \simeq f(x_1) + (x - x_1) f[x_1, x_0]$$

$\to$ Solve approx. for zero

$$x = \frac{-f(x_1) + x_1 f[x_1; x_0]}{}$$

$$f[x_1, x_0]$$

$$= x_1 - \frac{f(x_1)}{f[x_1, x_0]}$$

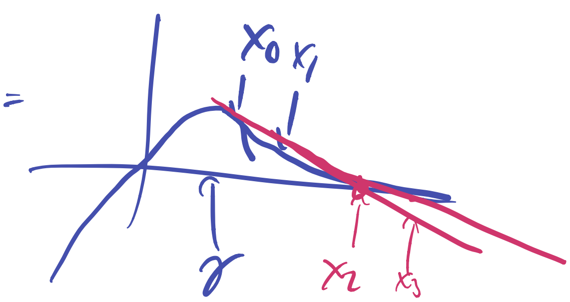
Generate Iteration:

$$x_{k+1} = x_k - \frac{f(x_k)}{f[x_k, x_{k-1}]} \qquad f[x_k, x_{k-1}] = \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}$$

Secant Iteration!

May diverge

Eg $f(x) = xe^{-x}$ =



If $x_0, x_1 > r$, then
$x_k \to \infty$

Newton's Method

2 ways to derive

↳ from Secant, $\lim_{x_1 \to x_0}$ =>

$$x_2 = x_1 - \frac{f(x_1)}{f[x_1, x_0]}$$

$\lim x_1 \to x_0$

$$\lim_{x_1 \to x_0} f[x_1, x_0] = f'(x_0)$$

$$x_2 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Alternative: Use Taylor approx.

$$f(x_0 + h) \approx f(x_0) + h\, f'(x_0)$$

$\Rightarrow$ solve for zero $\Rightarrow$

$$h = \frac{-f(x_0)}{f'(x_0)} \quad \to \quad \text{same iteration}$$

Newton: given x_0,

Compute $x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)}$

Why? Look at Routines like compute $\frac{1}{x}$.
w/o division

Look @ roots of $f(x) = \frac{1}{x} - c$

and Newton iterate is

Simplifies to! division

$$x_{k+1} = 2x_k - cx_k^2 = \boxed{x_k - \frac{f(x_k)}{f'(x_k)}}$$

$\Rightarrow$ No division