- My handwriting is NOT terrific!
- Thank you for "understanding"

- Sehr!

$\mathbb{R}$eal #'s

$\mathbb{R}$ contain infinite information.

- Computers / machines have finite capacity

$$\underset{\text{reals}}{\mathbb{R}} \longrightarrow \underset{\text{machine \#'s}}{\mathbb{M}}$$

Cause Error!

We need to

- deeply
- precisely
- quantitatively
- analytically
- ...

understand how this finite representing impacts sequences of ops on $\mathbb{R}$ vs. $\mathbb{M}$.

Another prob

- We may not even be able to represent our input!
- Quick case study with π.

3

π

3.14159265358979…

$\sin(\pi) = 0$

π_M - Machine rep. of π.

2-Floats

π_M - 3.1415927

$\sin(\pi_M) = ?$

≈ 0 but not quite!

Suppose we take

$\sin(10^9 x)$

where $x = \pi$

vs $x = \pi_M$.

$\Rightarrow$ very differen answers!

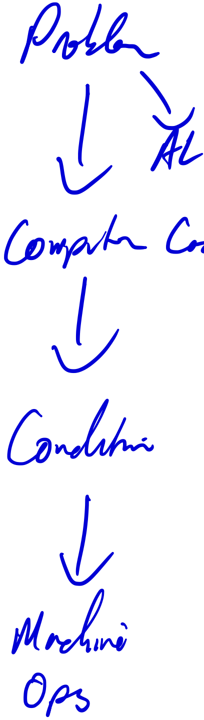
$\sin(10^9 \pi) = 0$

$\sin(10^9 \pi_M) =$

0.64

Where Errors arise

- Machine: approx to ℝ → roundoff Error!
- The impact of an input Error on the Solution. → Conditioning
- Numerical method give a 3rd Error: the finite approx. of a continuous problem w/ an Algorithm → Discretization Error.



G

le

$$\int_a^b p(x)\,dx$$

$\downarrow$

$$\text{ALG}(a, b, p, M)$$

function / approx / tol

→ Math ALG!

$$\sum_{\substack{y = a..b \text{ in}\\ M \text{ equal steps}}} p(y)\,\frac{b-a}{M-1}$$

If $\text{ALG}(a, b, p, m)$ is well-cond.

$$\text{Alg}(a_M, b_M, p_M, M) \rightarrow$$

Machine ops for yet more error!

$$= \text{Alg}(a, b, p, M)$$

$$\approx \int_a^b p(x)\,dx$$

ℝeal Numbers

- Cauchy Sequences
- Dedekind Cuts
- See Wikipedia for lovely notes!

Rational Numbers ℚ

(Ratios of Integers / Natural #'s)

We use a slig
informal defn. or
"Slang."

We use an
infinite binary seq.
to represent ℝ.

$$x \in \mathbb{R} \iff$$

$$x = \pm \left[b_n 2^n + b_{n-1} \right.$$

$$b_{n-2} 2^{n-2} + \dots$$

$$b_0 2^0 + b_{-1} 2^{-1} +$$

lity

$\dots_{-1} 2^{n-1} +$

$+$

$b_{-2} 2^{-2} + \dots$

- Not always unique

Notation:

$(b_n b_{n-1} \dots b_0 . b_{-1} b_{-2} \dots)_2$

Eg.

$(\overset{2^4}{1}0\overset{2}{0}\overset{2^1}{1}\overset{2^0}{1}.0\overset{1/2}{1}\overset{1/4}{0}0\dots)_2 =$

$16 + 3 + .25 =$

$(19.25)_{10}$

Suppose we ...

finite binary

Eg.

$(b_n \dots b_0) b$ ...

is 0 for ...

$-\infty$. Then ...

Show there is a ...

decimal repres...

for that # !

have a

#

$-1 \ldots b_{-k+1}$

$-k$ to

finite

entation

Soln

Each term 2^{-1} to 2^{-k} has a finite decimal rep.

Finite Sum of finite = finite!

→ Finite product of finite is finite!

$(2^{-1}) = (0.5)_{10}$ ↓ grade school mult!

Machine #s!

Fixed point

Map integer to

i/k

Eg. $1925 \xrightarrow{k=100} 19.25$

Subset of $\mathbb{Z}$ → Subset of $\mathbb{Q}$

- Used in Banking

Floating pt #s!

like 1.01×10^{5}

or $\underbrace{\boxed{1.01}}_{\text{fraction part}} \times \underbrace{10^{\boxed{-5}}}_{\text{Exponent}}$

10 order of magnitude!

t - # of bits for fraction

S - # of bits for Exponent.

$x \in \mathbb{R}(t,s)$ → IEEE Float pt!